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MiniZinc is a free and open-source constraint modeling language. You can use MiniZinc to model constraint satisfaction and optimization problems in a high-level, solver-independent way, taking advantage of a large library of pre-defined constraints.

This handbook consists of four parts: Section 1 covers installation and basic steps; Section 2 is a tutorial-style introduction into modelling with MiniZinc; Section 3 is a user manual for the individual tools in the MiniZinc tool chain; and Section 4 is a reference to the language.

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Part 1

Overview
MiniZinc is a language for specifying constrained optimization and decision problems over integers and real numbers. A MiniZinc model does not dictate how to solve the problem - the MiniZinc compiler can translate it into different forms suitable for a wide range of solvers, such as Constraint Programming (CP), Mixed Integer Linear Programming (MIP) or Boolean Satisfiability (SAT) solvers.

The MiniZinc language lets users write models in a way that is close to a mathematical formulation of the problem, using familiar notation such as existential and universal quantifiers, sums over index sets, or logical connectives like implications and if-then-else statements. Furthermore, MiniZinc supports defining predicates and functions that let users structure their models (similar to procedures and functions in regular programming languages).

MiniZinc models are usually parametric, i.e., they describe a whole class of problems rather than an individual problem instance. That way, a model of, say, a vehicle routing problem could be reused to generate weekly plans, by instantiating it with the updated customer demands for the upcoming week.

MiniZinc is designed to interface easily to different backend solvers. It does this by transforming an input MiniZinc model and data file into a FlatZinc model. FlatZinc models consist of variable declarations and constraint definitions as well as a definition of the objective function if the problem is an optimization problem. The translation from MiniZinc to FlatZinc makes use of a library of function and predicate definitions for the particular target solver, which allows the MiniZinc compiler to produce specialised FlatZinc that only contains the types of variables and constraints that are supported by the target. In particular, MiniZinc allows the specification of
global constraints by decomposition. Furthermore, annotations of the model let the user fine tune the behaviour of the solver, independent of the declarative meaning of the model.

1.1.1 Structure

This documentation consists of four parts. The first part (page 5) includes this introduction and then describes how to download and install MiniZinc and how to make your first steps using the MiniZinc IDE and the command line tools. The second part (page 23) is a tutorial introduction to modelling with MiniZinc, from basic syntax and simple modelling techniques to more advanced topics. It also explains how MiniZinc is compiled to FlatZinc. The third part (page 147) is a user manual for the tools that make up the MiniZinc tool chain. Finally, The fourth part (page 199) contains the reference documentation for MiniZinc, including a definition of the MiniZinc language, documentation on how to interface a solver to FlatZinc, and an annotated list of all predicates and functions in the MiniZinc standard library.

1.1.2 How to Read This

If you are new to MiniZinc, follow the installation instructions and the introduction to the IDE and then work your way through the tutorial. Most example code can be downloaded, but it is sometimes more useful to type it in yourself to get the language into your muscle memory! If you need help, visit the MiniZinc web site at http://www.minizinc.org where you find a discussion forum.

Some of the code examples are shown in boxes like the one below. If a code box has a heading, it usually lists the name of a file that can be downloaded from http://minizinc.org/doc-latest/en/downloads/index.html.
Listing 1.1.1: A code example (dummy.mzn)

```plaintext
% Just an example
var int: x;
solve satisfy;
```

Throughout the documentation, some concepts are defined slightly more formally in special sections like this one.

**More details**

These sections can be skipped over if you just want to work through the tutorial for the first time, but they contain important information for any serious MiniZinc user!

Finally, if you find a mistake in this documentation, please report it through our GitHub issue tracker at https://github.com/minizinc/minizinc-doc/issues.
A complete installation of the MiniZinc system comprises the MiniZinc compiler tool chain, one or more solvers, and (optionally) the MiniZinc IDE.

The easiest way to get a full, working MiniZinc system is to use the bundled binary packages, available from http://www.minizinc.org/software.html.

The bundled binary packages contain the compiler and IDE, as well as the following solvers: Gecode, Chuffed, COIN-OR CBC, and a interfaces to the Gurobi and CPLEX solvers (the Gurobi and CPLEX libraries are not included).

Alternatively, it is possible to compile all components from source code, as described in ch-installation_detailed.

### 1.2.1 Microsoft Windows

To install the bundled binary packages, simply download the installer, double-click to execute it, and follow the prompts. **Note:** 32 bit Windows installations are no longer supported in the bundled binary packages.

After installation is complete, you can find the MiniZinc IDE installed as a Windows application. The file extensions .mzn, .dzn, and .fzn are linked to the IDE, so double-clicking any MiniZinc file should open it in the IDE.
If you want to use MiniZinc from a command prompt, you need to add the installation directory to the PATH environment variable. In a Windows command prompt you could use the following command:

C:\>setx PATH "%PATH%;C:\Program Files\MiniZinc 2.4.3 (bundled)\"

### 1.2.2 Linux

The MiniZinc bundled binary distribution for Linux is provided using three different distribution methods: *Snap*, *AppImage*, and an *archive* containing all necessary files. The capabilities of each package is the same, but it depends on your Linux distribution which method is the easiest to use. **Note:** linux bundles are compiled to be compatible with many Linux distributions using a x86_64 architecture, but some dependencies on low level libraries cannot be avoided. Make use of the MiniZinc issue trackers if you encounter any problems.

#### 1.2.2.1 Snap

The MiniZinc Snap is the easiest way to install the MiniZinc bundle if Snap ([https://snapcraft.io/](https://snapcraft.io/)) is available on your system. The installation will provide the minizinc command within terminal environments and will add an application entry for the MiniZincIDE. The IDE can be started from the command line using minizinc.ide. The following command will commence the installation the MiniZinc snap:

```
$ snap install minizinc --classic
```

**Note:** the MiniZinc snap requires the *classic* permission model to access solvers that are installed somewhere else on your system.

An advantage of using the MiniZinc snap is that it will be automatically updated whenever a new version of the snap is available. Using snap channels it is also possible to automatically follow the developmental versions of MiniZinc. More information about the MiniZinc snap can be found in the Snap Store.

#### 1.2.2.2 AppImage

The MiniZinc AppImage provides a way of installing the MiniZinc bundles without any added requirements. AppImages ([https://appimage.org/](https://appimage.org/)) provide self-contained versions of applications with all their required dependencies. To use the MiniZinc AppImage you simply download the AppImage, make it executable, and run:
AppImages have no standard integration with the desktop environment. A script has been added to the MiniZinc AppImage to integrate the application into both the terminal and the desktop environment. The one requirement for this script is that the AppImage CANNOT be moved after the install script has been run. A suggestion would be to store the AppImages in ~/Applications/ or /opt/Applications/. The following commands move the AppImage to the second location and install minizinc and MiniZincIDE to /usr/local/bin and will add a desktop entry and its corresponding icon to $HOME/.local/share/{applications,icons}:

```bash
$ mv ./MiniZincIDE-2.4.3-x86_64.AppImage /opt/Applications/
$ /opt/Applications/MiniZincIDE-2.4.3-x86_64.AppImage install
```

**Note:** to change the installation locations can be changed using the environmental variables `$BIN_LOCATION` and `$DESKTOP_LOCATION`. The default behaviour can be achieved using the following command:

```bash
$ BIN_LOCATION=/usr/local/bin DESKTOP_LOCATION=$HOME/.local/share ./MiniZincIDE-2.4.3-x86_64.AppImage install
```

### 1.2.2.3 Archive

The MiniZinc archive bundle is straightforward, but doesn’t offer any automatic integrations with the desktop environment. After downloading, uncompress the archive, for example in your home directory or any other location where you want to install it:

```bash
$ tar xf MiniZincIDE-2.4.3-bundle-linux-x86_64.tgz
```

This will unpack MiniZinc into a directory that is called the same as the archive file (without the `.tgz`). You can start the MiniZinc IDE using the MiniZincIDE.sh shell script, which sets up a number of paths and environment variables.

In order to use the command line tools, after unpacking, make the following changes to some environment variables (assuming a bash-like shell):

```bash
$ export PATH=MiniZincIDE-2.4.3-bundle-linux-x86_64/bin:$PATH
$ export LD_LIBRARY_PATH=MiniZincIDE-2.4.3-bundle-linux-x86_64/lib:$LD_LIBRARY_PATH:$PATH
$ export QT_PLUGIN_PATH=MiniZincIDE-2.4.3-bundle-linux-x86_64/plugins:$QT_PLUGIN_PATH:$PATH
```
1.2.3 Apple macOS

The macOS bundled binary distribution works with any version of OS X starting from 10.9. After
downloading the disk image (.dmg) file, double click it if it doesn’t open automatically. You will
see an icon for the MiniZinc IDE that you can drag into your Applications folder (or anywhere
else you want to install MiniZinc).

In order to use the MiniZinc tools from a terminal, you need to add the path to the MiniZinc
installation to the PATH environment variable. If you installed the MiniZinc IDE in the standard
Applications folder, the following command will add the correct path:

```
$ export PATH=/Applications/MiniZincIDE.app/Contents/Resources:$PATH
```

1.2.4 Adding Third-party Solvers

Third party solvers for MiniZinc typically consist of two parts: a solver executable, and a solver-
specific MiniZinc library. MiniZinc must be aware of the location of both the executable and
the library in order to compile and run a model with that solver. Each solver therefore needs to
provide a configuration file in a location where the MiniZinc toolchain can find it.

The easiest way to add a solver to the MiniZinc system is via the MiniZinc IDE. This is explained
in Section 3.2.5.2. You can also add configuration files manually, as explained in Section 4.3.5.
We recommend using the bundled binary distribution of MiniZinc introduced in Section 1.2. It contains the MiniZinc IDE, the MiniZinc compiler, and several pre-configured solvers so you can get started straight away.

This section introduces the MiniZinc IDE and the command line tool minizinc using some very basic examples. This should be enough to get you started with the MiniZinc tutorial in Section 2 (in fact, you only need to be able to use one of the two, for instance just stick to the IDE if you are not comfortable with command line tools, or just use the minizinc command if you suffer from fear of mice).

### 1.3.1 The MiniZinc IDE

The MiniZinc IDE provides a simple interface to most of MiniZinc’s functionality. It lets you edit model and data files, solve them with any of the solvers supported by MiniZinc, run debugging and profiling tools, and submit solutions to online courses (such as the MiniZinc Coursera courses).

When you open the MiniZinc IDE for the first time, it will ask you whether you want to be notified when an update is available. If you installed the IDE from sources, it may next ask you to locate your installation of the MiniZinc compiler. Please refer to Section 3.2.5 for more details on this.

The IDE will then greet you with the *MiniZinc Playground*, a window that will look like this:
You can start writing your first MiniZinc model! Let’s try something very simple:

```
1 % Use this editor as a MiniZinc scratch book
2
3 var 1..3: x;
4 var 1..3: y;
5 constraint x+y > 2;
6 solve satisfy;
```

In order to solve the model, you click on the Run button in the toolbar, or use the keyboard shortcut Ctrl+R (or command+R on macOS):
As you can see, an output window pops up that displays a solution to the problem you entered. Let us now try a model that requires some additional data.

When you run this model, the IDE will ask you to enter a value for the parameter $n$:  

1.3.1. The MiniZinc IDE
After entering, for example, the value 4 and clicking Ok, the solver will execute the model for \( n=4 \):

Alternatively, data can also come from a file. Let’s create a new file with the data and save it as data.dzn:

```
% Use this editor as a MiniZinc scratch book
int: n;
var 1..n: x;
var 1..n: y;
constraint x+y > n;
solve satisfy;
```

```
Running untitled_model.mzn, additional arguments n=4;
x = 4;
y = 1;
----------
Finished in 100msec
```
When you now go back to the Playground tab and click Run, the IDE will give you the option to select a data file:

Click on the data.dzn entry, then on Ok, and the model will be run with the given data file:
Of course you can save your model to a file, and load it from a file, and the editor supports the usual functionality.

If you want to know more about the MiniZinc IDE, continue reading from Section 3.2.

1.3.2 The MiniZinc command line tool

The MiniZinc command line tool, minizinc, combines the functionality of the MiniZinc compiler, different solver interfaces, and the MiniZinc output processor. After installing MiniZinc from the bundled binary distribution, you may have to set up your PATH in order to use the command line tool (see Section 1.2).

Let’s assume we have a file model.mzn with the following contents:

```
var 1..3: x;
var 1..3: y;
constraint x+y > 3;
solve satisfy;
```

You can simply invoke minizinc on that file to solve the model and produce some output:
$ minizinc model.mzn
x = 3;
y = 1;
--------
$

If you have a model that requires a data file (like the one we used in the IDE example above), you pass both files to minizinc:

$ minizinc model.mzn data.dzn
x = 5;
y = 1;
--------
$

The minizinc tool supports numerous command line options. One of the most useful options is -a, which switches between one solution mode and all solutions mode. For example, for the first model above, it would result in the following output:

$ minizinc -a model.mzn
x = 3;
y = 1;
--------
x = 2;
y = 2;
--------
x = 3;
y = 2;
--------
x = 1;
y = 3;
--------
x = 2;
y = 3;
--------
x = 3;
y = 3;
--------
To learn more about the minizinc command, explore the output of minizinc --help or continue reading in Section 3.1.
Part 2

Minizinc 指南
在此节中，我们利用两个简单的例子来介绍一个 MiniZinc 模型的基本结构。

### 2.1.1 第一个实例

![图 2.1.1：澳大利亚各州](image-url)

Fig. 2.1.1: 澳大利亚各州
作为我们的第一个例子，假设我们要去给 Fig. 2.1.1 中的澳大利亚地图涂色。它包含了七个不同的州和地区，而每一块都要被涂一个颜色来保证相邻的区域有不同的颜色。

Listing 2.1.1: 一个用来给澳大利亚的州和地区涂色的 MiniZinc 模型 aust.mzn

```mini
% 用 nc 个颜色来涂澳大利亚
int: nc = 3;

var 1..nc: wa;  var 1..nc: nt;  var 1..nc: sa;  var 1..nc: q;
var 1..nc: nsw; var 1..nc: v;  var 1..nc: t;

constraint wa != nt;
constraint wa != sa;
constraint nt != sa;
constraint nt != q;
constraint sa != q;
constraint sa != nsw;
constraint sa != v;
constraint q != nsw;
constraint nsw != v;
solve satisfy;

output ["wa="(wa)", nt="(nt)", t="(t)", sa="(sa)",
         "q="(q)", nsw="(nsw)", v="(v)
         "t=",, show(t), "\n"];
```

我们可以很容易的用 MiniZinc 给此问题建模。此模型在 Listing 2.1.1 中给出。

模型中的第一行是注释。注释开始于 % 来表明此行剩下的部分是注释。MiniZinc 同时也含有 C 语言风格的由 /* 开始和 */ 结束的块注释。

模型中的下一部分声明了模型中的变量。此行

```mini
int: nc = 3;
```

定义了一个问题的 参数来代表可用的颜色个数。在很多编程语言中，参数和变量是类似的。它们必须被声明并且指定一个类型 type。在此例中，类型是 int。通过 赋值，它们被设了值。MiniZinc 允许变量声明时被赋值（就像上面那一行）或者单独给出一个赋值语句。因此下面的表示跟上面的只一行表示是相等的

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int: nc;
nc = 3;

和很多编程语言中的变量不一样的是，这里的参数只可以被赋唯一 的值。如果一个参数出现在了多于一个的赋值中，就会出现错误。

基本的 参数类型包括 整型 (int)、浮点型 (float)、布尔型 (bool) 以及 字符串型 (string)。同时也支持数组和集合。

MiniZinc 模型同时也可能包含另一种类型的变量 决策变量。决策变量是数学的或者逻辑的变量。和一般的编程语言中的参数和变量不同，建模者不需要给决策变量一个值。而是在开始时，一个决策变量的值是不知道的。只有当 MiniZinc 模型被执行的时候，求解系统才来决定决策变量是否可以被赋值从而满足模型中的约束。若满足，则被赋值。

在我们的模型例子中，我们给每一个区域一个 决策变量 wa, nt, sa, q, nsw, v 和 t。它们代表了 会被用来填充区域的（未知）颜色。

对于每一个决策变量，我们需要给出变量可能的取值集合。这个被称为变量的 定义域。定义域部分可以在 变量声明的时候同时给出，这时决策变量的 类型就会从定义域中的数值的类型推断出。

MiniZinc 中的决策变量的类型可以为布尔型，整型，浮点型或者集合。同时也可以是元素为决策变量的数组。在我们的 MiniZinc 模型例子中，我们使用整型去给不用的颜色建模。通过使用 var 声明，我们的每一个决策变量被声明为定义域为一个整数类型的范围表示 1..nc，来表明集合 {1,2,...,nc}。所有数值的类型为整型，所以模型中的所有的变量是整型决策变量。

### 标识符

用来命名参数和变量的标识符是一列由大小写字母，数字以及下划线 _ 字符组成的字符串。它们必须开始于一个字母字符。因此 myName_2 是一个有效的标识符。MiniZinc（和 Zinc）的关键字 不允许被用为标识符名字。它们在 Identifiers (page 204) 中被列出。所有的 MiniZinc 操作符 都不能被用做标识符名字。它们在 Operators (page 220) 中被列出。

MiniZinc 仔细地区别了以下两种模型变量：参数和决策变量。利用决策变量创建的表达式类型比利用参数可以创建的表达式类型更局限。但是，在任何可以用决策变量的地方，同类型的参数变量也可以被应用。
整型变量声明
一个整型参数变量可以被声明为以下两种方式:

```plaintext
int : <变量名>
<l> .. <u> : <变量名>
```

<l> 和 <u> 是固定的整型表达式。
一个整型决策变量被声明为以下两种方式:

```plaintext
var int : <变量名>
var <l>..<u> : <变量名>
```

<l> 和 <u> 是固定的整型表达式。

参数和决策变量形式上的区别在于对变量的实例化。变量的实例化和类型的结合被叫为类型-实例化。既然你已经开始使用 MiniZinc，毫无疑问的你会看到很多类型-实例化的错误例子。

模型的下一部分是约束。它们详细说明了决策变量想要组成一个模型的有效解必须要满足的布尔型表达式。在这个例子中我们有一些决策变量之间的不等式。它们规定如果两个区域是相邻的，则它们必须有不同的颜色。

关系操作符
MiniZinc 提供了以下关系操作符 关系操作符:
相同 (= or ==), 不等 (!=), 小于 (<), 大于 (>), 小于等于 (<=), and 大于等于 (>=).

模型中的下一行:

```plaintext
solve satisfy;
```

表明了它是什么类型的问题。在这个例子中，它是一个满足问题：我们希望给决策变量找到一个值使得约束被满足，但具体是哪一个值却没有所谓。

模型的最后一个部分是 solve 语句。它告诉 MiniZinc 当模型被运行并且找到一个解的时候，要输出什么。
输出和字符串

一个输出语句跟着一个字符。它们通常或者是在双引号之间的字符串或者字符串常量并且对特殊字符用类似C语言的语法。例如对于字符串常量。

数字的show有多种形式的表示：show_int(n,X)在至少n个字符里输出整型X的值，若n>0则右对齐，否则则左对齐；show_float(n,d,X)在至少n个字符里输出浮点型X的值，若n>0则右对齐，否则则左对齐，并且小数点后有d个字符。

字符串常量必须在同一行中。长的字符串常量可以利用字符串连接符""来分成几行。例如，字符串常量

"Invalid datafile: Amount of flour is non-negative"

为字符串常量表达式

"Invalid datafile: " ++
"Amount of flour is non-negative"

是相等的。

MiniZinc支持内插字符串内插字符串。表达式可以直接插入字符串常量中。"\(e\)"形式的字符串会被替换成show(e)。例如，"t=" ++ show(t) +"\n"一样的字符串。

一个模型可以包含多个输出语句。在这种情况下，所有输出会根据它们在模型中出现的顺序连接。

我们可以通过点击MiniZinc IDE中的Run按钮，或者输入

```
$ minizinc --solver Gecode aust.mzn
```

来评估我们的模型。其中aust.mzn是包含我们的MiniZinc模型的文件名字。我们必须使用文件扩展名.mzn来表明一个MiniZinc模型。带有--solver Gecode选项的命令minizinc使用Gecode有限域求解器去评估我们的模型。如果你使用的是MiniZinc二进制发布，这个求解器实际上是预设的，所以你也可以运行minizinc aust.mzn。

当我们运行上面的命令后，我们得到如下的结果：

```
wa=2 nt=3 sa=1
q=2 nsw=3 v=2
t=1
----------
```

2.1.1. 第一个实例
10 个破折号 --------- 这行是自动被 MiniZinc 输出的，用来表明一个解已经被找到。

### 2.1.2 算术优化实例

我们的第二个例子来自于要为了本地的校园游乐会烤一些蛋糕的需求。我们知道如何制作两种蛋糕。footnote{警告: 请不要在家里使用这些配方制作} 一个香蕉蛋糕的制作需要 250 克自发酵的面粉，2 个捣碎的香蕉，75 克糖和 100 克黄油。一个巧克力蛋糕的制作需要 200 克自发酵的面粉，75 克可可粉，150 克糖和 150 克黄油。一个巧克力蛋糕可以卖 \$4.50，一个香蕉蛋糕可以卖 \$4.00。我们一共有 4 千克的自发酵面粉，6 个香蕉，2 千克的糖，500 克的黄油和 500 克的可可粉。问题是对于每一种类型的蛋糕，我们需要烤多少给游乐会来得到最大的利润。一个可能的 MiniZinc 模型在 Listing 2.1.2 中给出。

#### Listing 2.1.2: 决定为了校园游乐会要烤多少香蕉和巧克力蛋糕的模型。(cakes.mzn)

```mini泽恩
% 为校园游乐会做蛋糕

var 0..100: b; % 香蕉蛋糕的个数
var 0..100: c; % 巧克力蛋糕的个数

% 面粉
constraint 250*b + 200*c <= 4000;
% 香蕉
constraint 2*b <= 6;
% 糖
constraint 75*b + 150*c <= 2000;
% 黄油
constraint 100*b + 150*c <= 500;
% 可可粉
constraint 75*c <= 500;

% 最大化我们的利润
solve maximize 400*b + 450*c;

output ["no. of banana cakes = \(b)\n",
       "no. of chocolate cakes = \(c)\n"];
```

第一个新特征是 arithmetic expressions 的使用。
整数算术操作符

MiniZinc 提供了标准的整数算术操作符。加 (+)，减 (-)，乘 (×)，整数除 (div) 和整数模 (mod)，同时也提供了一元操作符 + 和 −。
整数模被定义为输出和被除数 a 一样正负的 a mod b 值。整数除被定义为使得 a = b (a div b) + (a mod b) 值。
MiniZinc 提供了标准的整数函数用来取绝对值 (abs) 和幂函数 (pow)。例如 abs(−4) 和 pow(2, 5) 分别求得数值 4 和 32。
算术常量的语法是相当标准的。整数常量可以是十进制，十六进制或者八进制。例如 0, 5, 123, 0x1b7, 0o777。

例子中的第二个新特征是优化。这行

```
solve maximize 400 * b + 450 * c;
```

指出我们想找一个可以使 solve 语句中的表达式（我们叫做 目标 ）最大化化的解。这个目标可以为任何类型的算术表达式。我们可以把关键字 maximize 换为 minimize 来表明一个最小化问题。

当我们运行上面这个模型时，我们得到以下的结果:

```
no. of banana cakes = 2
no. of chocolate cakes = 2

-----------
```

一旦系统证明了一个解是最优解，这一行 =========== 在最优化问题中会自动被输出。

2.1.3 数据文件和谓词

此模型的一个缺点是如果下一次我们希望解决一个相似的问题，即我们需要为学校烤蛋糕（这是经常发生的），我们需要改变模型中的约束来表明食品间拥有的原料数量。如果我们想重新利用此模型，我们最好使得每个原料的数量作为模型的参数，然后在模型的最上层设置它们的值。

更好的办法是在一个单独的 数据文件 中设置这些参数的值。MiniZinc（就像很多其他的建模语言一样）允许使用数据文件来设置在原始模型中声明的参数的值。通过运行不同的数据文件，使得同样的模型可以很容易地和不同的数据一起使用。

数据文件的文件扩展名必须是 .dzn ，来表明它是一个 MiniZinc 数据文件。一个模型可以被任何多个数据文件运行（但是每个变量/参数在每个文件中只能被赋一个值）
独立于数据的用来决定为了校园游乐会要烤多少香蕉和巧克力蛋糕的模型。
(cakes2.mzn)

```
% 为校园游乐会做蛋糕 (和数据文件一起)

int: flour; % 拥有的面粉克数
int: banana; % 拥有的香蕉个数
int: sugar; % 拥有的糖克数
int: butter; % 拥有的黄油克数
int: cocoa; % 拥有的可可粉克数

constraint assert(flour >= 0, "Invalid datafile: " ++
    "Amount of flour should be non-negative");
constraint assert(banana >= 0, "Invalid datafile: " ++
    "Amount of banana should be non-negative");
constraint assert(sugar >= 0, "Invalid datafile: " ++
    "Amount of sugar should be non-negative");
constraint assert(butter >= 0, "Invalid datafile: " ++
    "Amount of butter should be non-negative");
constraint assert(cocoa >= 0, "Invalid datafile: " ++
    "Amount of cocoa should be non-negative");

var 0..100: b; % 香蕉蛋糕的个数
var 0..100: c; % 巧克力蛋糕的个数

% 面粉
constraint 250*b + 200*c <= flour;
% 香蕉
constraint 2*b <= banana;
% 糖
constraint 75*b + 150*c <= sugar;
% 黄油
constraint 100*b + 150*c <= butter;
% 可可粉
constraint 75*c <= cocoa;

% 最大化我们的利润
solve maximize 400*b + 450*c;
```
输出

```plaintext
output ["no. of banana cakes = \(b\)\n,
    "no. of chocolate cakes = \(c\)\n";]
```

我们的新模型在 Listing 2.1.3 中给出。我们可以用下面的命令来运行

数据文件 pantry.dzn 在 Listing 2.1.4 中给出。我们得到和 cakes.mzn 同样的结果。运行下面的命令

```
$ minizinc cakes2.mzn pantry2.dzn
```

利用另外一个 Listing 2.1.5 中定义的数据集，我们得到如下结果

```
no. of banana cakes = 3
no. of chocolate cakes = 8
-------------
```

如果我们从 cakes.mzn 中去掉输出语句，MiniZinc 会使用默认的输出。这种情况下得到的输出是

```
b = 3;
c = 8;
-------------
```

**默认输出**

一个没有输出语句的 MiniZinc 模型会给每一个决策变量以及它的值一个输出行，除非决策变量已经在声明的时候被赋了一个表达式。注意观察此输出是如何呈现一个正确的数据文件格式的。

**Listing 2.1.4: cakes2.mzn 的数据文件例子 (pantry.dzn)**

```plaintext
flour = 4000;
banana = 6;
sugar = 2000;
butter = 500;
cocoa = 500;
```
通过使用命令行标示 -D string，小的数据文件可以被直接输入而不是必须要创建一个.dzn 文件，其中 string 是数据文件里面的内容。

```bash
$ minizinc cakes2.mzn -D \
   "flour=4000;banana=6;sugar=2000;butter=500;cocoa=500;"
```

会给出和

```bash
$ minizinc cakes2.mzn pantry.dzn
```

一模一样的结果。

数据文件只能包含给模型中的决策变量和参数赋值的语句。

防御性编程建议我们应该检查数据文件中的数值是否合理。在我们的例子中，检查所有原料的份量是否是非负的并且若不正确则产生一个运行错误，这是明智的。MiniZinc 提供了一个内置的布尔型操作符 断言 用来检查参数值。格式是 assert(B,S)。布尔型表达式 B 被检测。若它是假的，运行中断。此时字符串表达式 S 作为错误信息被输出。如果我们想当面粉的份量是负值的时候去检测出并且产生合适的错误信息，我们可以直接加入下面的一行

```minizinc
constraint assert(flour >= 0,"Amount of flour is non-negative");
```

到我们的模型中。注意 断言 表达式是一个布尔型表达式，所以它被看做是一种类型的约束。我们可以加入类似的行来检测其他原料的份量是否是非负值。

### 2.1.4 实数求解

MiniZinc also supports “real number” constraint solving using floating point variables and constraints. Consider a problem of taking out a short loan for one year to be repaid in 4 quarterly
instalments. A model for this is shown in Listing 2.1.6. It uses a simple interest calculation to calculate the balance after each quarter.

通过使用浮点数求解，MiniZinc 也支持“实数”约束求解。考虑一个要在 4 季度分期偿还的一年短期贷款问题。此问题的一个模型在 Listing 2.1.6 中给出。它使用了一个简单的计算每季度结束后所欠款的利息计算方式。

### Listing 2.1.6: 确定一年借款每季度还款关系的模型 (loan.mzn)

```plaintext
% 变量
var float: R; % 季度还款
var float: P; % 初始借贷本金
var 0.0 .. 10.0: I; % 利率

% 中间变量
var float: B1; % 一个季度后的欠款
var float: B2; % 两个季度后的欠款
var float: B3; % 三个季度后的欠款
var float: B4; % 最后欠款

constraint B1 = P * (1.0 + I) - R;
constraint B2 = B1 * (1.0 + I) - R;
constraint B3 = B2 * (1.0 + I) - R;
constraint B4 = B3 * (1.0 + I) - R;

solve satisfy;

output [
"Borrowing ", show_float(0, 2, P), " at ", show(I*100.0),
"% interest, and repaying ", show_float(0, 2, R),
" \nper quarter for 1 year leaves ", show_float(0, 2, B4), " owing\n"
];
```

注意我们声明了一个浮点型变量 $f$，它和整型变量很类似。只是这里我们使用关键字 `float` 而不是 `int`。

我们可以用同样的模型来回答一系列不同的问题。第一个问题是：如果我以利息 4% 借款 $1000$ 并且每季度还款 $260$，我最终还欠款多少？这个问题在数据文件 `loan1.dzn` 中被编码。

由于我们希望用实数求解，我们需要使用一个可以支持这种问题类型的求解器。Gecode(Minizinc 捆绑二进制发布预设的求解器) 支持浮点型变量，一个混合整数线性求解器。
器可能更加适合这种类型的问题。MiniZinc 发布包含了一个这样的求解器。我们可以通过求解器 IDE 菜单（Run 按钮下面的三角形）选择 COIN-BC 来使用，或者在命令行中运行命令

```
minizinc --solver cbc
```

```bash
$ minizinc --solver cbc loan.mzn loan1.dzn
```

输出是

```
Borrowing 1000.00 at 4.0% interest, and repaying 260.00
per quarter for 1 year leaves 65.78 owing
----------
```

第二个问题是如果我希望用 4% 的利息来借款 $1000 并且在最后的时候一点都不欠款，我需要在每季度还款多少？这个问题在数据文件 loan2.dzn 中被编码。运行命令

```bash
$ minizinc --solver cbc loan.mzn loan2.dzn
```

后的输出是

```
Borrowing 1000.00 at 4.0% interest, and repaying 275.49
per quarter for 1 year leaves 0.00 owing
----------
```

第三个问题是如果我可以每个季度返还 $250, 我可以用 4% 的利息来借款多少并且在最后的时候一点都不欠款？这个问题在数据文件 loan3.dzn 中被编码。运行命令

```bash
$ minizinc --solver cbc loan.mzn loan3.dzn
```

后的输出是

```
Borrowing 907.47 at 4.0% interest, and repaying 250.00
per quarter for 1 year leaves 0.00 owing
----------
```

Listing 2.1.7: loan.mzn 的数据文件例子 (loan1.dzn)

```minizinc
I = 0.04;
P = 1000.0;
```

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R = 260.0;

Listing 2.1.8: loan.mzn 的数据文件例子 (loan2.dzn)

I = 0.04;
P = 1000.0;
B4 = 0.0;

Listing 2.1.9: loan.mzn 的数据文件例子 (loan3.dzn)

I = 0.04;
R = 250.0;
B4 = 0.0;

浮点算术操作符

MiniZinc 提供了标准的浮点算术操作符: 加 (+), 减 (-), 乘 (*) 和浮点除 (/)。同时也提供了一元操作符 + 和 -。

MiniZinc 不会自动地强制转换整数为浮点数。内建函数 `int2float` 被用来达到此目的。注意强制转换的一个后果是表达式 a / b 总是被认为是一个浮点除。如果你需要一个整数除，请确定使用 `div` 操作符。

MiniZinc 同时也包含浮点函数来计算: 绝对值 (abs), 平方根 (sqrt), 自然对数 (ln), 底数为 2 的对数 (log2), 底数为 10 的对数 (log10), $e$ 的幂 (exp), 正弦 (sin), 余弦 (cos), 正切 (tan), 反正弦 (asin), 反余弦 (acos), 反正切 (atan), 双曲正弦 (sinh), 双曲余弦 (cosh), 双曲正切 (tanh), 双曲反正弦 (asinh), 双曲反余弦 (acosh), 双曲反正切 (atanh)，和唯一的二元函数次方 (pow)。其余的都是一元函数。

算术常量的语法是相当标准的。浮点数常量的例子有 1.05, 1.3e-5 和 1.3E+5。

2.1.5 模型的基本结构

我们现在可以去总结 MiniZinc 模型的基本结构了。它由多个项组成，每一个在其最后都有一个分号，。项可以按照任何顺序出现。例如，标识符在被使用之前不需要被声明。

有八种类型的项 items。

- 引用项允许另外一个文件的内容被插入模型中。它们有以下形式:

```
include <文件名>;
```
其中 `<文件名>` 是一个字符串常量。它们使得大的模型可以被分为小的子模型以及包含库文件中定义的约束。我们会在 Listing 2.2.4 中看到一个例子。

- 变量声明声明新的变量。这种变量是全局变量，可以在模型中的任何地方被提到。变量有两种。在模型中被赋一个固定值的参数变量以及只有在模型被求解的时候才会被赋值的决策变量。我们称参数是 固定的，决策变量是 不固定的。变量可以选择性地被赋一个值来作为声明的一部分。形式是:

  `<类型-实例化 表达式>`: `<变量>[ = ]<表达式>;

`<类型-实例化 表达式>` 给了变量的类型和实例化。这些是 MiniZinc 比较复杂的其中一面。用 par 来实例化声明参数，用 var 来实例化声明决策变量。如果没有明确的实例化声明，
则变量是一个参数。类型可以为基类型，一个 数组或者浮点数范围，或者一个数组或集合。
基类型有 float, int, string, bool, ann。其中只有 float, int and bool 可以被决策变量
使用。基类型 ann 是一个 注解 - 我们会在 约束 (page 95) 中讨论注解。整数范围表达式可
被用来代替类型 int。类似的，浮点数范围表达式可以被用来代替类型 float。这些通
常被用来定义一个整型决策变量的定义域，但也可以被用来限制一个整型参数的范围。变量
声明的另外一个用处是定义 枚举类型，- 我们会在 枚举类型 (page 54) 中讨论。

- 赋值项给一个变量赋一个值。它们有以下形式:

  `<变量> = <表达式>;

数值可以被赋予决策变量。在这种情况下，赋值相当于加入
`<类型-实例化 表达式>`: `<变量>[ = ]<表达式>;

- 约束项是模型的核心。它们有以下形式:

  `constraint <布尔型表达式>;

我们已经看到了使用算术比较的简单约束以及内建函数 assert 操作符。在下一节我们会看到
约束变量的例子。

- 求解项详细说明了到底要找哪种类型的解。正如我们看到的，它们有以下三种形式:

  `solve satisfy;
solve maximize <算术表达式>;
solve minimize <算术表达式>;

一个模型必须有且只有一个求解项。`
• 输出项用来恰当的呈现模型运行后的结果。它们有下面的形式:

```plaintext
output [ <字符串表达式>, ..., <字符串表达式> ];
```

如果没有输出项，MiniZinc 会默认输出所有没有被赋值项的形式赋值的决策变量值。

• 枚举类型声明。我们会在 数组和集合 (page 39) 和 枚举类型 (page 54) 中讨论。

• 谓词函数和测试项被用来定义新的约束，函数和布尔测试。我们会在 谓词和函数 (page 69) 中讨论。

• 注解项用来定义一个新的注解。我们会在 模型 (page 95) 中讨论。
更用复杂模型

在上一节中，我们介绍了 MiniZinc 模型的基本结构。在这一节中，我们介绍数组和集合数据结构，枚举类型，以及更加复杂的约束。

### 2.2.1 数组和集合

在绝大多数情况下，我们都是有兴趣建一个约束和变量的个数依赖于输入数据的模型。为了达到此目的，我们通常会使用 数组。

考虑一个关于金属矩形板温度的简单有限元模型。通过把矩形板在 2 维的矩阵上分成有限个的元素，我们近似计算矩形板上的温度。一个模型在 Listing 2.2.1 中给出。它声明了有限元模型的宽 w 和高 h。

声明

```plaintext
set of int: HEIGHT = 0..h;
set of int: CHEIGHT = 1..h-1;
set of int: WIDTH = 0..w;
set of int: CWIDTH = 1..w-1;
array[HEIGHT,WIDTH] of var float: t; % 在点 (i,j) 处的温度
```

声明了四个固定的整型集合来描述有限元模型的尺寸： HEIGHT 是整个模型的整体高度，而 CHEIGHT 是省略了顶部和底部的中心高度，WIDTH 是模型的整体宽度，而 CWIDTH 是省略了左侧
和右侧的中心宽度。最后，声明了一个浮点型变量组成的行编号从 0 到 w，列编号从 0 到 h 的
二维数组 t 用来表示金属板上每一点的温度。我们可以通过表达式 t[i,j] 来得到数组中第 i^{th}
行和第 j^{th} 列的元素。

拉普拉氏方程规定当金属板达到一个稳定状态时，每一个内部点的温度是它的正交相邻点的平
均值。约束

```plaintext
constraint forall(i in CHEIGHT, j in CWIDTH)(
    4.0 * t[i,j] = t[i-1,j] + t[i,j-1] + t[i+1,j] + t[i,j+1]);
```

保证了每一个内部点 (i,j) 是它的四个正交相邻点的平均值。约束

```plaintext
% 边约束
constraint forall(i in CHEIGHT)(t[i,0] = left);
constraint forall(i in CHEIGHT)(t[i,w] = right);
constraint forall(j in CWIDTH)(t[0,j] = top);
constraint forall(j in CWIDTH)(t[h,j] = bottom);
```

限制了每一个边的温度必须是相等的，并且给这些温度名字：left, right, top 和 bottom。而约束

```plaintext
% 角约束
constraint t[0,0] = 0.0;
constraint t[0,w] = 0.0;
constraint t[h,0] = 0.0;
constraint t[h,w] = 0.0;
```

确保了角的温度（这些是不相干的）被设置为 0.0。我们可以用 Listing 2.2.1 中给出的模型来决
定一个被分成 5 × 5 个元素的金属板的温度。其中左右下侧的温度为 0，上侧的温度为 100。

**Listing 2.2.1:** 决定稳定状态温度的有限元平板模型 (laplace.mzn).

```plaintext
int: w = 4;
int: h = 4;

% arraydec
set of int: HEIGHT = 0..h;
set of int: CHEIGHT = 1..h-1;
set of int: WIDTH = 0..w;
set of int: CWIDTH = 1..w-1;
```
array[HEIGHT,WIDTH] of var float: t; % 在点 (i,j) 处的温度
var float: left; % 左侧温度
var float: right; % 右侧温度
var float: top; % 顶部温度
var float: bottom; % 底部温度

% 拉普拉斯方程：每一个内部点温度是它相邻点的平均值
constraint forall (i in CHEIGHT, j in CWIDTH)(
    4.0 * t[i,j] = t[i-1,j] + t[i,j-1] + t[i+1,j] + t[i,j+1]);

% sides
% 边约束
constraint forall (i in CHEIGHT)(t[i,0] = left);
constraint forall (i in CHEIGHT)(t[i,w] = right);
constraint forall (j in CWIDTH)(t[0,j] = top);
constraint forall (j in CWIDTH)(t[h,j] = bottom);

% 角约束
constraint t[0,0]=0.0;
constraint t[0,w]=0.0;
constraint t[h,0]=0.0;
constraint t[h,w]=0.0;
left = 0.0;
right = 0.0;
top = 100.0;
bottom = 0.0;

solve satisfy;

output [ show_float(6, 2, t[i,j]) ++
    if j == w then "\n" else " " endif |
    i in HEIGHT, j in WIDTH ];

运行命令

$ minizinc --solver cbc laplace.mzn

得到输出
集合

集合变量用以下方式声明

```mzn
set of <类型-实例化> : <变量名> ;
```

整型，枚举型（参见后面），浮点型和布尔型集合都可以定义。决策变量集合只可以是类型为整型或者枚举型的变量集合。集合常量有以下形式

```mzn
{ <表达式-1>, ..., <表达式-n> }
```

或者是以下形式的整型，枚举型或浮点型范围表达式

```mzn
<表达式-1> .. <表达式-2>
```

标准的集合操作符号: 元素属于 (in), (非严格的) 集合包含 (subset), (非严格的) 超集关系 (superset), 并集 (union), 交集 (intersect), 集合差运算 (diff), 集合对称差 (symdiff) 和集合元素的个数 (card).

我们已经看到集合变量和集合常量（包含范围）可以被用来作为变量声明时的隐式类型。在这种情况下变量拥有集合元素中的类型并且被隐式地约束为集合中的一个成员。

我们的烤蛋糕问题是一个非常简单的批量生产计划问题例子。在这些问题中，我们希望去决定每种类型的产品要制造多少来最大化利润。同时制造一个产品会消耗不同数量固定的资源。我们可以扩展 Listing 2.1.3 中的 MiniZinc 模型为一个不限制资源和产品类型的模型去处理这种类型的问题。这个模型在 Listing 2.2.2 中给出。一个 (烤蛋糕问题的) 数据文件例子在 Listing 2.2.3 中给出。

### Listing 2.2.2: 简单批量生产计划模型 (simple-prod-planning.mzn).

```mzn
% 要制造的产品
decision
% 每种产品的单位利润
enum Products;
% 每种类型的单位利润
array[Products] of int: profit;
% 用到的资源
```
enum Resources;
% 每种资源可获得的数量
array[Resources] of int: capacity;

% 制造一个单位的产品需要的资源单位量
array[Products, Resources] of int: consumption;
constraint assert (forall (r in Resources, p in Products)
  (consumption[p, r] >= 0), "Error: negative consumption");

% 产品数量的界
int: mproducts = max (p in Products)
  (min (r in Resources where consumption[p, r] > 0)
   (capacity[r] div consumption[p, r]));

% 变量：每种产品我们需要制造多少
array[Products] of var 0..mproducts: produce;
array[Resources] of var 0..max(capacity): used;

% 产量不可以使用超过可获得的资源量：
constraint forall (r in Resources) (used[r] = sum (p in Products)(consumption[p, r] * produce[p]));
constraint forall (r in Resources) (used[r] <= capacity[r]);

% 最大化利润
solve maximize sum (p in Products) (profit[p]*produce[p]);

output [ "\(p\) = \(\text{produce}[p]\);\n" | p in Products ] ++
  [ "\(r\) = \(\text{used}[r]\);\n" | r in Resources ];

Listing 2.2.3: 简单批量生产计划模型的数据文件例子
(simple-prod-planning-data.dzn).

% 简单批量生产计划模型的数据文件
Products = { BananaCake, ChocolateCake };
profit = [400, 450]; % 以分为单位
Resources = {Flour, Banana, Sugar, Butter, Cocoa};
capacity = [4000, 6, 2000, 500, 500];
consumption= [|
  250, 2, 75, 100, 0,
  200, 0, 150, 150, 75 |];

这个模型的新特征是只用枚举类型枚举类型。这使得我们可以把资源和产品的选择作为模型的参数。模型的第一个项

enum Products;

声明 Products 为未知的产品集合。

枚举类型
枚举类型，我们称为 enums，用以下方式声明

    enum <变量名> ;

一个枚举类型用以下赋值的方式定义

    enum <变量名> = { <变量名-1>, ..., <变量名-n> };

其中 <变量名-1>, ..., <变量名-n> 是名为 <变量名> 的枚举类型中的元素。通过这个定义，枚举类型中的每个元素也被有效地声明为这个类型的一个新的常量。声明和定义可以像往常一样结合为一行。

第二个项声明了一个整型数组:

    array[Products] of int: profit;

profit 数组的下标集合是 Products。理想情况下，这种声明方式表明只有集合 Products 中的元素才能被用来做数组的下标。

有 n 个元素组成的枚举类型中的元素的行为方式和整数 1...n 的行为方式很像。它们可以被比较，它们可以按照它们出现在枚举类型定义中的顺序被排序，它们可以遍历，它们可以作为数组的下标，实际上，它们可以出现在一个整数可能出现的任何地方。

在数据文件例子中，我们用一列整数来初始化数组
MiniZinc Handbook, Release 2.4.3

```plaintext
Products = { BananaCake, ChocolateCake };
profit = [400, 450];
```

意思是香蕉蛋糕的利润是 400，而巧克力蛋糕的利润是 450。在内部，BananaCake 会被看成是整数 1 一样，而 ChocolateCake 会被看成整数 2 一样。MiniZinc 虽然不提供明确的列表类型，但用 1..n 为下标集合的一维数组表现起来就像列表。我们有时候也会称它们为列表 lists。

根据同样的方法，接下来的两项中我们声明了一个资源集合 Resources，一个表明每种资源可获得量的数组 capacity。

更有趣的是项

```plaintext
array[Products, Resources] of int: consumption;
```

声明了一个二维数组 consumption。consumption[p,r] 的值是制造一单位的产品 p 所需要的资源 r 的数量。其中第一个下标是行下标，而第二个下标是列下标。

数据文件包含了一个二维数组的初始化例子:

```plaintext
consumption = [
  [250, 2, 75, 100, 0, 
  | 200, 0, 150, 150, 75 ]];
```

注意分隔符 | 是怎样被用来分隔行的。
数组

因此，MiniZinc 提供一维和多维数组。它们用以下类型来声明：

```
array [ <下标集合-1>, ..., <下标集合-n> ] of <类型-实例化>
```

MiniZinc 要求数组声明在每一维的下标集合。下标集合或者是一个整型范围，一个被初始化为整型范围的集合变量，或者是一个枚举类型。数组可以是所有的基类型：整型，枚举型，布尔型，浮点型或者字符串型。这些可以是固定的或者不固定的。除了字符串型，它只可以是参数。数组也可以作用于集合但是不可以作用于数值。

一维数组常量有以下格式

```
[ <表达式-1>, ..., <表达式-n> ]
```

二维数组常量有以下格式

```
[ ] <表达式-1>, ..., <表达式-1-n> |  
  ... |  
<表达式-m-1>, ..., <表达式-m-n> ]
```

其中这个数组有 m 行 n 列。

内建函数 array1d, array2d 等家族可以被用来从一个列表（或者更准确的说是一个一维数组）去实例化任何维度的数组。调用

```
array<n>d(<下标集合-1>, ..., <下标集合-n>, <列表>)
```

返回一个 n 维的数组，它的下标集合在前 n 个参数给出，最后一个参数包含了数组的元素。例如 array2d(1..3, 1..2, [1, 2, 3, 4, 5, 6]) 和 [1, 2 [1, 3, 4] [5, 6]] 是相等的。

数组元素按照通常的方式获取：array[i,j] 给出第 i\text{th} 行第 j\text{th} 列的元素。

串联操作符 ++ 可以被用来串联两个一维的数组。结果得到一个列表，即一个元素从 1 索引的一维数组。例如 [4000, 6] ++ [2000, 500, 500] 求得 [4000, 6, 2000, 500, 500]。

内建函数 length 返回一维数组的长度。

模型的下一项定义了参数 mproducts，它被设为可以生产出的任何类型产品的数量上限。这个确实是一个复杂的内嵌数组推导式和聚合操作符例子。在我们试图理解这些项和剩下的模型之前，我们应该先介绍一下它们。

首先，MiniZinc 提供了在很多函数式编程语言都提供的列表推导式。例如，列表推导式

```
[i + j | i, j in 1..3 where j < i]
```

算得 [2 + 1, 3 + 1, 3 + 2] 等同于 [3, 4, 5]。[3, 4, 5] 只是一个下标集合为 1..3 的数组。

MiniZinc 同时也提供了集合推导式，它有类似的语法：例如
\{i + j \mid i, j \in 1..3\text{ where } j < i\} 计算得到集合 \{3, 4, 5\}。

### 列表和集合推导式

列表推导式的一般格式是

\[
\begin{bmatrix}
\text{<表达式>} & \text{<生成元表达式>}
\end{bmatrix}
\]

- **<表达式>** 指明了如何从 **<生成元表达式>** 产生的元素输出列表中创建元素。生成元
- **<generator-exp>** 由逗号分开的一列生成元表达式组成，选择性地跟着一个布尔型表达式。两种格式是

\[
\begin{bmatrix}
\text{<生成元>} & \ldots & \text{<生成元>}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{<生成元>} & \ldots & \text{<生成元> \text{ where } <布尔表达式>}
\end{bmatrix}
\]

第二种格式中的可选择的 **<布尔表达式>** 被用作生成元表达式的过滤器：只有满足布尔表达式的输出列表中的元素才被用来构建元素。生成元 **<generator>** 有以下格式

\[
\begin{bmatrix}
\text{<标识符>} & \ldots & \text{<标识符> \text{ in } <数组表达式>}
\end{bmatrix}
\]

每一个标识符是一个迭代器，轮流从数值表达式中取值，最后一个标识符变化的最迅速。列表推导式的生成元和 **<布尔表达式>** 通常不会涉及决策变量。如果它们确实涉及了决策变量，那么产生的列表是一列 var opt <T>，其中 $\$T\$ 是 <表达式> 的类型。更多细节，请参考 [选项类型 (page 91)](#) 中有关选项类型 option types 的论述。

集合推导式几乎和列表推导式一样：唯一的不同是这里使用 {和} 括住表达式而不是 [和]。集合推导式生成的元素必须是固定的 fixed，即不能是决策变量。类似的，集合推导式的生成元和可选择的 **<布尔表达式>** 必须是固定的。

第二，MiniZinc 提供了一系列的可以把一组数组的元素聚合起来的内建函数。它们中最有用的可能是:mzn:forall。它接收一个布尔型表达式数组（即，约束），返回单个布尔型表达式，它是对数组中的布尔型表达式的逻辑合取。

例如，以下表达式

```plaintext
forall([a[i] != a[j] \mid i, j in 1..3\text{ where } i < j])
```

聚合函数

算术数组的聚合函数有：sum 把元素加起来，product 把元素乘起来，和 min 跟 max 各自返回数组中的最小和最大元素。当作用于一个空的数组时，min 和 max 返回一个运行错误，sum 返回 0，product 返回 1。

MiniZinc 为数组提供了包含有布尔型表达式的四个聚合函数。正如我们看到的，它们中的第一个是forall，它返回一个等于多个约束的逻辑合取的单个约束。第二个函数，exists，返回多个约束的逻辑析取。因此 forall 强制数组中的所有约束都满足，而 exists 确保至少有一个约束满足。第三个函数，forall 强制奇数个约束满足。第四个函数，iffall 确保偶数个约束满足。

第三个，也是难点的最后一个部分是当使用数组推导式时，MiniZinc 允许使用一个特别的聚合函数的语法。建模者不仅仅可以用

```plaintext
forall([a[i] != a[j] | i, j in 1..3 where i < j])
```

也可以用一个更加数学的表示

```plaintext
forall (i, j in 1..3 where i < j) (a[i] != a[j])
```

两种表达方式是完全相等的：建模者可以自由使用任何一个他们认为更自然的表达方式。

生成表达式

一个生成表达式有以下格式

```plaintext
<聚合函数> ( <生成元表达式> ) ( <表达式> )
```

圆括号内的生成元表达式和构造表达式与表达式是选择性的：它们必须存在。它等同于

```plaintext
<聚合函数> ( [ <表达式> | <生成元表达式> ] )
```

<聚合函数> aggregation function 可以是 MiniZinc 的任何由单个数组作为其参数的函数。

接下来我们就来了解 Listing 2.2.2 中的简单批量生产计划模型剩余的部分。现在请暂时忽略定义 mproducts 的这部分。接下来的项:

```plaintext
array[Products] of var 0..mproducts: produce;
```

定义了一个一维的决策变量数组 produce。produce[p] 的值代表了最优解中产品 p 的数量。下一项
array[Resources] of var 0..max(capacity): used;

定义了一个辅助变量集合来记录每一种资源的使用量。下面的两个约束

```
constraint forall (r in Resources)
    (used[r] = sum (p in Products) (consumption[p, r] * produce[p]));
constraint forall (r in Resources)(used[r] <= capacity[r]);
```

使用 used[r] 计算资源 r 的总体消耗以及保证它是小于可获得的资源 r 的量。最后，项

```
solve maximize sum (p in Products) (profit[p]*produce[p]);
```

表明这是一个最大化问题以及最大化的目标是全部利润。

现在我们回到 mproducts 的定义。对每个产品 p，表达式

```
(min (r in Resources where consumption[p,r] > 0)
    (capacity[r] div consumption[p,r]))
```

决定了在考虑了每种资源 r 的数量以及制造产品 p 需要的 r 量的情况下，p 可以生产的最大量。注意过滤器 where consumption[p,r] > 0 的使用保证了只有此产品需要的资源才会考虑，因此避免了出现除数为零的错误。所以，完整的表达式

```
int: mproducts = max (p in Products)
    (min (r in Resources where consumption[p,r] > 0)
    (capacity[r] div consumption[p,r]));
```

计算 任何 产品可以被制造的最大量，因此它可以被作为 produce 中的决策变量定义域的上限。

最后，注意输出项是比较复杂的，并且使用了列表推导式 列表推导式去创建一个易于理解的输出。运行

```
$ minizinc --solver gecode simple-prod-planning.mzn simple-prod-planning-
data.dzn
```

输出得到如下结果
MiniZinc Handbook, Release 2.4.3

BananaCake = 2;
ChocolateCake = 2;
Flour = 900;
Banana = 4;
Sugar = 450;
Butter = 500;
Cocoa = 150;

--------
=-=-=-=-=

2.2.2 全局约束

MiniZinc 包含了一个全局约束的库，这些全局约束也可以被用来定义模型。一个例子是 `alldifferent` 约束，它要求所有参数中的变量都必须是互不相等的。

```
Listing 2.2.4: SEND+MORE=MONEY 算式谜题模型 (send-more-money.mzn)

include "alldifferent.mzn";

var 1..9: S;
var 0..9: E;
var 0..9: N;
var 0..9: D;
var 1..9: M;
var 0..9: O;
var 0..9: R;
var 0..9: Y;

constraint 1000 * S + 100 * E + 10 * N + D
            + 1000 * M + 100 * O + 10 * R + E
            = 10000 * M + 1000 * O + 100 * N + 10 * E + Y;

constraint alldifferent([S,E,N,D,M,O,R,Y]);

solve satisfy;

output [" (S) (E) (N) (D) \n",]
```

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SEND+MORE=MONEY 问题要求给每一个字母赋不同的数值使得此算术约束满足。Listing 2.2.4 中的模型使用 alldifferent([S,E,N,D,M,O,R,Y]) 约束表达式来保证每个字母有不同的数字值。在使用了引用项

```
include "alldifferent.mzn";
```

后，此全局约束 alldifferent 可以在模型中使用。我们可以用以下代替此行

```
include "globals.mzn";
```

它包含了所有的全局约束。

一系列所有在 MiniZinc 中定义了的全局约束都被包含在了发布的文档中。对一些重要的全局约束的描述，请参见全局约束 (page 69)。

2.2.3 条件表达式

MiniZinc 提供了一个条件表达式 if-then-else-endif。它的一个使用例子如下

```
int: r = if y != 0 then x div y else 0 endif;
```

若 y 不是零，则 r 设为 x 除以 y，否则则设为零。
Listing 2.2.5: 广义数独问题的模型 (sudoku.mzn)

```mini
include "alldifferent.mzn"

int: S;
int: N = S * S;
int: digs = ceil(log(10.0,int2float(N))); % 输出的数字

set of int: PuzzleRange = 1..N;
set of int: SubSquareRange = 1..S;

array[1..N,1..N] of 0..N: start; %% 板初始 \( \emptyset = \) 空
array[1..N,1..N] of var PuzzleRange: puzzle;

% 填充初始板
constraint forall(i,j in PuzzleRange)(
    if start[i,j] > 0 then puzzle[i,j] = start[i,j] else true endif);

% 每行中取值各不相同
constraint forall (i in PuzzleRange) (  
    alldifferent( [ puzzle[i,j] | j in PuzzleRange ] ));

% 每列中取值各不相同
constraint forall (j in PuzzleRange) (  
    alldifferent( [ puzzle[i,j] | i in PuzzleRange ] ));

% 每个子方格块中取值各不相同
constraint  
    forall (a, o in SubSquareRange)(  
        alldifferent( [ puzzle[(a-1) *S + a1, (o-1)*S + o1] |  
                        a1, o1 in SubSquareRange ] ));

solve satisfy;

output [ show_int(digs,puzzle[i,j]) ++ " " ++  
            if j mod S == 0 then " " else "" endif ++  
            if j == N then  
                if i != N then 
```
if i mod S == 0 then "\n\n" else "\n" endif
else "" endif else "" endif
| i,j in PuzzleRange | ++ ["\n"];

Listing 2.2.6: 广义数独问题的数据文件例子 (sudoku.dzn)

S=3;
start=[]
0, 0, 0, 0, 0, 0, 0, 0, 0|
0, 6, 8, 4, 0, 1, 0, 7, 0|
0, 0, 0, 0, 8, 5, 0, 3, 0|
0, 2, 6, 8, 0, 9, 0, 4, 0|
0, 0, 7, 0, 0, 0, 9, 0, 0|
0, 5, 0, 1, 0, 6, 3, 2, 0|
0, 4, 0, 6, 1, 0, 0, 0, 0|
0, 3, 0, 2, 0, 7, 6, 9, 0|
0, 0, 0, 0, 0, 0, 0, 0, 0|

Fig. 2.2.1: sudoku.dzn 代表的问题。

在创建复杂模型或者复杂输出时，条件表达式是非常有用的。我们来看下 Listing 2.2.5 中的数独问题模型。板的初始位置在参数 start 中给出，其中 0 代表了一个空的板位置。通过使用以下条件表达式

constraint forall(i,j in PuzzleRange)(
    if start[i,j] > 0 then puzzle[i,j] = start[i,j] else true endif);

它被转换为对决策变量 puzzle 的约束。
在定义复杂输出时，条件表达式也很有用。`output` 在数独模型 Listing 2.2.5 中，表达式

```minizinc
if j mod S == 0 then " " else "" endif
```

在大小为 $S$ 的组群之间插入了一个额外的空格。输出表达式同时也使用条件表达式来在每 $S$ 行后面加入一个空白行。这样得到的输出有很高的可读性。

剩下的约束保证了每行中，每列中以及每 $S \times S$ 子方格块中的值都是互相不相同的。

通过使用标示 `-a` 或 `--all-solutions`，我们可以使用 MiniZinc 求解得到一个满足问题 (solve satisfy) 的所有解。运行

```shell
$ minizinc --all-solutions sudoku.mzn sudoku.dzn
```

得到

```
5 9 3 7 6 2 8 1 4
2 6 8 4 3 1 5 7 9
7 1 4 9 8 5 2 3 6

3 2 6 8 5 9 1 4 7
1 8 7 3 2 4 9 6 5
4 5 9 1 7 6 3 2 8

9 4 2 6 1 8 7 5 3
8 3 5 2 4 7 6 9 1
6 7 1 5 9 3 4 8 2
----------
==========
```

当系统输出完所有可能的解之后，此行 `==========` 被输出。在这里则表明了此问题只有一个解。

### 2.2.4 枚举类型

枚举类型允许我们根据一个或者数据中的一部分，或者在模型中被命名的对象集合来创建模型。这样一来，模型就更容易被理解和调试。我们之前已经简单介绍了枚举类型或者 `enums`。在这一小分段，我们会探索如何可以全面地使用它们，并且给出一些处理枚举类型的内建函数。

让我们重新回顾一下 *MiniZinc 基本模型 (page 23)* 中的给澳大利亚涂色问题。
**Listing 2.2.7**: 使用枚举类型的澳大利亚涂色模型（aust-enum.mzn）。

```minizinc
enum Color;
var Color: wa;
var Color: nt;
var Color: sa;
var Color: q;
var Color: nsw;
var Color: v;
var Color: t;
constraint wa != nt \ wa != sa \ nt != sa \ nt != q \ sa != q;
constraint sa != nsw \ sa != v \ q != nsw \ nsw != v;
solve satisfy;
```

**Listing 2.2.7** 中的模型声明了一个枚举类型 `Color`，而它必须在数据文件中被定义。每一个州变量被声明为从此枚举类型中取一个值。使用以下方式运行这个程序

```
$ minizinc -D "Color = { red, yellow, blue };" aust-enum.mzn
```

可能会得到输出

```minizinc
wa = yellow;
nt = blue;
sa = red;
q = yellow;
nsw = blue;
v = yellow;
t = red;
```

### 枚举类型变量声明

一个枚举类型参数变量被声明为以下两种方式:

- `<枚举名>` : `<变量名>`
- `<l>..<u>` : `<变量名>`

其中 `<枚举名>` 是枚举类型的名字，`<l>` 和 `<u>` 是此枚举类型的固定枚举类型表达式。

枚举类型一个重要的行为是，当它们出现的位置所期望的是整数时，它们会自动地强制转换为
整数。这样一来，这就允许我们使用定义在整数上的全局变量，例如

```mini
global_cardinality_low_up([wa, nt, sa, q, nsw, v, t],
                         [red, yellow, blue], [2, 2, 2], [2, 2, 3]);
```

要求每种颜色至少有两个州涂上并且有三个州被涂了蓝色。

### 枚举类型操作符

有一系列关于枚举类型的内部操作符：

- `enum_next(X, x)`: 返回枚举类型 `X` 中 `x` 后的下一个值。这是一个部分函数，如果 `x` 是枚举类型 `X` 最后一个值，则函数会返回 `false`。
- `enum_prev(X, x)` `enum_prev(X, x)`: 返回枚举类型 `X` 中 `x` 的上一个值。`enum_prev` 同样是一个部分函数。
- `to_enum(X, i)`: 映射一个整型表达式 `i` 到一个在 `X` 的枚举类型值，或者如果 `i` 是小于等于 0 或大于 `mzn:X` 中元素的个数，则返回 `math:bot`。

注意，一些标准函数也可以应用于枚举类型：

- `card(X)`: 返回枚举类型 `X` 的势。
- `min(X)`: 返回枚举类型 `X` 中最小的元素。
- `max(X)`: 返回枚举类型 `X` 中最大的元素。

### 2.2.5 复杂约束

约束是 MiniZinc 模型的核心。我们已经看到了简单关系表达式，但是约束其实是比这更加强大的。一个约束可以是任何布尔表达式。想象一个包含两个时间上不能重叠的任务的调度问题。如果 `s1` 和 `s2` 是相对应的起始时间，`d1` 和 `d2` 是相对应的持续时间，我们可以表达约束为:

```mini
constraint s1 + d1 <= s2 \ / s2 + d2 <= s1;
```

来保证任务之间互相不会重叠。

### 布尔型

MiniZinc 中的布尔型表达式可以按照标准的数学语法来书写。布尔常量是 `true` 或 `false`，布尔型操作符有合取（`\land`），析取（`\lor`），或（`\lor`），必要条件蕴含（`\implies`），充分条件蕴含（`\implies`），充分必要蕴含（`\iff`）以及非（`\neg`）。内建函数 `bool2int` 强制转换布尔型为整型：如果参数为真，它返回 1，否则返回 0。
Listing 2.2.8: 车间作业调度问题模型 (jobshop.mzn).

```miniZinc
enum JOB;
enum TASK;
TASK: last = max(TASK);
array [JOB,TASK] of int: d; % 任务持续时间
int: total = sum(i in JOB, j in TASK)(d[i,j]); % 总持续时间
int: digs = ceil(log(10.0,int2float(total))); % 输出的数值
array [JOB,TASK] of var 0..total: s; % 起始时间
var 0..total: end; % 总结束时间

constraint %% 保证任务按照顺序出现
forall(i in JOB) (    
    forall(j in TASK where j < last)  
    (s[i,j] + d[i,j] <= s[i,enum_next(TASK,j)]) \  
    s[i,last] + d[i,last] <= end  
);

constraint %% 保证任务之间没有重叠
forall(j in TASK) (    
    forall(i,k in JOB where i < k) (  
        s[i,j] + d[i,j] <= s[k,j] \  
        s[k,j] + d[k,j] <= s[i,j]  
    )  
);

solve minimize end;

output ["end = \(end)\n"] ++
    [ show_int(digs,s[i,j]) ++ " " ++
    if j == last then "\n" else "" endif |  
    i in JOB, j in TASK ];
```

Listing 2.2.9: 车间作业调度问题数据 (jdata.dzn).

```miniZinc
JOB = anon_enum(5);
TASK = anon_enum(5);
```
$ minizinc --all-solutions jobshop.mzn jdata.dzn

求解了一个小的车间作业调度问题，并且显示了优化问题在 all-solutions 下的表现。在这里，求解器只有当找到一个更好的解时才会输出它，而不是输出所有的可能最优解。这个命令下的（部分）输出是：

```plaintext
end = 39
 5 9 13 22 30
 6 13 18 25 36
 0 4 8 12 16
 4 8 12 16 22
 9 16 25 27 38

----------
end = 37
 4 8 12 17 20
 5 13 18 26 34
 0 4 8 12 16
 8 12 17 20 26
 9 16 25 27 36

----------
end = 34
 0 1 5 10 13
 6 10 15 23 31
 2 6 11 19 27
 1 5 10 13 19
```
表示一个结束时间为 30 的最优解终于被找到，并且被证明为是最优的。通过加一个约束
end = 30，并且把求解项改为 solve satisfy，然后运行

```bash
$ minizinc --all-solutions jobshop.mzn jobshop.dzn
```

我们可以得到所有的最优解。这个问题有 3,444,375 个最优解。

**Listing 2.2.10:** 稳定婚姻问题模型 (stable-marriage.mzn).

```minizinc
int: n;

enum Men = anon_enum(n);
enum Women = anon_enum(n);

array[Women, Men] of int: rankWomen;
array[Men, Women] of int: rankMen;

array[Men] of var Women: wife;
array[Women] of var Men: husband;

% assignment
constraint forall (m in Men) (husband[wife[m]] = m);
constraint forall (w in Women) (wife[husband[w]] = w);
% ranking
constraint forall (m in Men, o in Women) (  
  rankMen[m, o] < rankMen[m, wife[m]] ->  
  rankWomen[o, husband[o]] < rankWomen[o, m]);
```

2.2.5. 复杂约束
constraint forall (w in Women, o in Men) (  
    rankWomen[w,o] < rankWomen[w,husband[w]] ->  
    rankMen[o,wife[o]] < rankMen[o,w] );

solve satisfy;

output ["wives= \(wife)\nhusbands= \(husband)\n"];

Listing 2.2.11: 稳定婚姻问题模型的数据文件例子。(stable-marriage.dzn).

n = 5;

rankWomen =
[[ 1, 2, 4, 3, 5,  
  3, 5, 1, 2, 4,  
  5, 4, 2, 1, 3,  
  1, 3, 5, 4, 2,  
  4, 2, 3, 5, 1 ]];

rankMen =
[[ 5, 1, 2, 4, 3,  
  4, 1, 3, 2, 5,  
  5, 3, 2, 4, 1,  
  1, 5, 4, 3, 2,  
  4, 3, 2, 1, 5 ]];

MiniZinc 中的另外一个强大的建模特征是决策变量可以被用来访问数组 array access。作为一个例子，考虑（老式的）稳定婚姻问题。我们有 n 个（直）女以及 n 个（直）男。每一个男士有一个女士排行榜，女士也是如此。我们想给每一个女士/男士找一个丈夫/妻子来使得所有的婚姻按意义上来说都是 稳定的：

- 每当 m 喜欢另外一个女士 o 多过他的妻子 w 时，o 喜欢他的丈夫多过 m，以及
- 每当 w 喜欢另外一个男士 o 多过她的丈夫 m 时，o 喜欢他的妻子多过 w。

这个问题可以很优雅地在 MiniZinc 中建模。模型和数据例子在 Listing 2.2.10 和 Listing 2.2.11 中分别被给出。

模型中的前三项声明了男士/女士的数量以及男士和女士的集合。在这里我们介绍 名称枚举类型 的使用。Men 和 Women 都是大小为 n 的集合，但是我们不希望把它们混合到一起，所以我们使用了一个匿名枚举类型。这就允许 MiniZinc 检测到使用 Men 为 Women 或者反之的建模错误。

矩阵 rankWomen 和 rankMen 分别给出了男士们的女士排行以及女士们的男士排行。因此，项
rankWomen[w,m] 给出了女士 texttt{w} 的关于男士 texttt{m} 的排行。在排行中的数不大，此男士或女士被选择的倾向越大。

有两个决策变量的数组： wife 和 husband。这两个分别代表了每个男士的妻子和每个女士的丈夫。

前两个约束

\[
\text{constraint forall (m in Men) (husband[wife[m]] = m);}\\
\text{constraint forall (w in Women) (wife[husband[w]] = w);}\\
\]

确保了丈夫和妻子的分配是一致的：w 是 m 的妻子蕴含了 m 是 w 的丈夫，反之亦然。注意在 husband[wife[m]] 中，下标表达式 wife[m] 是一个决策变量，而不是一个参数。

接下来的两个约束是稳定条件的直接编码：

\[
\text{constraint forall (m in Men, o in Women) (}\\
\text{rankMen[m,o] < rankMen[m,wife[m]] \rightarrow}\\
\text{rankWomen[o,husband[o]] < rankWomen[o,m] );}\\
\text{constraint forall (w in Women, o in Men) (}\\
\text{rankWomen[w,o] < rankWomen[w,husband[w]] \rightarrow}\\
\text{rankMen[o,wife[o]] < rankMen[o,w] );}\\
\]

在有了用决策变量作为数组的下标和用标准的布尔型连接符构建约束的功能后，稳定婚姻问题的自然建模才变得可行。敏锐的读者可能会在这时产生疑问，如果数组下标变量取了一个超出数组下标集合的值，会产生什么情况？MiniZinc 把这种情况看作失败：一个数组访问 a[e] 在其周围最近的布尔型语句中隐含地加入了约束 e in index_set(a)，其中 index_set(a) 给出了 a 的下标集合。

**匿名枚举类型**

一个匿名枚举类型表达式有格式 anon_enum(<n>)，其中 <n> 是一个固定的整型表达式，它定义了枚举类型的大小。除了其中的元素没有名字，匿名枚举类型和其他的枚举类型一样。当被输出时，它们根据枚举类型的名称被给定独有的名字。

例如，如下的变量声明

\[
\text{array[1..2] of int: a = [2,3];}\\
\text{var 0..2: x;}\\
\]

2.2.5. 复杂约束
约束 $a[x] = y$ 会在 $x = 1 \land y = 2$ 和 $x = 2 \land y = 3$ 时得到满足。约束 not $a[x] = y$ 会在 $x = 0 \land y = 2$, $x = 0 \land y = 3$, $x = 1 \land y = 3$ 和 $x = 2 \land y = 2$ 时得到满足。

当参数无效访问数组时，正式的 MiniZinc 语义会把此情况看成失败来确保参数和决策变量的处理方式是一致的，但是会发出警告，因为这种情况下几乎总是会有错误出现。

<table>
<thead>
<tr>
<th>Listing 2.2.12: 魔术串问题模型 (magic-series.mzn).</th>
</tr>
</thead>
<tbody>
<tr>
<td>int: n;</td>
</tr>
<tr>
<td>array[0..n-1] of var 0..n: s;</td>
</tr>
<tr>
<td>constraint forall(i in 0..n-1) (</td>
</tr>
<tr>
<td>s[i] = (sum(j in 0..n-1)(bool2int(s[j]=i))));</td>
</tr>
<tr>
<td>solve satisfy;</td>
</tr>
<tr>
<td>output [ &quot;s = (s)\n&quot; ];</td>
</tr>
</tbody>
</table>

强制转换函数 bool2int 可以被任何布尔型表达式调用。这就使得 MiniZinc 建模者可以使用所谓的合逻辑约束。举个简单的例子，请看魔术串问题：找到一列数字 $s = [s_0, \ldots, s_{n-1}]$ 使得 $\mathbf{math}: s_i$ 是数字 $i$ 出现在 $s$ 的次数。一个解的例子是 $s = [1, 2, 1, 0]$。

这个问题的一个 MiniZinc 模型在 Listing 2.2.12 中给出。Listing 2.2.12 的使用使得我们可以把函数 $s[j]=i$ 满足的次数加起来。运行命令

```
$ minizinc --all-solutions magic-series.mzn -D "n=4;"
```

得到输出

```
s = [1, 2, 1, 0];
----------
s = [2, 0, 2, 0];
----------
----------
```

确切地显示出这个问题的两个解。

注意当有需要的时候，MiniZinc 会自动地强制转换布尔型为整型以及整型为浮点型。我们可以
把 **Listing 2.2.12** 中的约束项替换成

```plaintext
constraint forall(i in 0..n-1) ( 
    s[i] = (sum(j in 0..n-1)(s[j]=i)));
```

由于 MiniZinc 系统实际上会自动地加入缺失的 bool2int，布尔型表达式 s[j] = i 会被自动地强制转换为整型，所以会得到同样的结果。

### 强制转换

MiniZinc 中，通过使用函数 bool2int，我们可以把一个布尔型数值 emph(强制转换) 为一个整型数值。同样地，通过使用函数 int2float，我们也可以把一个整型数值强制转换为一个浮点型数值。被强制转换的数值的实例化和原数值一样。例如，par bool 被强制转换为 par int，而 var bool 被强制转换为 var int。

通过适当地在模型中加入 bool2int 和 int2float，MiniZinc 会自动地强制转换布尔型表达式为整型表达式，以及整型表达式为浮点型表达式。注意通过两步转换，它也会强制转换布尔型为浮点型。

### 2.2.6 集合约束

MiniZinc 另外一个强大的建模特征是它允许包含整数的集合是决策变量：这表示当模型被评估时，求解器会查找哪些元素在集合中。

举个简单的例子，0/1 背包问题。这个问题是背包问题的极限版本，即我们或者选择把物品放入背包或者不放。每一个物品有一个重量和一个利润。在限制于背包不能太满的条件下，我们想找到选取哪些物品会得到最大化利润。

很自然地，我们在 MiniZinc 中使用单个的决策变量来建模：

```plaintext
var set of ITEM: knapsack;
```

在 **Listing 2.2.13** 中给出了一个自然的模型。

**Listing 2.2.13**: 0/1 背包问题模型 (knapsack.mzn).

```plaintext
enum ITEM;
int: capacity;

array[ITEM] of int: profits;
array[ITEM] of int: weights;

var set of ITEM: knapsack;
```
Listing 2.2.14: 高尔夫联谊问题模型 (social-golfers.mzn).

```plaintext
include "partition_set.mzn";
int: weeks; set of int: WEEK = 1..weeks;
int: groups; set of int: GROUP = 1..groups;
int: size; set of int: SIZE = 1..size;
int: ngolfers = groups*size;
set of int: GOLFER = 1..ngolfers;

array[WEEK,GROUP] of var set of GOLFER: Sched;

% constraints
constraint
  forall (i in 1..weeks-1) (  
    Sched[i,1] < Sched[i+1,1]
  ) \/
  forall (i in WEEK, j in GROUP) (  
    card(Sched[i,j]) = size  
    \forall (k in j+1..groups) (  
      Sched[i,j] < Sched[i,k]
    )  
    \forall (j in 1..groups-1) (  
      Sched[i,j] < Sched[i,j+1]
    )
  )
```

注意，关键字 `var` 出现在 `set` 声明之前，表明这个集合本身是决策变量。这就和一个 `var` 关键字描述其中元素而不是数组自身的数组形成对比，因为此时数组的基本结构，即它的下标集合，是固定了的。
%  
) /
forall (i in 1..weeks-1, j in i+1..weeks) (  
    forall (x,y in GROUP) (  
        card(Sched[i,x] intersect Sched[j,y]) <= 1  
    )  
);  
% symmetry  
constraint  
% Fix the first week %  
forall (i in GROUP, j in SIZE) (  
    ((i-1)*size + j) in Sched[1,i]  
) /
% Fix first group of second week %  
forall (i in SIZE) (  
    ((i-1)*size + 1) in Sched[2,1]  
) /
% Fix first 'size' players  
forall (w in 2..weeks, p in SIZE) (  
    p in Sched[w,p]  
);  
solve satisfy;

output [ show(Sched[i,j]) ++ " " ++  
    if j == groups then "\n" else "" endif |  
    i in WEEK, j in GROUP ];

我们来看一个更复杂的关于集合约束的例子，Listing 2.2.14 中给出的高尔夫联誼问题。这个问题的目的是给 groups × size 个高尔夫手在 weeks 时间内安排一个高尔夫联赛。每一周我们需要安排 groups 个大小为 size 的不同的组。任何一对高尔夫手都不能一起出现于两个组中进行比赛。

模型中的变量是第 i\textsuperscript{th} 周第 j\textsuperscript{th} 组的高尔夫手: \texttt{mzn:Sched[i,j]} 组成的集合。

11-32 行中的约束首先对每一周的第一个集合进行一个排序来去除掉周之间可以互相调换的对称。然后它对每一周内的集合进行了一个排序，同时使得每一个集合的势为 size。接下来通过使用全局约束 partition_set，确保了每一周都是对高尔夫手集合的一个划分。最后一个约束确保了任何两个高尔夫手都不会一起在两个组内比赛（因为任何两个组的交集的势最多都是 1)。
我们也有
在 34-46 行中，我们也给出了去对称初始化约束：第一周被固定为所有的高尔夫手都按顺序排列；第二周的第一组被规定为是由第一周的前几组的第一个选手组成；最后，对于剩下的周，模型规定第一个 size 内的高尔夫手们出现在他们相对应的组数中。

运行命令

$ minizinc social-golfers.mzn social-golfers.dzn

其中数据文件定义了一个周数为 4，大小为 3，组数为 4 的问题，得到如下结果

1..3 4..6 7..9 10..12
{ 1, 4, 7 } { 2, 5, 10 } { 3, 9, 11 } { 6, 8, 12 }
{ 1, 5, 8 } { 2, 6, 11 } { 3, 7, 12 } { 4, 9, 10 }
{ 1, 6, 9 } { 2, 4, 12 } { 3, 8, 10 } { 5, 7, 11 }

注意范围集合是如何以范围格式输出的。

2.2.7 汇总

我们以一个可以阐释这一章介绍的大部分特征的复杂例子来结束这一节，包括枚举类型，复杂约束，全局约束以及复杂输出。

### Listing 2.2.15: 使用枚举类型规划婚礼座位（wedding.mzn）。

```minizinc
enum Guests = { bride, groom, bestman, bridesmaid, bob, carol,
    ted, alice, ron, rona, ed, clara};
set of int: Seats = 1..12;
set of int: Hatreds = 1..5;
array[Hatreds] of Guests: h1 = [groom, carol, ed, bride, ted];
array[Hatreds] of Guests: h2 = [clara, bestman, ted, alice, ron];
set of Guests: Males = {groom, bestman, bob, ted, ron, ed};
set of Guests: Females = {bride, bridesmaid, carol, alice, rona, clara};

array[Guests] of var Seats: pos; % 客人的座位
array[Hatreds] of var Seats: p1; % 互相憎恶的客人 1 的座位
array[Hatreds] of var Seats: p2; % 互相憎恶的客人 2 的座位
```
array[Hatreds] of var 0..1: sameside; % 互相憎恶的客人是否坐在同一边
array[Hatreds] of var Seats: cost; % 互相憎恶的客人的距离

include "alldifferent.mzn";
constraint alldifferent(pos);
constraint forall(g in Males) ( pos[g] mod 2 == 1 );
constraint forall(g in Females) ( pos[g] mod 2 == 0 );
constraint not (pos[ed] in {1,6,7,12});
constraint abs(pos[bride] - pos[groom]) <= 1 /
  (pos[bride] <= 6 <-> pos[groom] <= 6);
constraint forall(h in Hatreds) (p1[h] = pos[h1[h]] /
  p2[h] = pos[h2[h]] /
  sameside[h] = bool2int(p1[h] <= 6 <-> p2[h] <= 6) /
  cost[h] = sameside[h] * abs(p1[h] - p2[h]) +
  (1 - sameside[h]) * (abs(13 - p1[h] - p2[h]) + 1));
solve maximize sum(h in Hatreds)(cost[h]);

output [ show(g) ++ " | s in Seats, g in Guests where fix(pos[g]) == s]
  ++ ["\n"];

Listing 2.2.15 中的模型安排婚礼桌上的座位。这个桌子有 12 个编码的顺序排列的座位，每边有 6 个。男士必须坐奇数号码的座位，女士坐偶数。Ed 由于恐惧症不能坐在桌子的边缘，新郎和新娘必须坐在彼此旁边。我们的目的是最大化已知的互相憎恶的人之间的距离。如果在同一边，座位之间的距离是座位号码之间的差，否则则是和其对面座位的距离 + 1。

注意在输出语句中我们观察每个座位 s 来找一个客人 g 分配给此座位。我们利用内建函数 fix，它检查一个决策变量是否是固定的以及输出它的固定值，否则的话中断。在输出语句中使用此函数总是安全的，因为当输出语句被运行的时候，所有的决策变量都应该是固定了的。

运行

$ minizinc wedding.mzn

得到输出

ted bride groom rona ed carol ron alice bob bridesmaid bestman clara
----------
最终得到的座位安排在 Fig. 2.2.2 中给出。其中连线表示互相憎恶，总的鎮距是 22.

![Fig. 2.2.2: 婚礼桌上座位的安排](image)

输出项中，內建函数 `fix` 检查一个决策变量的值是否固定，然后把决策变量的实例化强制转换为参数。
MiniZinc 中的谓词允许我们用简洁的方法来表达模型中的复杂约束。MiniZinc 中的谓词利用预先定义好的全局约束建模，同时也让建模者获取以及定义新的复杂约束。MiniZinc 中的函数用来捕捉模型中的共同结构。实际上，一个谓词就是一个输出类型为 var bool 的函数。

### 2.3.1 全局约束

MiniZinc 中定义了很多可以在建模中使用的全局约束。由于全局约束的列表一直在慢慢增加，最终确定的列表可以在发布的文档中找到。下面我们讨论一些最重要的全局约束。

#### 2.3.1.1 Alldifferent

约束 alldifferent 的输入为一个变量数组，它约束了这些变量取不同的值。alldifferent 的使用有以下格式

```plaintext
alldifferent(array[int] of var int: x)
```

即，参数是一个整型变量数组。

alldifferent 是约束规划中被最多研究以及使用的全局约束之一。它被用来定义分配子问题，人们也给出了 alldifferent 的高效全局传播器。send-more-money.mzn (Listing 2.2.4) 和 sudoku.mzn (Listing 2.2.5) 是使用 alldifferent 的模型例子。
2.3.1.2 Cumulative

约束 cumulative 被用来描述资源累积使用情况。

```
cumulative(array[int] of var int: s, array[int] of var int: d,
           array[int] of var int: r, var int: b)
```

规定对于一个起始时间为 $s$，持续时间为 $d$ 以及资源需求量为 $r$ 的任务集合，在任何时间对资源的需求量都不能超过一个全局资源量界限 $b$。

Listing 2.3.1: 使用 cumulative 来建模搬运家具问题的模型 (moving.mzn).
```
include "cumulative.mzn";

enum OBJECTS;
array[OBJECTS] of int: duration; % 移动的持续时间
array[OBJECTS] of int: handlers; % 需要的搬运工的数量
array[OBJECTS] of int: trolleys; % 需要的手推车的数量
int: available_handlers;
int: available_trolleys;
int: available_time;
array[OBJECTS] of var 0..available_time: start;
var 0..available_time: end;
constraint cumulative(start, duration, handlers, available_handlers);
constraint cumulative(start, duration, trolleys, available_trolleys);
constraint forall(o in OBJECTS)(start[o] + duration[o] <= end);
solve minimize end;
output [ "start = "(start)\nend = \(end)\n"];
```

Listing 2.3.2: 使用 cumulative 来建模搬运家具问题的数据 (moving.dzn).
```
OBJECTS = { piano, fridge, doublebed, singlebed,
             wardrobe, chair1, chair2, table };
Listing 2.3.1 中的模型为搬运家具规划一个行程表使得每一份家具在搬运的过程中都有足够的搬运工和足够的手推车可以使用。允许的时间，可以使用的搬运工以及手推车被给出，每个物体的搬运持续时间，需要的搬运工和手推车的数量等数据也被给出。使用 Listing 2.3.2 中的数据，命令

```minizinc
$ minizinc moving.mzn moving.dzn
```

可能会得到如下输出

```minizinc
start = [0, 60, 60, 90, 120, 0, 15, 105]
end = 140
----------
```

Fig. 2.3.1 and Fig. 2.3.2 给出了这个解中搬运时每个时间点所需要的搬运工和手推车。

![Fig. 2.3.1: 搬运时搬运工使用量直方图](image)

### 2.3.1.3 Table

约束 table 强制变量元组从一个元组集合中取值。由于 MiniZinc 中没有元组，我们用数组来描述它。根据元组是布尔型还是整型，table 的使用有以下两种格式

#### 2.3.1. 全局约束
强制约束了 $x \in t$，其中 $x$ 和 $t$ 中的每一行是元组，$t$ 是一个元组集合。

Listing 2.3.3: 使用 table 约束来建模食物规划问题的模型 (meal.mzn).

```plaintext
% 规划均衡的膳食
include "table.mzn";
int: min_energy;
int: min_protein;
int: max_salt;
int: max_fat;
set of FOOD: desserts;
set of FOOD: mains;
set of FOOD: sides;
enum FEATURE = { name, energy, protein, salt, fat, cost};
enum FOOD;
array[FOOD,FEATURE] of int: dd; % 食物数据库

array[FEATURE] of var int: main;
array[FEATURE] of var int: side;
array[FEATURE] of var int: dessert;
var int: budget;

calendar constraint main[name] in mains;
calendar constraint side[name] in sides;
calendar constraint dessert[name] in desserts;
calendar constraint table(main, dd);
calendar constraint table(side, dd);
```
constraint table(dessert, dd);
constraint main[energy] + side[energy] + dessert[energy] >= min_energy;
constraint main[salt] + side[salt] + dessert[salt] <= max_salt;
constraint budget = main[cost] + side[cost] + dessert[cost];
solve minimize budget;

output ["main = ",show(to_enum(FOOD,main[name])),",
  ", side = ",show(to_enum(FOOD,side[name])),
  ", dessert = ",show(to_enum(FOOD,dessert[name])),
  ", cost = ",show(budget), "\n"];

Listing 2.3.4: 定义 table 的食物规划的数据 (meal.dzn).

FOODS = { icecream, banana, chocolatecake, lasagna,
  steak, rice, chips, brocolli, beans };

dd = [| icecream, 1200, 50, 10, 120, 400  % 冰淇淋
  | banana, 800, 120, 5, 20, 120  % 香蕉
  | chocolatecake, 2500, 400, 20, 100, 600  % 巧克力蛋糕
  | lasagna, 3000, 200, 100, 250, 450  % 千层面
  | steak, 1800, 800, 50, 100, 1200  % 牛排
  | rice, 1200, 50, 5, 20, 100  % 米饭
  | chips, 2000, 50, 200, 200, 250  % 薯条
  | brocolli, 700, 100, 10, 10, 125  % 花椰菜
  | beans, 1900, 250, 60, 90, 150 |];  % 黄豆

min_energy = 3300;
min_protein = 500;
max_salt = 180;
max_fat = 320;
desserts = { icecream, banana, chocolatecake }; 
 mains = { lasagna, steak, rice }; 
sides = { chips, brocolli, beans }; 

Listing 2.3.3 中的模型寻找均衡的膳食。每一个食物项都有一个名字（用整数表示），卡路里数，蛋白质克数，盐毫克数，脂肪克数以及单位为分的价钱。这些个项之间的关系用一个 table 约
束来描述。模型寻找拥有最小花费，最少卡路里数 min_energy，最少蛋白质 min_protein，最大盐分 max_salt 以及脂肪 max_fat 的膳食。

2.3.1.4 Regular

约束 regular 用来约束一系列的变量取有限自动机定义的值。regular 的使用有以下方式

```plaintext
regular(array[int] of var int: x, int: Q, int: S,
array[int,int] of int: d, int: q0, set of int: F)
```

它约束了 x 中的一列值（它们必须是在范围 range 1..S 内）被一个有 Q 个状态，输入为 1..S，转换函数为 d (<1..Q, 1..S> 映射到 0..Q），初始状态为 q0（必须在 1..Q 中）和接受状态为 F'（必须在 :mzn:`1..Q` 中）的 DFA 接受。状态 0 被保留为总是会失败的状态。

![DFA Diagram](image)

Fig. 2.3.3: 判定正确排班的 DFA。

我们来看下护士排班问题。每一个护士每一天被安排为以下其中一种：(d) 白班 (n) 夜班或者 (o) 休息。每四天，护士必须有至少一天的休息。每个护士都不可以被安排为连续三天夜班。这个问题可以使用 Fig. 2.3.3 中的不完全 DFA 来表示。我们可以把这个 DFA 表示为初始状态是 1，结束状态是 1..6，转换函数为
注意状态表中的状态 0 代表一个错误状态。Listing 2.3.5 中给出的模型为 num_nurses 个护士 num_days 天寻找一个排班，其中我们要求白天有 req_day 个护士值班，晚上有 req_night 个护士值班，以及每个护士至少有 min_night 个夜班。

Listing 2.3.5: 使用 regular 约束来建模的护士排班问题模型 (nurse.mzn)

```mzn
% 简单护士排班问题
include "regular.mzn";
enum NURSE;
enum DAY;
int: req_day;
int: req_night;
int: min_night;

enum SHIFT = { d, n, o };  
int: S = card(SHIFT);

int: Q = 6; int: q0 = 1; set of int: STATE = 1..Q;
array[STATE,SHIFT] of int: t =
[| 2, 3, 1 | % 状态 1
 | 4, 4, 1 | % 状态 2
 | 4, 5, 1 | % 状态 3
 | 6, 6, 1 | % 状态 4
 | 6, 0, 1 | % 状态 5
 | 0, 0, 1 |]; % 状态 6

array[NURSE,DAY] of var SHIFT: roster;

constraint forall(j in DAY)(
    sum(i in NURSE)(roster[i,j] == d) == req_day /
    sum(i in NURSE)(roster[i,j] == n) == req_night
```

2.3.1. 全局约束
constraint forall(i in NURSE)
    regular([roster[i,j] | j in DAY], Q, S, t, q0, STATE) \/
    sum(j in DAY)(roster[i,j] == n) >= min_night
);

solve satisfy;

output [ show(roster[i,j]) ++ if j==card(DAY) then "\n" else " " endif
    | i in NURSE, j in DAY ];

运行命令

$ minizinc nurse.mzn nurse.dzn

找到一个给 7 个护士 10 天的排班，要求白天有 3 个人值班，夜晚有 2 个人值班，以及每个护士最少有 2 个夜班。一个可能的结果是

```
od n n n o n d o o
  d o n d o d n n o n
  o d d o d o d n n o
  d d d o n d o n n
  d o d n n o d o d d
  n n o d d d o d d d
  n n o d d d o d d d
  ---------
```

另外一种 regular 约束是 regular_nfa。它使用 NFA（没有 \epsilon 矩）来定义 regular 表达式。此约束有以下格式

```
regular_nfa(array[int] of var int: x, int: Q, int: S,
        array[int,int] of set of int: d, int: q0, set of int: F)
```

它约束了数组 x 中的数值序列（必须在范围 1..S 中）被含有 Q 个状态，输入为 1..S，转换函数为 d （映射 <1..Q, 1..S> 到 1..Q 的子集），初始状态为 q0 （必须在范围 1..Q 中）以及接受状态为 F （必须在范围 1..Q 中的 NFA 接受。在这里，我们没必要再给出失败状态 0，因为转换函数可以映射到一个状态的空集。
2.3.2 定义谓词

MiniZinc 的其中一个最强大的建模特征是建模者可以定义他们自己的高级约束。这就使得他们可以对模型进行抽象化和模块化。也允许了在不同的模型之间重新利用约束以及促使了用来定义标准约束和类型的特殊库应用的发展。

Listing 2.3.6: 使用谓词的车间作业调度问题模型 (jobshop2.mzn)

```plaintext
int: jobs;                             % 作业的数量
set of int: JOB = 1..jobs;
int: tasks;                            % 每个作业的任务数量
set of int: TASK = 1..tasks;
array [JOB,TASK] of int: d;            % 任务持续时间
int: total = sum(i in JOB, j in TASK)(d[i,j]); % 总持续时间
int: digs = ceil(log(10.0,total));     % 输出的数值
array [JOB,TASK] of var 0..total: s;   % 起始时间
var 0..total: end;                     % 总结束时间

% nooverlap
predicate no_overlap(var int:s1, int:d1, var int:s2, int:d2) =
    s1 + d1 <= s2 \ s2 + d2 <= s1;

constraint %% 保证任务按照顺序出现
    forall(i in JOB) (  
        forall(j in 1..tasks-1)  
            (s[i,j] + d[i,j] <= s[i,j+1]) \  
            s[i,tasks] + d[i,tasks] <= end  
    );

constraint %% 保证任务之间没有重叠
    forall(j in TASK) (  
        forall(i,k in JOB where i < k) (  
            no_overlap(s[i,j], d[i,j], s[k,j], d[k,j])  
        )  
    );

solve minimize end;

output ["end = \(end)\n"] ++
```

2.3.2. 定义谓词
我们用一个简单的例子开始，回顾下前面章节中的车间作业调度问题。这个模型在 Listing 2.3.6 中给出。我们感兴趣的项是 **谓词** 项:

```mini
predicate no_overlap(var int:s1, int:d1, var int:s2, int:d2) =
    s1 + d1 <= s2 \/ s2 + d2 <= s1;
```

它定义了一个新的约束用来约束起始时间为 \( s_1 \)，持续时间为 \( d_1 \) 的任务不能和起始时间为 \( s_2 \)，持续时间为 \( d_2 \) 的任务重叠。它可以在模型的任何（包含决策变量的）布尔型表达式可以出现的地方使用。

和谓词一样，建模者也可以定义只涉及到参数的新的约束。和谓词不一样的是，它们可以被用在条件表达式的测试中。它们被关键字 **test** 定义。例如

```mini
test even(int:x) = x mod 2 = 0;
```

### 谓词定义

使用以下形式的语句，我们可以定义谓词

```mini
predicate <谓词名> ( <参数定义>, ..., <参数定义> ) = <布尔表达式>
```

其中的**<布尔表达式>** 必须是固定的。

另外我们介绍一个谓词中使用到的 **assert** 命令的新形式。

```mini
assert ( <布尔表达式>, <字符串表达式>, <表达式> )
```

**assert** 表达式的类型和最后一个参数的类型一样。**assert** 表达式检测第一个参数是否为假，如果是则输出第二个参数字符串。如果第一个参数是真，则输出第三个参数。

注意 **assert** 表达式中的第三个参数是延迟的，即如果第一个参数是假，它就不会被评估。所以
它可以被用来检查

```plaintext
predicate lookup(array[int] of var int:x, int: i, var int: y) =
    assert(i in index_set(x), "index out of range in lookup"
    y = x[i]
)
```

此代码在 i 超出数组 x 的范围时不会计算 x[i]。

### 2.3.3 定义函数

MiniZinc 中的函数和谓词一样定义，但是它有一个更一般的返回类型。

下面的函数定义了一个数独矩阵中的第 $a^{th}$ 个子方块的第 $a1^{th}$ 行。

```plaintext
function int: posn(int: a, int: a1) = (a-1) * S + a1;
```

有了这个定义之后，我们可以把 Listing 2.2.5 中的数独问题的最后一个约束替换为

```plaintext
constraint forall(a, o in SubSquareRange)(
    alldifferent([ puzzle [ posn(a,a0), posn(o,o1) ] |
        a1,o1 in SubSquareRange ] ));
```

函数对于描述模型中经常用到的复杂表达式非常有用。例如，想象下在 $n \times n$ 的方格的不同位置上放置数字 1 到 $n$ 使得任何两个数字 $i$ 和 $j$ 之间的曼哈顿距离比这两个数字其中最大的值减一还要大。我们的目的是最小化数组对之间的总的曼哈顿距离。曼哈顿距离函数可以表达为:

```plaintext
function var int: manhattan(var int: x1, var int: y1,
    var int: x2, var int: y2) =
    abs(x1 - x2) + abs(y1 - y2);
```

完整的模型在 Listing 2.3.7 中给出。

```plaintext
Listing 2.3.7: 例释如何使用函数的数字放置问题模型 (manhattan.mzn).
int: n;
set of int: NUM = 1..n;
```
array[NUM] of var NUM: x;
array[NUM] of var NUM: y;
array[NUM,NUM] of var 0..2*n-2: dist =
array2d(NUM,NUM,[]
    if i < j then manhattan(x[i],y[i],x[j],y[j]) else 0 endif
    | i,j in NUM ]);

% manf
function var int: manhattan(var int: x1, var int: y1,
    var int: x2, var int: y2) =
    abs(x1 - x2) + abs(y1 - y2);

constraint forall(i,j in NUM where i < j)
    (dist[i,j] >= max(i,j)-1);

var int: obj = sum(i,j in NUM where i < j)(dist[i,j]);
solve minimize obj;

% 简单地显示结果
include "alldifferent_except_0.mzn";
array[NUM,NUM] of var 0..n: grid;
constraint forall(i in NUM)(grid[x[i],y[i]] = i);
constraint alldifferent_except_0([grid[i,j] | i,j in NUM]);

output ["obj = 
(obj);\n"] ++
    [ if fix(grid[i,j]) > 0 then show(grid[i,j]) else "." endif
    ++ if j = n then "\n" else " " endif
    | i,j in NUM ];

函数定义

函数用以下格式的语句定义

```
function <返回类型> : <函数名> ( <参数定义>, ... , <参数定义> ) =
    <表达式>
```

<函数名> 必须是一个合法的 MiniZinc 标识符。每一个 <参数定义> 是一个合法的 MiniZinc 类型声明。<返回类型> 是函数的返回类型，它必须是 <表达式> 的类型。参数和谓词定义中的参数有一样的限制。
MiniZinc 中的函数可以有任何返回类型，而不只是固定的返回类型。在定义和记录多次出现在模型中的复杂表达式时，函数是非常有用的。

### 2.3.4 反射函数

为了方便写出一般性的测试和谓词，各种反射函数会返回数组的下标集合，var 集合的定义域以及决策变量范围的信息。关于下标集合的有以下反射函数 index_set(<1-D array>)，index_set_1of2(<2-D array>)，index_set_2of2(<2-D array>)，以及关于更高维数组的反射函数。

车间作业问题的一个更好的模型是把所有的对于同一个机器上的不重叠约束结合为一个单个的析取约束。这个方法的一个优点是虽然我们只是初步地把它建模成一个 non-overlap 约束的连接，但是如果不求解器对于解决析取约束有一个更好的方法，在对我们的模型最小改变的情况下，我们可以直接使用它。这个模型在 Listing 2.3.8 中给出。

#### Listing 2.3.8: 使用 disjunctive 谓词的车间作业调度问题模型 (jobshop3.mzn).

```plaintext
include "disjunctive.mzn";

int: jobs; % 作业的数量
set of int: JOB = 1..jobs;
int: tasks; % 每个作业的任务数量
set of int: TASK = 1..tasks;
array [JOB,TASK] of int: d; % 任务持续时间
int: total = sum(i in JOB, j in TASK)(d[i,j]);% 总持续时间
int: digs = ceil(log(10.0,total)); % 输出的数值
array [JOB,TASK] of var 0..total: s; % 起始时间
var 0..total: end; % 总结束时间

constraint %% 保证任务按照顺序出现
forall(i in JOB) ( %
  forall(j in 1..tasks-1)
    (s[i,j] + d[i,j] <= s[i,j+1]) /\ %
    s[i,tasks] + d[i,tasks] <= end %
  );

constraint %% 保证任务之间没有重叠
forall(j in TASK) ( %
  disjunctive([s[i,j] | i in JOB], [d[i,j] | i in JOB]) %
)
```

#### 2.3.4 反射函数

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约束 disjunctive 获取每个任务的开始时间数组以及它们的持续时间数组，确保每次只有一个任务是被激活的。我们定义析取约束为一个有以下特征的谓词

```plaintext
predicate disjunctive(array[int] of var int:s, array[int] of int:d);
```

在 Listing 2.3.8 中，我们可以用这个析取约束定义任务之间不重叠。我们假设 disjunctive 谓词的定义已经在模型中引用的文件 disjunctive.mzn 中给出。

如果下层的系统直接支持 disjunctive，则会在它的全局目录下包含一个 disjunctive.mzn 文件（拥有上述特征定义内容）。如果我们使用的系统不直接支持析取，通过创建文件 disjunctive.mzn，我们可以给出我们自己的定义。最简单的实现是单独使用上面定义的 no_overlap 谓词。一个更好的实现是利用全局约束 cumulative，假如下层求解器支持它的话。Listing 2.3.9 给出了一个 disjunctive 的实现。注意我们使用 index_set 反射函数来（a）检查 disjunctive 的参数是有意义的，以及（b）构建 cumulative 的合适大小的资源利用数组。另外注意这里我们使用了 assert 的三元组版本。
### 2.3.5 局部变量

在谓词，函数或者测试中，引进 **局部变量** 总是非常有用的。表达式 let 允许你去这样做。它可以被用来引进决策变量 决策变量和 参数，但是参数必须被初始化。例如：

```plaintext
var s..e: x;
let {int: l = s div 2; int: u = e div 2; var l .. u: y;} in x = 2*y
```

引进了参数 l 和 u 以及变量 y。let 表达式虽然在 谓词，函数和测试定义中最有用，它也可以被用在其他的表达式中。例如，来消除共同的子表达式：

```plaintext
constraint let { var int: s = x1 + x2 + x3 + x4 } in
   l <= s /\ s <= u;
```

局部变量可以被用在任何地方，在简化复杂表达式时也很有用。通过使用局部变量来定义目标 function 而不是显式地加入很多个变量，Listing 2.3.10 给出了稳定婚姻模型的一个改进版本。

#### Listing 2.3.10: 使用局部变量来定义一个复杂的目标函数 (wedding2.mzn).

```plaintext
enum Guests = { bride, groom, bestman, bridesmaid, bob, carol,
               ted, alice, ron, rona, ed, clara};
set of int: Seats = 1..12;
set of int: Hatreds = 1..5;
array[Hatreds] of Guests: h1 = [groom, carol, ed, bride, ted];
array[Hatreds] of Guests: h2 = [clara, bestman, ted, alice, ron];
set of Guests: Males = {groom, bestman, bob, ted, ron, ed};
```
set of Guests: Females = {bride, bridesmaid, carol, alice, rona, clara};

array[Guests] of var Seats: pos; % 客人的座位

include "alldifferent.mzn";
constraint alldifferent(pos);

constraint forall(g in Males) ( pos[g] mod 2 == 1 );
constraint forall(g in Females) ( pos[g] mod 2 == 0 );

constraint not (pos[ed] in {1, 6, 7, 12});
constraint abs(pos[bride] - pos[groom]) <= 1 /
    (pos[bride] <= 6 <-> pos[groom] <= 6);

solve maximize sum(h in Hatreds)(
    let { var Seats: p1 = pos[h1[h]]; 
    var Seats: p2 = pos[h2[h]]; 
    var 0..1: same = bool2int(p1 <= 6 <-> p2 <= 6); } in 
    same * abs(p1 - p2) + (1-same) * (abs(13 - p1 - p2) + 1));

output [ show(g)++" " | s in Seats, g in Guests where fix(pos[g]) == s] 
    ++ ["\n"];

### 2.3.6 语境

有一个局限，即含有决策变量并且在声明时没有初始化的谓词和函数不可以被用在一个否定语境下。下面例子是非法的

```mzn
predicate even(var int:x) =
    let { var int: y } in x = 2 * y;

constraint not even(z);
```

原因是求解器只解决存在约束的问题。如果我们在否定语境下引入了一个局部变量，则此变量是 普遍地量化了，因此超出下层求解器的解决范围。例如，\neg even(z) 等价于 \neg\exists y. z = 2y ，然后等价于 \forall y. z \neq 2y 。

如果局部变量被赋了值，则它们可以被用在否定语境中。下面的例子是合法的

```mzn
predicate even(var int:x) =
    let { var int: y } in x = 2 * y;

constraint not even(z);
```
注意，现在 even 的意思是正确的，因为如果 x 是偶数，则 x = 2*(x div 2)。由于 math:y 被 z 功能性定义了，¬even(z) 等价于 ¬∃y. y = z div 2 ∧ z = 2y ，同时等价于 ∃y. y = z div 2 ∧ ¬z # 2y。

MiniZinc 中的任意表达式都在以下四种语境中的一种中：根，肯定，否定，或者 混合。非布尔型表达式的语境直接地为包含其最近的布尔型表达式的语境。唯一的例外是目标表达式出现在一个根语境下（由于它没有包含其的布尔型表达式）。

为了方便定义语境，我们把蕴含表达式 e1 -> e2 等价地写为 not e1 \/ e2, e1 <- e2 等价地写为 e1 \/ not e2。

一个布尔型表达式的语境可能有：

**根** 根语境是任何作为 constraint 的参数或者作为一个赋值项 assignment 出现的表达式 e 的语境，或者作为一个出现在根语境中的 e1 ∧ e2 的子表达式 e1 或 e2 的语境。

根语境下的布尔型表达式必须在问题的任何模型中都满足。

**肯定** 肯定语境是任何作为一个出现在根语境或者肯定语境中的 e1 ∧ e2 的子表达式 e1 或 e2 的语境，或者是一个出现在肯定语境中的 e1 ∧ e2 的子表达式 e1 或 e2 的语境，或者是作为一个出现在肯定语境中的 :mzn:`not e` 的子表达式 e 的语境。

肯定语境下的布尔型表达式不必要在模型中满足，但是满足它们会增加包含其的约束被满足的可能性。对于一个肯定语境下的表达式，从包含其的根语境到此表达式有偶数个肯定。

**否定** 否定语境是任何作为一个出现在根语境或者否定语境中的 e1 ∧ e2 或 e1 \/ e2 的子表达式 e1 或 e2，或者是一个出现在否定语境中的 not e 的子表达式 e 的语境。

否定语境下的布尔型表达式不必要满足，但是让它们成假会增加包含其的约束被满足的可能性。对于一个否定语境下的表达式，从包含其的根语境到此表达式有奇数个否定。

**混合** 混合语境是任何作为一个出现在 e1 <- e2, e1 = e2 或者 bool2int(e) 中的子表达式 e1 或 e2 的语境。

混合语境下的表达式实际上既是肯定也是否定的。通过以下可以看出：e1 <- e2 等价于 (e1 ∧ e2) \( (not e1 ∧ not e2) \) 以及 x = bool2int(e) 等价于 (e \( x=1) \) \( (not e ∧ x=0) \)。
constraint \( x > 0 \) /\ (i <= 4 -> x + bool2int(x > i) = 5); 

其中 \( x > 0 \) 在根语境中，\( i <= 4 \) 在否定语境中，\( x + \text{bool2int}(x > i) = 5 \) 在肯定语境中，\( x > i \) 在混合语境中。

### 2.3.7 局部约束

**Let** 表达式也可以被用来引入局部约束，通常用来约束局部变量的行为。例如，考虑只利用乘法来定义开根号函数:

```plaintext
function var float: mysqrt(var float:x) =
  let { var float: y;
    constraint y >= 0;
    constraint x = y * y; } in y;

局部约束确保了 \( y \) 取正确的值，而此值则会被函数返回。

局部约束可以在 **Let** 表达式中使用，尽管最普遍的应用是在定义函数时。

#### Let 表达式

局部变量可以在任何以下格式的 **emph(let 表达式)** 中引入:

```plaintext
let { <声明>; ... <声明>; } in <表达式>
```

声明 **<dec>** 可以是决策变量或者参数（此时必须被初始化）或者约束项的声明。任何声明都不能在一个新的声明变量还没有引进时使用它。

注意局部变量和约束不可以出现在测试中。局部变量不可以出现在否定或者混合语境下的谓词和函数中，除非这个变量是用表达式定义的。

### 2.3.8 定义域反射函数

其他重要的反射函数有允许我们对变量定义域进行访问的函数。表达式 **lb(x)** 返回一个小于等于 \( x \) 在一个问题的解中可能取的值的数值。通常它会是 \( x \) 声明的下限。如果 \( x \) 被声明为一个非有限类型，例如，只是 **var int**，则它是错误的。类似地，表达式 **dom(x)** 返回一个 **SxS** 在问题的任何解中的可能值的（非严格）超集。再次，它通常是声明的值，如果它不是被声明为有限则会出现错误。
**Listing 2.3.11:** 使用反射谓词 (reflection.mzn).

```mini
var -10..10: x;
constraint x in 0..4;
int: y = lb(x);
set of int: D = dom(x);
solve satisfy;
output ["y = ", show(y), 
\nD = ", show(D), 
\n];
```

例如，**Listing 2.3.11** 中的模型或者输出

```
y = -10
D = -10..10
----------
```

或

```
y = 0
D = {0, 1, 2, 3, 4}
----------
```

或任何满足 \(-10 \leq y \leq 0\) 和 \(\{0, \ldots, 4\} \subseteq D \subseteq \{-10, \ldots, 10\}\) 的答案。

变量定义域反射表达式应该在任何安全近似下都正确的使用。但是注意这个是没有被检查的！例如加入额外的代码

```mini
var -10..10: z;
constraint z <= y;
```

不是一个定义域信息的正确应用，因为使用更紧密（正确的）近似会比使用更弱的初始近似产生更多的解。

### 2.3.8. 定义域反射函数

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定义域反射

我们有查询包含变量的表达式的可能值的反射函数：
- `dom(<表达式>)` 返回 `<表达式>` 所有可能值的安全近似。
- `lb(<表达式>)` 返回 `<表达式>` 下限值的安全近似。
- `ub(<表达式>)` 返回 `<表达式>` 上限值的安全近似。

`lb` 和 `ub` 的表达式必须是 `int`, `bool`, `float` 或者 `set of int` 类型。`dom` 中表达式的类型不能是 `float`。如果 `<表达式>` 中的一个变量有一个非有限声明类型（例如, `var int` 或 `var float` 类型），则会出一个错误。

我们也有直接作用于表达式数组的版本（有类似的限制）：
- `dom_array(<数组表达式>)`：返回数组中出现的表达式的所有可能值的并集的一个安全近似。
- `lb_array(<数组表达式>)` 返回数组中出现的所有表达式的下限的安全近似。
- `ub_array(<数组表达式>)` 返回数组中出现的所有表达式的下限的安全近似。

谓词，局部变量和定义域反射的结合使得复杂全局约束通过分解定义变为可能。利用 Listing 2.3.12 中的代码，我们可以定义 cumulative 约束根据时间的分解。

Listing 2.3.12: 利用分解来定义一个谓词 (cumulative.mzn).

```zinc
%--------------------------------------------------------------------%
% 需要给出一个任务集合，其中起始时间为's'
% 持续时间'd' 以及资源需求量'r'
% 任何时候资源量都不能超过
% 一个全局资源极限'b'。
% 假设:
% - forall i, d[i] >= 0 and r[i] >= 0
%--------------------------------------------------------------------%
predicate cumulative(array[int] of var int: s,
                        array[int] of var int: d,
                        array[int] of var int: r, var int: b) =
  assert(index_set(s) == index_set(d) /
        index_set(s) == index_set(r),
        "cumulative: the array arguments must have identical index sets",
  assert(lb_array(d) >= 0 /
        lb_array(r) >= 0,
        "cumulative: durations and resource usages must be non-negative",
  let {
    set of int: times =
    lb_array(s) ..
    max([ ub(s[i]) + ub(d[i]) | i in index_set(s) ]) });
```

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MiniZinc Handbook, Release 2.4.3

```plaintext
Listing 2.3.13: 阐述变量作用域的模型 (scope.mzn).

int: x = 3;
int: y = 4;
predicate smallx(var int:y) = -x <= y \&\& y <= x;
predicate p(int: u, var bool: y) =
    exists(x in 1..u)(y \&\& smallx(x));
constraint p(x,false);
solve satisfy;
```

2.3.9 作用域

MiniZinc 中声明的作用域值得我们简单地介绍下。MiniZinc 只有一个作用域，所以出现在声明中的所有变量都可以在模型中的每个表达式中可见。用以下几个方式，MiniZinc 引进局部作用域变量:

- 推导式表达式中的迭代器
- 使用 let 表达式
- 谓词和函数中的参数

任何局部作用域变量都会覆盖同名称的外部作用域变量。例如，在 Listing 2.3.13 中给出的模型中，-x <= y 中的 x 是全局 x , smallx(x) 中的 x 是迭代器 x in 1..u ，而析取中的 y 是谓词的第二个参数。

这个分解利用 lb 和 ub 来决定任务可以执行的时间范围集合。接下来，它对 times 中的每个时间 times 都断言在此时间 t 激活的所有任务所需要的资源量总和小于界限 b。
CHAPTER 2.4

选项类型

选项类型是一个强大的抽象，使得简洁建模成为可能。一个选项类型决策变量代表了一个有其他可能 T 的变量，在 MiniZinc 中表达为 <>，代表了这个变量是 缺失的。选项类型变量在建模一个包含在其他变量没做决定之前不会有意义的变量的问题时是很有用的。

2.4.1 声明和使用选项类型

<table>
<thead>
<tr>
<th>选项类型变量</th>
</tr>
</thead>
<tbody>
<tr>
<td>一个选项类型变量被声明为：</td>
</tr>
<tr>
<td>var opt &lt;类型&gt; : &lt;变量名&gt;；</td>
</tr>
</tbody>
</table>

其中 <类型> 是 int，float 或 bool 中的一个，或者是一个固定范围的表达式。选项类型变量可以是参数，但是这个不常用。
一个选项类型变量可以有附加值 <> 表明它是 缺失的。
三个内建函数被提供给选项类型变量：absent(ν) 只有在选项类型变量 ν 取值 <> 时返回 true，occurs(ν) 只有在选项类型变量 ν 不取值 <> 时返回 true，以及 <> 返回 ν 的正常值或者当它取值 <> 时返回失败。

选项类型最常被用到的地方是调度中的可选择任务。在灵活的车间作业调度问题中，我们有 n 个在 k 个机器上执行的任务，其中完成每个机器上每个任务的时间可能是不一样的。我们的目标是最小化所有任务的总完成时间。一个使用选项类型来描述问题的模型在 Listing 2.4.1 中给
在建模这个问题的时候，我们使用 $n \times k$ 个可选择的任务来代表每个机器上每个任务的可能性。我们使用 alternative 全局约束来要求任务的起始时间和它的持续时间跨越了组成它的可选择任务的时间，同时要求只有一个会实际运行。我们使用 disjunctive 全局变量在每个机器上最多有一个任务在运行，这里我们延伸到可选择的任务。最后我们约束任何时候最多有 $Sk$ 个任务在运行，利用一个在实际（不是可选择的）任务上作用的冗余约束。

### Listing 2.4.1: 使用选项类型的灵活车间作业调度模型 (flexible-js.mzn).

```mzn
int: horizon;                   % 时间范围
set of int: Time = 0..horizon;
enum Task;
enum Machine;

array[Task,Machine] of int: d; % 每个机器上的持续时间
int: maxd = max([ d[t,m] | t in Task, m in Machine ]); 
int: mind = min([ d[t,m] | t in Task, m in Machine ]); 

array[Task] of var Time: S;     % 起始时间
array[Task] of var mind..maxd: D; % 持续时间
array[Task,Machine] of var opt Time: O; % 可选择的任务起始

constraint forall(t in Task)(alternative(S[t],D[t], 
    [O[t,m]|m in Machine],[d[t,m]|m in Machine]));
constraint forall(m in Machine)  
    (disjunctive([O[t,m]|t in Task],[d[t,m]|t in Task]));
constraint cumulative(S,D,[1]|i in Task),k;
solve minimize max(t in Task)(S[t] + D[t]);
```

### 2.4.2 隐藏选项类型

当列表推导式是从在变量集合迭代上创建而来，或者 where 从句中的表达式还没有固定时，选项类型变量会隐式地出现。

例如，模型片段

```mzn
var set of 1..n: x;
constraint sum(i in x)(i) <= limit;
```
是以下的语法糖

```mini
var set of 1..n: x;
constraint sum(i in 1..n)(if i in x then i else <> endif) <= limit;
```

内建函数 sum 实际上在一列类型-实例化 var opt int 上操作。由于 <> 在 + 中表现为标识 0，我们会得到期望的结果。

类似地，模型片段

```mini
array[1..n] of var int: x;
constraint forall(i in 1..n where x[i] >= 0)(x[i] <= limit);
```

是以下的语法糖

```mini
array[1..n] of var int: x;
constraint forall(i in 1..n)(if x[i] >= 0 then x[i] <= limit else <> endif);
```

同样地，函数 forall 实际上在一列类型-实例化 var opt bool 上操作。由于 <> 在 && 上表现为标识 true，我们可以得到期望的结果。

尽管我们已经很小心了，隐式的使用可能会导致意外的行为。观察

```mini
var set of 1..9: x;
constraint card(x) <= 4;
constraint length([ i | i in x]) > 5;
solve satisfy;
```

它本应该是一个不可满足的问题。它返回 x = {1,2,3,4} 作为一个解例子。这个是正确的因为第二个约束等于

```mini
constraint length([ if i in x then i else <> endif | i in 1..9 ]) > 5;
```

而可选择整数列表的长度总会是 9，所以这个约束总是会满足。

我们可以通过不在变量集合上创建迭代或者使用不固定的 where 从句来避免隐式的选项类型。例如，上面的两个例子可以不使用选项类型重写为

```mini
var set of 1..n: x;
constraint sum(i in 1..n)(bool2int(i in x)*i) <= limit;
```

2.4.2. 隐藏选项类型
array[1..n] of var int: x;
constraint forall(i in 1..n)(x[i] >= 0 -> x[i] <= limit);
MiniZinc 默认没有我们想如何搜索解的声明。这就把搜索全部都留给下层的求解器了。但是有些时候，尤其是对组合整数问题，我们或许想规定搜索应该如何去进行。这就需要我们和求解器沟通出一个搜索策略 search。注意，搜索策略不应是模型的一部分。实际上，我们不要求每个求解器把所有可能的求解策略都实现了。MiniZinc 通过使用 annotations 来用一个稳定的方法跟约束求解器沟通额外的信息。

### 2.5.1 有限域搜索

利用有限域求解器搜索涉及到检查变量剩余的可能值以及选择进一步地约束一些变量。搜索则会加一个新的约束来限制这个变量的剩余值（实际上猜测解可能存在于哪里），然后使用传播来确定其他的值是否可能存在于解中。为了确保完全性，搜索会保留另外一个个选择，而它是新约束的否定。搜索会当有限域求解器发现所有的约束都被满足，此时一个解已经被找到，或者有约束不被满足时停止。当不可满足出现的时候，搜索必须换另外一个不同的选择集合继续下去。通常有限域求解器使用深度优先搜索，它会撤销最后一个做的选择然后尝试做一个新的选择。

**Listing 2.5.1: n 皇后问题模型 (nqueens.mzn).**

```plaintext
int: n;
array [1..n] of var 1..n: q; % i 列的皇后在行 q[i]

include "alldifferent.mzn";
```
有限域问题的一个简单例子是 n 皇后问题，它要求我们放置 n 个皇后在 n × n 棋盘上使得任何两个都不会互相攻击。变量 q[i] 记录了在 i 列的皇后放置在哪一行上。alldifferent 约束确保了任何两个皇后都不会在同一行或者对角线上。Fig. 2.5.1 的左边给出了 n = 9 的典型（部分）搜索树。我们首先设置 q[1] = 1，这样就可以从其他变量的定义域里移除一些数值，例如 q[2] 不能取值 1 或者 2，我们接下来设置 q[2] = 3，然后进一步地从其他变量的定义域里移除一些数值。我们设置 q[3] = 5（它最早的可能值）。在这三个决策后，棋盘的状态显示为 Fig. 2.5.2。其中皇后表示已经固定的皇后位置。星星表示此处放置的皇后会攻击到已经放置的皇后，所以我们不能在此处放置皇后。


很多有限域搜索被定义为这种方式：选择一个变量来进一步约束，然后选择如何进一步地约束它。

### 2.5.2 搜索注解

MiniZinc 中的搜索注解注解了为了找到一个问题的解应如何去搜索。注解附在求解例，在关键字 solve 之后。搜索注解：

```plaintext
constraint alldifferent(q); % 不同行
constraint alldifferent([ q[i] + i | i in 1..n]); % 不对角线
constraint alldifferent([ q[i] - i | i in 1..n]); % 上 + 下

% 搜索
solve :: int_search(q, first_fail, indomain_min, complete)
satisfy;
output [ if fix(q[j]) == i then "Q" else "." endif ++
  if j == n then "\n" else " " endif | i,j in 1..n]
```
Fig. 2.5.1: 9 皇后问题的部分搜索树
Fig. 2.5.2: In the state $q[1] = 1$, $q[2] = 4$, $q[3] = 5$. 


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Fig. 2.5.3: 在进一步加入 q[6] = 4 后的初始传播
出现在求解项中。注解使用连接符 :: 附为模型的一部分。这个搜索注解意思是我们应该按照从
整型变量数组 q 中选择拥有最小现行定义域的变量（这个是 first_fail 规则），然后尝试设置
其为它的最小可能值 (indomain_min 值选择)，纵观整个搜索树来搜索 (complete 搜索)。

### 基本搜索注解

我们有三个基本搜索注解，相适应于不同的基本搜索类型:

- int_search( <变量>, <变量选择>, <约束选择>, <策略> ) 其中 <变量> 是一个
  var int 类型的一维数组，<变量选择> 是一个接下来会讨论的变量选择注解，
  <约束选择> 是一个接下来会讨论的如何约束一个变量的选择，<策略> 是一个搜索
  策略，我们暂时假设为 complete
- bool_search( <变量>, <变量选择>, <约束选择>, <策略> ) 其中 <变量> 是一个
  var bool 类型的一维数组，剩余的和上面一样。
- set_search( <变量>, <变量选择>, <约束选择>, <策略> ) 其中 <变量> 是一个
  var set of int 类型的一维数组，剩余的和上面一样。
- float_search( <变量>, <精度>, <变量选择>, <约束选择>, <策略> ) 其中
  <变量> 是一个一维 var float 数组，<precision> 是一个固定的用于表示 ε 浮点数,
  其中两个数之差低于这个浮点数时被认为相等。剩余的和上面一样。

变量选择注解的例子有:

- input_order: 从数组中按照顺序选择
- first_fail: 选择拥有最小定义域大小的变量，以及
- smallest: 选择拥有最小值的变量。

约束一个变量的方式有:

- indomain_min: 赋最小的定义域内的值给变量。
- indomain_median: 赋定义域内的中间值给变量。
- indomain_random: 从定义域中取一个随机的值赋给变量，以及
- indomain_split 把变量定义域一分为二然后去除掉上半部分。

对于完全搜索，<策略> 基本都是 complete。关于一份完整的变量和约束选择注解，请参
看 MiniZinc 参考文档中的 FlatZinc 说明书。

利用搜索构造注解，我们可以创建更加复杂的搜索策略。目前我们只有一个这样的注解。

```plaintext
seq_search([ [ <搜索注解>, ..., <搜索注解> ]])
```

顺序搜索构造首先执行对列表中的第一个注解所指定的变量的搜索，当这个注解中的所有的变
量都固定后，它执行第二个搜索注解，等等。直到所有的搜索注解都完成。
我们来看一下 Listing 2.3.8 中给出的车间作业调度模型。我们可以替换成解项为

```plaintext
solve :: seq_search([ 
    int_search(s, smallest, indomain_min, complete),
    int_search([end], input_order, indomain_min, complete)])
minimize end
```

通过选择可以最早开始的作业并设置其为 s，起始时间被设置为 s。当所有的起始时间都设置完后，终止时间 end 或许还没有固定。因此我们设置其为它的最小可能取值。

### 2.5.3 注解

在 MiniZinc 中，注解是第一类对象。我们可以在模型中声明新的注解，以及声明和赋值给注解变量。

#### 注解

注解有一个类型 `ann`。你可以声明一个注解参数 `parameter`（拥有可选择的赋值）：

```plaintext
ann : <标识符>;
ann : <标识符> = <注解表达式> ;
```

对注解变量赋值和对其他参数赋值一样操作。

表达式、变量声明，和 `solve` 项都可以通过使用 `::` 操作符来成为注解。

使用注解项 `annotation`，我们可以声明一个新的注解 `annotation` 项：

```plaintext
annotation <注解名> ( <参数定义>, ... , <参数定义> ) ;
```

#### Listing 2.5.2: n 皇后问题的注解模型 (nqueens-ann.mzn).

```plaintext
annotation bitdomain(int:nwords);

include "alldifferent.mzn";

int: n;
array [1..n] of var 1..n: q :: bitdomain(n div 32);

constraint alldifferent(q) :: domain;
constraint alldifferent([ q[i] + i | i in 1..n]) :: domain;
```
constraint alldifferent([ q[i] - i | i in 1..n]) :: domain;

ann: search_ann;

solve :: search_ann satisfy;

output [ if fix(q[j]) == i then "Q" else "." endif ++
  if j == n then "\n" else " " endif | i,j in 1..n]

Listing 2.5.2 中的程序阐述了注解声明，注解和注解变量的使用。我们声明一个新的注解 bitdomain，意思是来告诉求解器变量定义域应该通过大小为 nwords 的比特数组来表示。注解附注在变量 q 的声明之后。每一个 alldifferent 约束都被注解为内部注解 domain，而它指导求解器去使用 alldifferent 的定义域传播版本（如果有的话）。一个注解变量 search_ann 被声明和使用来定义搜索策略。我们可以在一个单独的数据文件中来给出搜索策略的值。

搜索注解的例子或许有以下几种（我们假设每一行都在一个单独的数据文件中）

search_ann = int_search(q, input_order, indomain_min, complete);
ssearch_ann = int_search(q, input_order, indomain_median, complete);
ssearch_ann = int_search(q, first_fail, indomain_min, complete);
ssearch_ann = int_search(q, first_fail, indomain_median, complete);

第一个只是按顺序来选择皇后然后设置其为最小值。第二个按顺序来选择皇后，但是设置中间值给出。第三个选择定义域大小最小的皇后，然后设置最小值给出。最后一个策略选择定义域大小最小的皇后，设置中间值给出。

不同的搜索策略对于能多容易找到解有显著的差异。下面的表格给出了一个简单的关于使用 4 种不同的搜索策略找到 n 皇后问题的第一个解所要做的决策个数（其中一表示超过 100,000 个决策）。很明显地看到，合适的搜索策略会产生显著的提高。

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</tbody>
</table>
对同一个问题，几乎总是存在多种方式来建模。其中一些产生的模型可以很有效地求解，另外一些则不是。通常情况下，我们很难提前判断哪个模型是对解决一个特定的问题最有效的。事实上，这或许十分依赖于我们使用的底层求解器。在这一章中，我们专注于建模实践，来避免产生模型的过程和产生的模型低效。

### 2.6.1 变量界限

有限域传播器，是 MiniZinc 所针对的求解器中的核心类型。在当其所涉及到的变量的界限越紧凑时，此传播器越有效。它也会当问题含有会取很大整型数值的子表达式时表现得很差，因为它可能也会导致模型整型变量的大小。

#### Listing 2.6.1: 没有无界整数的模型 (grocery.mzn).

```plaintext
var int: item1;
var int: item2;
var int: item3;
var int: item4;

constraint item1 + item2 + item3 + item4 == 711;
constraint item1 * item2 * item3 * item4 == 711 * 100 * 100 * 100;
```
constraint  0 < item1 \ item1 <= item2 \
\ item2 <= item3 \ item3 <= item4;

solve satisfy;

output ["\n", show(item1), ",", show(item2), ",", show(item3), ",", show(item4), "]\n];

在 Listing 2.6.1 中的中的杂货店问题要找寻 4 个物品使得它们的价格加起来有 7.11 元并且乘起来也有 7.11 元。变量被声明为无界限。运行

```
$ minizinc --solver g12fd grocery.mzn
```

得到

```
=====UNSATISFIABLE===== 
% /tmp/mznfile85EWzj.fzn:11: warning: model inconsistency detected before _
|_search.
```

这是因为乘法中的中间表达式的类型也会是 var int ，也会被求解器给一个默认的界限 −1,000,000...1,000,000 。但是这个范围太小了以至于不能承载住中间表达式所可能取的值。

更改模型使得初始变量都被声明为拥有更紧致的界限

```
var 1..711: item1;
var 1..711: item2;
var 1..711: item3;
var 1..711: item4;
```

我们得到一个更好的模型，因为现在 MiniZinc 可以推断出中间表达式的界限，并且使用此界限而不是默认的界限。在做此更改后，求解模型我们得到

```
{120,125,150,316}
----------
```

注意，就算是改善的模型也可能对于某些求解器来说会很难解决。运行

```
$ minizinc --solver g12lazy grocery.mzn
```
不能得到任何结果，因为求解器给中间产生的变量创建了巨大的表示。

### 给变量加界限
在模型中要尽量使用有界限的变量。当使用let声明来引进新的变量时，始终尽量给它们定义正确的和紧凑的界限。这会使得你的模型更有效率，避免出现意外溢出的可能性。一个例外是当你引进一个新的变量然后立刻定义它等于一个表达式，通常MiniZinc都可以从此表达式推断出此变量有效的界限。

### 2.6.2 有效的生成元

想象下我们想要计算在一个图中出现的三角形的个数（K₃子图）。假设此图由一个邻接矩阵定义：如果点Sᵢ$和Sⱼ$邻接，则adj[i,j] 为真。我们或许可以写成

```mini
int: count = sum ( [ 1 | i,j,k in NODES where i < j \&\& j < k \&\& adj[i,j] \&\& adj[i,k] \&\& adj[j,k] ]);  
```

这当然是对的，但是它检查了所有点可能组成的三元组。如果此图是稀疏的，在意识到一旦我们选择了i和j，就可以进行一些测试之后，我们可以做得更好。

```mini
int: count = sum ( [ 1 | i,j in NODES where i < j \&\& adj[i,j],  
  k in NODES where j < k \&\& adj[i,k] \&\& adj[j,k] ]);  
```

你可以使用内建trace函数来帮助决定在生成元内发生了什么。

### 追踪
函数trace(s,e)在对表达式e求值并返回它的值之前就输出字符串s。它可以在任何情况下使用。

例如，我们可以查看在两种计算方式下的内部循环中分别进行了多少次测试。

```mini
int: count = sum ( [ 1 | i,j,k in NODES where  
  trace("+", i<j \&\& j<k \&\& adj[i,j] \&\& adj[i,k] \&\& adj[j,k] ) ];  
adj = [ ] false, true, true, false  
| true, false, true, false  
| true, true, false, true  
| false, false, true, false | ];  
constraint trace("\n",true);  
solve satisfy;  
```

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得到输出:

```
+---------------------------------------------------+

----------
```

表示内部循环进行了 64 次，而

```
int: count = sum ([ 1 | i, j in NODES where i < j \ adj[i,j],
                  k in NODES where trace("+", j < k \ adj[i,k] \ adj[j,k]]));
```

得到输出

```
+--------------+

----------
```

表示内部循环进行了 16 次。

注意你可以在 trace 中使用单独的字符来帮助你理解模型创建过程中发生了什么。

```
int: count = sum ( i, j in NODES where i < j \ adj[i,j])(
                 sum([trace("++show(i)+", ++show(j)+", ++show(k)++"), 1] |
                     k in NODES where j < k \ adj[i,k] \ adj[j,k]]));
```

会输出在计算过程中找到的每个三角形。得到输出

```
(1,2,3)
----------
```

我们要承认这里我们有一点点作弊：在某些情况下, MiniZinc 编译器实际上会把 where 从句中的参数自动重新排序，所以他们会尽快地被计算。在这种情况下, 加入 trace 函数实际上阻止了这种优化。一般来说, 通过分离 where 从句把它们摆到尽量接近生成元, 这其实是一个很好的主意来帮助编译器运作正常。

### 2.6.3 冗余约束

模型的形式会影响约束求解器求解它的效率。在很多情况下加入冗余约束，即被现有的模型逻辑上隐含的约束，可能会让求解器在更早时候产生更多可用的信息从而提高寻找解的搜索效率。

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回顾第2.6.2节中的魔术串问题。

运行 n = 16 时的模型:

```minizinc
$ minizinc --all-solutions --statistics magic-series.mzn -D "n=16;"
```

```minizinc
n = 16

s = [12, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0];

Listing 2.6.2: 使用冗余约束求解魔术串问题模型 (magic-series2.mzn).
```

```minizinc
int: n;
array[0..n-1] of var 0..n: s;

constraint forall(i in 0..n-1) ( 
  s[i] = (sum(j in 0..n-1)(bool2int(s[j]=i))));
% redundant
constraint sum(i in 0..n-1)(s[i]) = n;
constraint sum(i in 0..n-1)(s[i] * i) = n;
solve satisfy;

output [ "s = ", show(s), ";n" ];
```

统计数据显示需要有 89 个失败。

我们可以在模型中加入冗余约束。由于序列中的每个数字是用来计算某一个数字出现的次数，我们知道它们的和肯定是 n。类似地，由于这个序列是魔术的，我们知道了 s[i] * i 的和肯定是 n。使用如下方式把这些约束加入我们的模型 Listing 2.6.2.

像之前那样求解同一个问题

```minizinc
$ minizinc --all-solutions --statistics magic-series2.mzn -D "n=16;"
```

产生了同样的输出。但是统计显示只搜索了 13 个决策点。这些冗余约束使得求解器更早地去剪枝。

2.6.3. 冗余约束

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2.6.4 模型选择

在 MiniZinc 中有很多方式去给同一个问题建模，尽管其中有些模型或许会比另外的一些模型更自然。不同的模型或许会产生不同的求解效率。更糟糕的是，在不同的求解后端中，不同的模型或许会更好或更差。但是，我们还是可以给出一些关于普遍情况下产生更好的模型的指导:

### 模型之间的选择

一个好的模型倾向于有以下特征
- 更少的变量，或者至少是更少的没有被其他变量功能上定义的变量。
- 更小的变量定义域范围
- 模型的约束定义更简洁或者直接
- 尽可能地使用全局约束

实际情况中，所有这些都需要通过检查这个模型的搜索到底多有效率来断定模型好坏。通常除了用实验之外，我们很难判断搜索是否高效。

观察如下问题，我们要找寻 1 到 n 这 n 个数字的排列，使得相邻数字的差值也形成一个 1 到 n 的排列。Listing 2.6.3 中给出了一种用直观的方式来建模此问题的模型。注意变量 u 被变量 x 功能性定义。所以最差情况下的搜索空间是 $n^n$。

Listing 2.6.3: 对于 CSPlib prob007 所有间隔序列问题的模型 (allinterval.mzn).

```mzn
include "alldifferent.mzn";

int: n;

array[1..n] of var 1..n: x;  % 数字的序列
array[1..n-1] of var 1..n-1: u;  % 差的序列

constraint alldifferent(x);
constraint alldifferent(u);
constraint forall(i in 1..n-1)(u[i] = abs(x[i+1] - x[i]));

solve :: int_search(x, first_fail, indomain_min, complete)
satisfy;
output ["x = ",show(x),"\n"];
```

在这个模型中，数组 x 代表 n 个数字的排序。约束自然地可用 alldifferent 来表示。

求解模型
$ \textit{minizinc} --\texttt{solver g12fd} --\texttt{all-solutions} --\texttt{statistics} \textit{allinterval.mzn} -D \n \text{"n=10;\text{"} ,

在 84598 个决策点和 3 秒的时间内找到了所有的解。

另外一个模型是使用数组 y，其中 y[i] 代表数字 i 在序列中的位置。我们同时也使用变量 v 来建模表示差的位置。v[i] 表示了绝对差值 i 在序列出现的位置。如果 y[i] 和 y[j] 差别为一，其中 j > i，则代表了它们的位置是相邻的。所以 v[j-i] 被约束为两个位置中最大的那个。我们可以给这个模型加入两个冗余约束：因为我们知道差值 n-1 一定会发生，我们就可以推断出 1 和 n 的位置必须是相邻的 abs( y[1] - y[n] ) = 1。同时也告诉我们差值 n-1 的位置就是在 y[1] 和 y[n] 中的最大的那个位置，即 v[n-1] = min(y[1], y[n])。有了这些之后，我们可以建模此问题为: \texttt{numref:ex-allint2}。输出语句从位置数组 y 里重现了原本的序列 x。

Listing 2.6.4: CSPlib 中区间序列问题 prob007 的一个逆向模型。 (allinterval2.mzn).

```MiniZinc
include "alldifferent.mzn";

int: n;

array[1..n] of var 1..n: y; % 每个数的位置
array[1..n-1] of var 1..n-1: v; % 差值 i 的位置

constraint alldifferent(y);
constraint alldifferent(v);
constraint forall(i,j in 1..n where i < j)(
    (y[i] - y[j] = 1 -> v[j-i] = y[j]) \/
    (y[j] - y[i] = 1 -> v[j-i] = y[i])
);

constraint abs(y[1] - y[n]) = 1 \/
    v[n-1] = min(y[1], y[n]);

solve :: int_search(y, first_fail, indomain_min, complete)
    satisfy;

output [ "x = \""," +
    [ show(i) ++ if j == n then "\n\" else ", " endif
    | j in 1..n, i in 1..n where j == fix(y[i]) ];
```

逆向模型跟初始模型有同样的变量和定义域大小。但是对于给变量 x 和 u 的关系建模，逆向模型使用了一个更加非直接的方式来给变量 y 和 v 的关系建模。所以我们或许期望初始模型更
好些。
命令

```minizinc
$ minizinc --solver g12fd --all-solutions --statistics allinterval2.mzn -D "n=10;"
```

在 75536 个决策点和 18 秒内找到了所有的解。有趣的是，尽管这个模型不是简洁的，在变量 y 上搜索比在变量 x 上搜索更加有效率。简洁的缺乏意味着尽管搜索需要更少的决策点，但是在时间上实质上会更慢。

### 2.6.5 多重建模和连通

当我们将同一个问题有两个模型时，由于每个模型可以给解器不同的信息，通过把两个模型中的变量系到一起从而同时使用两个模型或许对我们是有帮助的。

Listing 2.6.5: CSPlib 中全区间序列问题 prob007 的一个双重模型。(allinterval3.mzn).

```minizinc
include "inverse.mzn";

int: n;
array[1..n] of var 1..n: x;  % 数值的序列
array[1..n-1] of var 1..n-1: u;  % 差值的序列

constraint forall(i in 1..n-1)(u[i] = abs(x[i+1] - x[i]));

array[1..n] of var 1..n: y;  % 每个数值的位置
array[1..n-1] of var 1..n-1: v;  % 差值的位置

constraint inverse(x,y);
constraint inverse(u,v);

constraint abs(y[1] - y[n]) = 1 \&_ v[n-1] = min(y[1], y[n]);

solve :: int_search(y, first_fail, indomain_min, complete)
satisfy;

output ["x = ",show(x),"\n"];
```
MiniZinc Handbook, Release 2.4.3

Listing 2.6.5 给出了一个结合 allinterval.mzn 和 allinterval2.mzn 特征的双重模型。模型的开始来自于 allinterval.mzn。我们接着介绍了来自于 allinterval2.mzn 中的变量 y 和 v。我们使用全局约束 inverse 来把变量绑到一起: inverse(x, y) 约束 y 为 x 的逆向函数 (反之亦然), 即, x[i] = j <> y[j] = i。Listing 2.6.6 中给出了它的定义。这个模型没有包含把变量 y 和 v 关联起来的约束, 它们是冗余的 (实际上是传播冗余)。所以它们不会给基于传播的求解器多于的信息。alldifferent 也不见了。原因是它们被逆向约束变得冗余了 (传播冗余)。唯一的约束是关于变量 x 和 u 而关系的约束以及 y 和 v 的冗余约束。

<table>
<thead>
<tr>
<th>Listing 2.6.6: 全局约束 inverse 的一个定义 (inverse.mzn).</th>
</tr>
</thead>
<tbody>
<tr>
<td>predicate inverse(array[int] of var int: f,</td>
</tr>
<tr>
<td>array[int] of var int: invf) =</td>
</tr>
<tr>
<td>forall(j in index_set(invf))(invf[j] in index_set(f)) \</td>
</tr>
<tr>
<td>forall(i in index_set(f))(</td>
</tr>
<tr>
<td>f[i] in index_set(invf) \</td>
</tr>
<tr>
<td>forall(j in index_set(invf))(j == f[i] &lt;&gt; i == invf[j])</td>
</tr>
<tr>
<td>);</td>
</tr>
</tbody>
</table>

双重模型的一个优点是我们可以有更多的定义不同搜索策略的视角。运行双重模型,

```bash
$ minizinc --solver g12fd --all-solutions --statistics allinterval3.mzn -D
"n=10;"
```

注意它使用逆向模型的搜索策略，标记变量 y，在 1714 决策点和 0.5 秒内找到了所有的解。注意标记变量 x 来运行同样的模型，需要 13142 个决策点和 1.5 秒。

2.6.6 对称

对称在约束满足和优化问题中是常见的。我们可以再次通过 Listing 2.5.1 来看一下这个问题。在 Fig. 2.6.1 棋盘的左上方展示了一个 8 皇后问题的解 (标记为 "original")。剩下的棋盘展示了 7 个解的对称版本: 旋转 90 度, 180 度和 270 度, 还有垂直翻转。

如果我们想要穷举 8 皇后问题的 所有 解, 很明显我们需要通过穷举 彼此之间不对称的 解为求解器省下一些工作, 然后生成这些的对称版本。这是我们想要在约束模型中摆脱对称的一个理由。另外一个更重要的理由是, 我们的求解器也可能会 探索非状态的对称版本!

举个例子, 一个典型的约束求解器可能会尝试把第 1 列皇后放在第 1 行上 (这是可以的), 然后尝试把第 2 列的皇后放到第 3 行上。这在第一眼看是没有违反任何约束的。然而, 这种设置不能被完成成为一个解 (求解器会在一些搜索之后发现这一点)。Fig. 2.6.2 在左上方的棋盘展示了这种设置。现在没有任何东西会阻止求解器去尝试, 比如, 在 Fig. 2.6.2 最低一行的左边第二种设置。
其中第 1 列的皇后仍然在第 1 行，而第 3 列的皇后放在第 2 行。于是，即便是搜索一个解，求解器可能需要探索很多它已经看到并证明不可满足状态的对称状态！

2.6.6.1 静态的对称性破缺

解决对称问题的建模技巧叫做 对称性破缺，在它的最简单形式里，需要在模型中加入约束使一个（不完整的）赋值的所有对称变换去除掉而保留一个。这些约束称作 静态的对称性破缺约束。

对称性破缺背后基本的想法是加入 顺序。举个例子，我们可以通过简单地加入约束使第一列的皇后必须在棋盘的上半部分，从而去除掉所有棋盘垂直翻转的。

```MiniZinc
constraint q[1] <= n div 2;
```
请相信以上约束会去除掉在 Fig. 2.6.1 中所有对称变换的一半。为了去除 所有 对称, 我们需要更多工作。

当我们把所有对称都表示成数组变量的排列，一组 字典顺序约束 可以用于破坏所有对称。举个例子, 如果数组变量名为 x , 而翻转数组是这个问题的一种对称, 那么以下约束可以破坏那种对称:

```plaintext
constraint lex_lesseq(x, reverse(x));
```

那么二维数组又怎么样呢? 字典顺序同样适用, 我们只需要把数组转换成一维的。举个例子, 下面的约束破坏了沿着其中一个对角线翻转数组的对称性 (注意到第二个生成式里对换的数组下标):

```plaintext
array[1..n,1..n] of var int: x;
constraint lex_lesseq([ x[i,j] | i,j in 1..n ], [ x[j,i] | i,j in 1..n ]);  
```

字典排序约束的好处在于我们可以加入多个 (同时破坏几个对称性), 而不需要它们互相干扰, 只要我们保持第一个参数中的顺序一致即可。

对于 n 皇后问题, 很不幸的是这个技巧不能马上适用, 因为又一些对称不能被描述成数组 q 的排列。克服这个问题的技巧是把 n 皇后问题表示成布尔变量。对于每个棋盘的每个格子, 布尔变量表示是否有一个皇后在上面。现在所有的对称性都可以表示成这个数组的排列。因为主要的 n 皇后问题的主要约束在整型数组 q 里面更加容易表达, 我们只需要两个模型都用起来, 然后在它们之间加入连通约束。正如 重建模和连通 (page 110) 中解释的一样。

加入布尔变量, 连通约束和对称性破缺约束的完整模型展示在 Listing 2.6.7 里面。我们可以做一些小实验来检查它是否成功的破坏所有对称性。尝试用不断增加的 n 运行模型, 比如从 1 到 10, 数一下解的个数 (比如使用 Gecode 求解器的 "s 标志, 或者选择 IDE 中” Print all solutions”和” Statistics for solving” )。你应该可以获得以下数列的解: 1, 0, 0, 1, 2, 1, 6, 12, 46, 92。你可以搜索 On-Line Encyclopedia of Integer Sequences (http://oeis.org) 来校验这个序列。

### Listing 2.6.7: n 皇后问题对称性破缺的部分模型 (full model: nqueens_sym.mzn).

```plaintext
% 映射每一个位置 i,j 到一个布尔变量上来表示在 i,j 上是否有一个皇后
array[1..n,1..n] of var bool: qb;

% 连通约束
constraint forall (i,j in 1..n) ( qb[i,j] <-> (q[i]=j) );

% 字典排序对称性破缺
constraint
```
2.6.6.2 其他对称的例子

许多其他问题都有内在的对称性，破坏这些对称性常常会令求解表现不一样。以下是一些常见的例子:

- 装箱问题: 当尝试把物品装入箱子时，任意两个有相同容量的箱子都是对称的。

- 涂色问题: 当尝试为一个图涂色使得所有相邻的节点都有不同的颜色时，我们通常用整型变量对颜色建模。但是，对颜色的任意排列都是一种合法的涂色方案。

- 车辆路线问题: 如果任务是给顾客分配一些车辆，任何两辆有相同容量的车可能是对称的（这跟装箱问题是相似的）。

- 排班/时间表问题: 两个有相同能力的职员可能是可以相互交换的。就像两个有相同容量或者设备的的房间一样
在 MiniZinc 中对布尔可满足性问题建模

MiniZinc 可以被用来给布尔可满足性问题建模，这种问题的变量被限制为是布尔型 (bool)。MiniZinc 可以使用有效率的布尔可满足性求解器来有效地解决求得的模型。

### 2.7.1 整型建模

很多时候，尽管我们想要使用一个布尔可满足性求解器，我们可能也需要给问题的整数部分建模。有三种通用的方式使用布尔型变量对定义域为范围 0...m 内的整型变量 I 建模，其中 \( m = 2^k - 1 \)。

- 二元: \( I \) 被表示为 \( k \) 个二元变量 \( i_0, \ldots, i_{k-1} \)，其中 \( I = 2^{k-1}i_{k-1} + 2^{k-2}i_{k-2} + \cdots + 2i_1 + i_0 \)。在 MiniZinc 中，这可表示为

```plaintext
class MyModel:
  array[0..k-1] of var bool: i;
  var 0..pow(2,k)-1: I = sum(j in 0..k-1)(bool2int(i[j])*pow(2,j));
```

- 一元: \( I \) 被表示为 \( m \) 个二元变量 \( i_1, \ldots, i_m \) 且 \( i = \sum_{j=1}^{m} \text{bool2int}(i_j) \)。由于在一元表示中有大量的冗余表示，我们通常要求 \( i_j \rightarrow i_{j-1}, 1 < j \leq m \)。在 MiniZinc 中，这可表示为

```plaintext
class MyModel:
  array[1..m] of var bool: i;
  constraint forall(j in 2..m)(i[j] -> i[j-1]);
  var 0..m: I = sum(j in 1..m)(bool2int(i[j]));
```
• 值: 其中 $I$ 被表示为 $m+1$ 个二元变量 $i_0,\ldots,i_m$，其中 $i = k \iff i_k$ 并且 $i_0,\ldots,i_m$ 中最多有一个为真。在 MiniZinc 中，这可表示为

```plaintext
array[0..m] of var bool: i;
constraint sum(j in 0..m)(bool2int(i[j]) == 1);
var 0..m: I;
constraint foall(j in 0..m)(I == j <-> i[j]);
```

每种表示都有其优点和缺点。这取决于模型中需要对整数做什么样的操作，而这些操作在哪一种表示上更为方便。

### 2.7.2 非等式建模

接下来，让我们考虑如何为一个拉丁方问题建模。一个拉丁方问题是在 $n \times n$ 个网格上放置 $1..n$ 之间的数值使得每个数在每行每列都仅出现一次。图 Listing 2.7.1 中给出了拉丁方问题的一个整数模型。

**Listing 2.7.1:** 拉丁方问题的整数模型 (latin.mzn).

```plaintext
int: n; % 拉丁方的大小
array[1..n,1..n] of var 1..n: a;

include "alldifferent.mzn";
constraint forall(i in 1..n)(
   alldifferent(j in 1..n)(a[i,j]) \/
   alldifferent(j in 1..n)(a[j,i])
);
solve satisfy;
output [ show(a[i,j]) ++ if j == n then "\n" else " " endif |
   i in 1..n, j in 1..n ];
```

整数变量直接的唯一的约束实际上是非等式，而它在约束 alldifferent 中被编码。数值表示是表达非等式的最佳方式。图 Listing 2.7.2 给出了一个关于拉丁方问题的只含有布尔型变量的模型。注意每个整型数组元素 $a[i,j]$ 被替换为一个布尔型数组。我们使用谓词 exactlyone 来约束每个数值在每行每列都仅出现一次，也用来约束有且仅有一个布尔型变量对应于整型数组元素 $a[i,j]$ 为真。
2.7.3 势约束建模

让我们来看下如何对点灯游戏建模。这个游戏由一个矩形网格组成，每个网格为空白或者被填充。每个被填充的方格可能包含 1 到 4 之间的数字，或者没有数字。我们的目标是放置灯泡在空白网格使得

- 每个空白的网格是“被照亮的”，也就是说它可以透过一行没有被打断的空白网格看到光亮。
- 任何两个灯泡都不能看到彼此。
- 一个有数值的填充的网格相邻的灯泡个数必须等于这个网格中的数值。

Fig. 2.7.1 给出了点灯游戏的一个例子以及 Fig. 2.7.2 给出了它的解。

这个问题很自然地可以使用布尔型变量建模。布尔型变量用来决定哪一个网格包含一个点灯以及哪一个没有。同时我们也有一些作用于填充的网格上的整数算术运算要考虑。
Fig. 2.7.1: 点灯游戏的一个例子展示

Fig. 2.7.2: 点灯游戏的完整的解
Listing 2.7.3: 点灯游戏的 SAT 模型 (lightup.mzn).

```mini
int: h; set of int: H = 1..h; % 格的高度
int: w; set of int: W = 1..w; % 格的宽度
array[H,W] of -1..5: b; % 格
int: E = -1; % 空白格
set of int: N = 0..4; % 填充并含有数字的格子
int: F = 5; % 填充但不含有数字的格子

% 位置 (i1,j1) 对位置 (i2,j2) 可见
test visible(int: i1, int: j1, int: i2, int: j2) =
  ((i1 == i2) \ forall(j in min(j1,j2)..max(j1,j2))(b[i1,j] == E))
  \ ((j1 == j2) \ forall(i in min(i1,i2)..max(i1,i2))(b[i,j1] == E));

array[H,W] of var bool: l; % is there a light

% 填充的格子没有灯泡
constraint forall(i in H, j in W, where b[i,j] != E)(l[i,j] == false);
% lights next to filled numbered square agree
include "boolsum.mzn";
constraint forall(i in H, j in W where b[i,j] in N)(
  bool_sum_eq(! l[i1,j1] | i1 in i-1..i+1, j1 in j-1..j+1 where
  abs(i1 - i) + abs(j1 - j) == 1 \ /
  i1 in H \ j1 in W \ b[i,j]));
% 每个空白格子是被照亮的
constraint forall(i in H, j in W where b[i,j] == E)(
  exists(j1 in W where visible(i,j,i,j))(l[i,j1]) \ /
  exists(i1 in H where visible(i,j,i,j))(l[i1,j]));
% 任何两个灯泡看不到彼此
constraint forall(i1,i2 in H, j1,j2 in W where
  (i1 != i2 \ j1 != j2) \ b[i1,j1] == E
  \ b[i2,j2] == E \ visible(i1,j1,i2,j2))(not l[i1,j1] \ not l[i2,j2]);
solve satisfy;
output [ if b[i,j] != E then show(b[i,j])
```

2.7.3. 势约束建模
图 Listing 2.7.3 中给出了这个问题的一个模型。图 Fig. 2.7.1 中给出的问题的数据文件在图 Listing 2.7.4 中给出。

图 Listing 2.7.3 中给出了这个问题的一个模型。图 Fig. 2.7.1 中给出的问题的数据文件在图 Listing 2.7.4 中给出。

Listing 2.7.4: 点灯游戏的 Fig. 2.7.1 中实例的数据文件

```
h = 7;
w = 7;
b = [ | -1,-1,-1,-1, 0,-1,-1 |
     | -1,-1,-1,-1,-1,-1,-1 |
     | 0,-1,-1, 3,-1,-1,-1 |
     | -1,-1, 2,-1, 4,-1,-1 |
     | -1,-1,-1, 5,-1,-1, 1 |
     | -1,-1,-1,-1,-1,-1,-1 |
     |  1,-1, 2,-1,-1,-1,-1 | ];
```

模型利用了一个布尔型求和谓词

```
predicate bool_sum_eq(array[int] of var bool:x, int:s);
```

使得一个布尔型数组的和等于一个固定的整数。多种方式都能使用布尔型变量给 cardinality 约束建模。

- 加法器网络: 我们可以使用包含加法器的一个网络给布尔型总和建立一个二元布尔型表达式
- 排序网络: 我们可以通过使用一个排序网络去分类布尔型数组来创建一个布尔型总和的一元表达式
- 二元决策图: 我们可以创建一个二维决策图（BDD）来编码势约束。

Listing 2.7.5: 使用二元加法器网络表示势约束 (bboolsum.mzn).

```
% 布尔型变量 x 的总和 = s
predicate bool_sum_eq(array[int] of var bool:x, int:s) =
  let { int: c = length(x) } in
  if s < 0 then false
  elseif s == 0 then
```
forall(i in 1..c)(x[i] == false)
elseif s < c then
  let { % cp = number of bits required for representing 0..c
       int: cp = floor(log2(int2float(c))),
       % z is sum of x in binary
       array[0..cp] of var bool: z } in
  binary_sum(x, z) \/
  % z == s
  forall(i in 0..cp)(z[i] == ((s div pow(2,i)) mod 2 == 1))
elseif s == c then
  forall(i in 1..c)(x[i] == true)
else false endif;
include "binarysum.mzn";

Listing 2.7.6: 创建二元求和网络的代码 (binarysum.mzn).

% 位 x 的位的总和 = 二元表示的 s。
% s[0], s[1], ..., s[k] 其中 2^k >= length(x) > 2^(k-1)
predicate binary_sum(array[int] of var bool:x,
                        array[int] of var bool:s)=
  let { int: l = length(x) } in
  if l == 1 then s[0] = x[1]
  elseif l == 2 then
    s[0] = (x[1] xor x[2]) \/ s[1] = (x[1] \/ x[2])
  else let { int: ll = (l div 2),
             array[1..ll] of var bool: f = [ x[i] | i in 1..ll ],
             array[1..ll] of var bool: t = [x[i] | i in ll+1..2*ll],
             var bool: b = if ll*2 == l then false else x[1] endif,
             int: cp = floor(log2(int2float(ll)))},
             array[0..cp] of var bool: fs,
             array[0..cp] of var bool: ts } in
  binary_sum(f, fs) \/ binary_sum(t, ts) \/
  binary_add(fs, ts, b, s)
  endif;

% 把两个二元数值 x 和 y 加起来，位 ci 用来表示进位，来得到二元 s
predicate binary_add(array[int] of var bool: x,
                      array[int] of var bool: y,
var bool: ci,
array[int] of var bool: s) =

let { int:1 = length(x),
      int:n = length(s), } in
assert(1 == length(y),
      "length of binary_add input args must be same",
assert(n == 1 \ n == l+1, "length of binary_add output " ++
      "must be equal or one more than inputs",
let { array[0..1] of var bool: c } in
full_adder(x[0], y[0], ci, s[0], c[0]) \
forall(i in 1..l)(full_adder(x[i], y[i], c[i-1], s[i], c[i])) \
if n > l then s[n] = c[l] else c[l] == false endif);

predicate full_adder(var bool: x, var bool: y, var bool: ci, var bool: s, var bool: co) =
let { var bool: xy = x xor y } in
s = (xy xor ci) \ co = ((x \ y) \( ci \ xy));

我们可以使用图 Listing 2.7.5 给出的二元加法器网络代码实现 bool_sum_eq。图 Listing 2.7.6
中定义的谓词 binary_sum 创建了一个 x 总和的二维表示法。它把列表分为两部分，把每一部分
分别加起来得到它们的一个二元表示，然后用 binary_add 把这两个二元数值加起来。如果 x 列
大小是奇数，则最后一位被保存起来作为二元加法时的进位来使用。

Listing 2.7.7: 使用二元加法器网络表示势约束 (boolsum.mzn).

% 布尔型变量 x 的总和 = s
predicate bool_sum_eq(array[int] of var bool:x, int:s) =
let { int: c = length(x) } in
if s < 0 then false
elseif s == 0 then forall(i in 1..c)(x[i] == false)
elseif s < c then
  let { % cp = nearest power of 2 >= c
         int: cp = pow(2,ceil(log2(int2float(c)))),
         array[1..cp] of var bool:y, % y is padded version of x
         array[1..cp] of var bool:z } in
  forall(i in 1..c)(y[i] == x[i]) \ /
  forall(i in c+1..cp)(y[i] == false) \ /
  oesort(y, z) \ z[s] == true \ z[s+1] == false
  elseif s == c then forall(i in 1..c)(x[i] == true)
else false endif;

include "oesort.mzn";

Listing 2.7.8: 奇偶归并排序网络 (oesort.mzn).

%% 奇偶排序
%% y 是 x 的有序版本，所有的真都在假之前
predicate oesort(array[int] of var bool:x, array[int] of var bool:y) =
  let { int: c = card(index_set(x)) } in
  if c == 1 then x[1] == y[1]
  elseif c == 2 then comparator(x[1],x[2],y[1],y[2])
  else
    let {
      array[1..c div 2] of var bool:xf = [x[i] | i in 1..c div 2],
      array[1..c div 2] of var bool:xl = [x[i] | i in c div 2 +1..c],
      array[1..c div 2] of var bool:tf,
      array[1..c div 2] of var bool:tl } in
    oesort(xf,tf) /\ oesort(xl,tl) /\ oemerge(tf ++ tl, y)
  endif;

%% 奇偶合并
%% y 是 x 的有序版本，所有的真都在假之前
%% 假设 x 的前一半是有序的，后一半也是
predicate oemerge(array[int] of var bool:x, array[int] of var bool:y) =
  let { int: c = card(index_set(x)) } in
  if c == 1 then x[1] == y[1]
  elseif c == 2 then comparator(x[1],x[2],y[1],y[2])
  else
    let {
      array[1..c div 2] of var bool:xo =
        [ x[i] | i in 1..c where i mod 2 == 1],
      array[1..c div 2] of var bool:xe =
        [ x[i] | i in 1..c where i mod 2 == 0],
      array[1..c div 2] of var bool:to,
      array[1..c div 2] of var bool:te } in
      oemerge(xo, to) /\ oemerge(xe, te) /\
      y[1] = to[1] /\
      forall(i in 1..c div 2 -1)(
        comparator(te[i], to[i+1], y[2*i], y[2*i+1]) /\
MiniZinc Handbook, Release 2.4.3

```plaintext
y[c] = te[c div 2]
endif);

% 比较器 o1 = max(i1,i2), o2 = min(i1,i2)
predicate comparator(var bool:i1, var bool:i2, var bool:o1, var bool:o2)=
 (o1 = (i1 \ i2)) \ (o2 = (i1 \ i2));
```

我们可以使用图 Listing 2.7.7 中给出的一元排序网络代码来实现 bool_sum_eq。势约束通过扩展输入 x 长度为 2 的次幂，然后使用奇偶归并排序网络给得到的位排序来实现。奇偶归并排序多行方式在图 Listing 2.7.8 中给出，它递归地把输入列表拆为两部分，给每一部分排序，然后再把有序的两部分归并起来。

```plaintext
Listing 2.7.9: 使用二元加法器网络表示势约束 (bddsum.mzn).

% the sum of booleans x = s
predicate bool_sum_eq(array[int] of var bool:x, int:s) =
 let { int: c = length(x),
         array[1..c] of var bool: y = [x[i] | i in index_set(x)]
 } in
 rec_bool_sum_eq(y, 1, s);

predicate rec_bool_sum_eq(array[int] of var bool:x, int: f, int:s) =
 let { int: c = length(x) } in
 if s < 0 then false
 elseif s == 0 then
   forall(i in f..c)(x[i] == false)
 elseif s < c - f + 1 then
   (x[f] == true \ rec_bool_sum_eq(x,f+1,s-1)) \ /
   (x[f] == false \ rec_bool_sum_eq(x,f+1,s))
 elseif s == c - f + 1 then
   forall(i in f..c)(x[i] == true)
 else false endif;
```

我们可以使用图 Listing 2.7.9 中给出的二元加法器网络代码来实现 bool_sum_eq。势约束被分为两种情况：或者第一个元素 x[1] 为 true 并且剩下位的总和是 s-1，或者 x[1] 为 false 并且剩下位的总和是 s。它的效率的提高依赖于去除共同子表达式来避免产生太多的相同的约束。
约束求解器不会直接支持 MiniZinc 模型。为了运行一个 MiniZinc 模型，它被翻译成一个 MiniZinc 的简单子集叫 FlatZinc。FlatZinc 反映了大多数约束求解器只会求解具有 \( \exists! c_1 \land \cdots \land c_m \) 的满足问题或者有 minimize \( z \) subject to \( c_1 \land \cdots \land c_m \) 的优化问题，其中 \( c_i \) 是基本的约束面 \( z \) 是一个具有某些限定形式的整型或者浮点表达式。

minizinc 工具包含了 MiniZinc 编译器，它可以用一个 MiniZinc 模型和数据文件来创建一个展平后的 FlatZinc 模型，它等价于给定数据的 MiniZinc 模型表达为之前提到的受限制的形式。通常来说构建一个给求解器的 FlatZinc 模型是对用户隐藏的，不过你也可以通过以下命令查看结合数据 data.dzn 来展平一个模型 model.mzn 的结果:

```
minizinc -c model.mzn data.dzn
```

这会创建一个 FlatZinc 模型叫 model.fzn。

在这一章中我们探索把 MiniZinc 翻译成 FlatZinc 的过程。

### 2.8.1 展平表达式

底层求解器的限制意味着复杂的 MiniZinc 表达式需要被 展平 为内部不具有更复杂结构项的基本约束的合取式。

思考以下保证两个在长方形箱子的两个圆不会重叠的模型:
Listing 2.8.1: 两个不会重叠的圆的模型 (cnonoverlap.mzn).

```mzn
float: width;  % 包含圆的长方形的宽
float: height; % 包含圆的长方形的高
float: r1;
var r1..width-r1: x1; % (x1,y1) 是第一个圆的中心
var r1..height-r1: y1;
float: r2;
var r2..width-r2: x2; % (x2,y2) 是第二个圆的中心
var r2..height-r2: y2;
  % 中心之间至少有 r1 + r2 的距离
constraint (x1-x2)*(x1-x2) + (y1-y2)*(y1-y2) >= (r1+r2)*(r1+r2);
solve satisfy;
```

### 2.8.1.1 简化和求值

给定数据文件

```mzn
width = 10.0;
height = 8.0;
r1 = 2.0;
r2 = 3.0;
```

转换到 FlatZinc 首先通过替换所有参数为它们的值来简化模型，然后求出所有取值已经固定的表达式的值。在这步简化后参数的值将不再需要。一个例外是大型数组的参数值。如果它们被适用多于一次，那么为了避免重复大的表达式这个参数会被保留。

在简化后, Listing 2.8.1 的模型的变量和参数声明部分变为

```mzn
var 2.0 .. 8.0: x1;
var 2.0 .. 6.0: y1;
var 3.0 .. 7.0: x2;
var 3.0 .. 5.0: y2;
```
### 2.8.1.2 定义子表达式

现在没有约束求解器可以直接处理像在 Listing 2.8.1 里复杂的约束表达式。作为替代，我们可以命名表达式中的每一个子表达式，我们创建约束来构建及代表子表达式数值。让我们来看看约束表达式的子表达式。（x1 - x2）是一个子表达式，我们将它命名为 FLOAT01 我们可以定义它为 constraint FLOAT01 = x1 - x2;。注意到这个表达式只在模型中出现两次。我们只需要构建这个值一次，然后我们可以重复使用它。这就是共同子表达式消除。子表达式 (x1 - x2)*(x1 - x2) 可以命名为 FLOAT02，而我们可以定义它为 constraint FLOAT02 = FLOAT01 * FLOAT01;。我们可以命名 constraint FLOAT03 = y1 - y2; 和 constraint FLOAT04 = FLOAT03 * FLOAT03; 最后 constraint FLOAT05 = FLOAT02 * FLOAT04;。不等约束本身则变成 constraint FLOAT05 >= 25.0; 因为 (r1+r2)*(r1 + r2) 计算出结果为 25.0。于是这个约束被展平为

```plaintext
constraint FLOAT01 = x1 - x2;
constraint FLOAT02 = FLOAT01 * FLOAT01;
constraint FLOAT03 = y1 - y2;
constraint FLOAT04 = FLOAT03 * FLOAT03;
constraint FLOAT05 = FLOAT02 * FLOAT04;
constraint FLOAT05 >= 25.0
```

### 2.8.1.3 FlatZinc 约束形式

展平的最后步骤是把约束的形式转换成标准的 FlatZinc 形式，它总是以 p(a1, ..., an) 的形式出现。其中的 p 是某个基础约束，a1, ..., an 是参数。FlatZinc 尝试使用最少的不同约束形式。所以 FLOAT01 = x1 - x2 首先尝试重写为 FLOAT01 + x2 = x1 然后使用 float_plus 输出基本的约束。得出的约束形式如下：

```plaintext
constraint float_plus(FLOAT01, x2, x1);
constraint float_plus(FLOAT03, y2, y1);
constraint float_plus(FLOAT02, FLOAT04, FLOAT05);
constraint float_times(FLOAT01, FLOAT01, FLOAT02);
constraint float_times(FLOAT03, FLOAT03, FLOAT04);
```

2.8.1. 展平表达式
2.8.1.4 边界分析

我们仍然缺少一项, 声明引入的变量 FLOAT01, ..., FLOAT05, 这些可以被声明为 var float. 不过为了令求解器的任务更简单, MiniZinc 尝试通过简单的分析确定新引入变量的上界和下界. 比如因为 FLOAT01 = x1 * x2 和 2.0 ≤ x1 ≤ 8.0 还有 3.0 ≤ x2 ≤ 7.0, 所以可以得出 -5.0 ≤ FLOAT01 ≤ 5.0 然后我们可以看到 -25.0 ≤ FLOAT02 ≤ 25.0 (尽管注意到如果我们发现相乘实际上是方形, 我们可以给出更精确的边界 0.0 ≤ FLOAT02 ≤ 25.0).

细心的读者会发现有定义子表达式 (page 127) 和 FlatZinc 约束形式 (page 127) 里约束的展平形式的一点不同. 在后面没有非等约束. 因为一元的不等式可以完全被一个变量的边界表示出来,不等关系可以令 FLOAT05 的下界变为 25.0, 然后这会变得冗余. 最后 Listing 2.8.1 的展平后形式是:

```
% 变量
var 2.0 .. 8.0: x1;
v 2.0 .. 6.0: y1;
v 3.0 .. 7.0: x2;
v 3.0 .. 5.0: y2;
%
var -5.0 .. 5.0: FLOAT01;
var -25.0 .. 25.0: FLOAT02;
var -3.0 .. 3.0: FLOAT03;
var -9.0 .. 9.0: FLOAT04;
v 25.0 .. 34.0: FLOAT05;
%
constraint float_plus(FLOAT01, x2, x1);
constraint float_plus(FLOAT03, y2, y1);
constraint float_plus(FLOAT02, FLOAT04, FLOAT05);
constraint float_times(FLOAT01, FLOAT01, FLOAT02);
constraint float_times(FLOAT03, FLOAT03, FLOAT04);
%
solve satisfy;
```

2.8.1.5 目标函数

MiniZinc 就像展平约束一样, 展平最小化和最大化目标函数. 跟其他表达式一样, 目标表达式被展平时创建一个变量. 在 FlatZinc 输出求解项永远是单一变量. 看一下例子 Let 表达式 (page 139).
### 2.8.2 线性表达式

约束的一个最重要的而广泛用于建模的形式是线性约束

$$a_1 x_1 + \cdots + a_n x_n \leq a_0$$

其中 $a_i$ 是整数或者浮点数约束, 而 $x_i$ 是整数或者浮点数变量. 它们非常有表达能力, 同时也是（整数）线性规划约束求解器唯一支持的形式. 从 MiniZinc 到 FlatZinc 的翻译器尝试创建线性约束, 而不是把线性约束变成许多子表达式.

<table>
<thead>
<tr>
<th>Listing 2.8.2: 说明线性约束展平的 MiniZinc 模型 (linear.mzn).</th>
</tr>
</thead>
<tbody>
<tr>
<td>int: d = -1;</td>
</tr>
<tr>
<td>constraint 3<em>x - y + x * z &lt;= 19 + d * (x + y + z) - 4</em>d;</td>
</tr>
</tbody>
</table>

考虑在 Listing 2.8.2 中的模型. 这里并没有为所有子表达式 $3*x, 3*x - y, x*z, 3*x - y + x*z, x + y + z, d*(x + y + z), 19 + d*(x + y + z)$, 和 $19 + d*(x + y + z) - 4*d$ 创建一个变量. 这里的翻译在创建一个 FlatZinc 约束时, 会尝试创建一个尽可能记录大部分原始约束内容的线性约束.

展平会创建线性表达式并将其视为一个单位, 而不视为每个子表达式是构建中间变量. 这也使创建的表达式简单化. 从约束中抽取出线性表达式为

| var 0..80: INT01; |
| constraint 4*x + z + INT01 <= 23; |
| constraint INT01 = x * z; |

注意到 非线性表达式 $x \times z$ 是如何被抽出来作为一个新的子表达式并赋予名字的, 与此同时剩下的项会被收集起来从而使每个变量只出现一次 (的确变量 $y$ 的项被移除了)

最后每个约束被写到 FlatZinc 形式, 从而得到:

| var 0..80: INT01; |
| constraint int_lin_le([1,4,1],[INT01,x,z],23); |
| constraint int_times(x,z,INT01); |

#### 2.8.2 线性表达式
2.8.3 展开表达式

大多数的模型需要创建一些基于输入数据的约束。Minizinc 通过数组类型，列表和列生成解析还有聚合函数来支持这些模型。

考虑以下从生产调度例子 Listing 2.2.2 中出现的聚合函数表达式

```minizinc
int: mproducts = max (p in Products)
    (min (r in Resources where consumption[p,r] > 0)
     (capacity[r] div consumption[p,r]));
```

由于这用到生成器语法，我们可以把它重写成可以被编译器处理的相等的形式：

```minizinc
int: mproducts = max([ min [ capacity[r] div consumption[p,r]
    | r in Resources where consumption[p,r] > 0]
    | p in Products]);
```

given data

```minizinc
nproducts = 2;
nresources = 5;
capacity = [4000, 6, 2000, 500, 500];
consumption= [| 250, 2, 75, 100, 0,
    | 200, 0, 150, 150, 75 |];
```

这首先创建 p = 1 的数组

```minizinc
[ capacity[r] div consumption[p,r]
    | r in 1..5 where consumption[p,r] > 0]
```

也就是 [16, 3, 26, 5] 然后计算最小值为 3。它之后为 p = 2 建立相同的数组 [20, 13, 3, 6]，并计算最小的数值为 3。然后创建数组 [3, 3] 并计算最大值为 3。在 FlatZinc 里面没有 mproducts 的表示，这种通过计算数值 3 的方法会被用来代替 mproducts。

在约束模型中最常见的聚合表达式是 forall。Forall 表达式会被展开成为多个约束。

考虑以下在 SEND-MORE-MONEY 例子 Listing 2.2.4 中使用预设的分解 alldifferent 出现的 MiniZinc 片段。
array[1..8] of var 0..9: v = [S,E,N,D,M,O,R,Y];
constraint forall(i,j in 1..8 where i < j)(v[i] != v[j])

forall 表达式为每一对需要满足 $i < j$ 的 $i, j$ 创建一个约束, 所以创建

```
constraint v[1] != v[2]; % S != E
constraint v[1] != v[3]; % S != N
...
constraint v[1] != v[8]; % S != Y
constraint v[2] != v[3]; % E != N
...
constraint v[7] != v[8]; % R != Y
```

在 FlatZinc 中形成

```
constraint int_neq(S,E);
constraint int_neq(S,N);
...
constraint int_neq(S,Y);
constraint int_neq(E,N);
...
constraint int_neq(R,Y);
```

注意到临时的数组变量 $v[i]$ 是如何在 FlatZinc 的输出中被原来的变量替换的.

### 2.8.4 数组

一维变量在 MiniZinc 中可以有任意的下标, 只要它们是相邻的整数. 在 FlatZinc 所有数组都被 $1..1$ 标注, 其中 1 是数组的长度. 这意味着数组查询时需要被转换成 FlatZinc 下标的形式.

考虑以下 MiniZinc 模型来使用 $m$ 个 1kg 砝码来平衡一个长为 $2 \times 12$ 的跷跷板, 其中上面有一个重为 $cw$ kg 小孩.

```
int: cw; % 小孩重量
int: 12; % 一半跷跷板长度
int: m; % 1kg 砝码的数量
array[-12..12] of var 0..max(m,cw): w; % 在每个点的重量
```
var -12..12: p; % 孩子的位置
constraint sum(i in -12..12)(i * w[i]) = 0; % 平衡
constraint sum(i in -12..12)(w[i]) = m + cw; % 所有使用的砝码
constraint w[p] = cw; % 孩子在位置 p
solve satisfy;

给定 $cw = 2, 12 = 2, m = 3$，展开可以产生约束

array[-2..2] of var 0..3: w;
var -2..2: p
constraint w[p] = 2;

不过 FlatZinc 坚持 $w$ 数组从下标 1 开始，这意味着我们需要重写所有数组获取来使用新的下标数值。对于固定值数组查找这很简单，对于变量值数组查找我们可能需要创建一个新的变量。以上公式的结果为

array[1..5] of var 0..3: w;
var -2..2: p
var 1..5: INT01;
constraint w[INT01] = 2;
constraint INT01 = p + 3;

最后我们重写约束成 FlatZinc 的形式。注意到变量数组下标查找的形式是如何映射到 array_var_int_element 上的。

array[1..5] of var 0..3: w;
var -2..2: p
var 1..5: INT01;
constraint int_lin_eq([-2, 1, -1, -2], [w[1], w[2], w[4], w[5]], 0);
constraint int_lin_eq([1, 1, 1, 1], [w[1], w[2], w[3], w[4], w[5]], 5);
constraint array_var_int_element(INT01, w, 2);
constraint int_lin_eq([-1, 1], [INT01, p], -3);

MiniZinc 支持多维数组，但是（目前来说）FlatZinc 只支持单维度数组。这意味着多维度数组必须
映射到单维度数组上。而且多维度数组访问必须映射到单维度数组访问。

考虑在有限元平面模型 Listing 2.2.1: 的 Laplace 等式约束:

```plaintext
set of int: HEIGHT = 0..h;
set of int: CHEIGHT = 1..h-1;
set of int: WIDTH = 0..w;
set of int: CWIDTH = 1..w-1;
array[HEIGHT,WIDTH] of var float: t; % 在点 (i,j) 处的温度
var float: left; % 左侧温度
var float: right; % 右侧温度
var float: top; % 顶部温度
var float: bottom; % 底部温度

% 拉普拉斯方程：每一个内部点温度是它相邻点的平均值
constraint forall(i in CHEIGHT, j in CWIDTH)(
    4.0\times t[i,j] = t[i-1,j] + t[i,j-1] + t[i+1,j] + t[i,j+1]);
```

假设 w = 4 和 h = 4，这会创建约束

```plaintext
array[0..4,0..4] of var float: t; % temperature at point (i,j)
constraint 4.0\times t[1,1] = t[0,1] + t[1,0] + t[2,1] + t[1,2];
constraint 4.0\times t[1,2] = t[0,2] + t[1,1] + t[2,2] + t[1,3];
constraint 4.0\times t[1,3] = t[0,3] + t[1,2] + t[2,3] + t[1,4];
constraint 4.0\times t[2,1] = t[1,1] + t[2,0] + t[3,1] + t[2,2];
constraint 4.0\times t[2,2] = t[1,2] + t[2,1] + t[3,2] + t[2,3];
constraint 4.0\times t[2,3] = t[1,3] + t[2,2] + t[3,3] + t[2,4];
constraint 4.0\times t[3,1] = t[2,1] + t[3,0] + t[4,1] + t[3,2];
constraint 4.0\times t[3,2] = t[2,2] + t[3,1] + t[4,2] + t[3,3];
constraint 4.0\times t[3,3] = t[2,3] + t[3,2] + t[4,3] + t[3,4];
```

25 个元素的 2 维数组被转换成一维数组，而且下标也会相应改变：所以 [i,j] 下标会变成 [i \times 5 + j + 1]。

```plaintext
array [1..25] of var float: t;
```

2.8.4. 数组
2.8.5 具体化

FlatZinc 模型包含了只有变量和参数声明，和一系列原始的约束。所以当我们在 MiniZinc 用布尔连接符而不是析取式来建模时，需要进行一些处理。处理使用连接符而不过是析取式来构建的复杂公式，其核心的方法是具体化。具体化一个约束 \( c \) 创建新的约束等价于 \( b \iff c \)，即如果约束满足则布尔变量 \( b \) 的值是 true，否则为 false。

当我们有能力具体化约束，对待复杂公式的方式跟数学表达式并无不同。我们为子表达式创建了一个名称和一个展平的约束来约束子表达式的数值。

考虑以下任务调度例子 Listing 2.2.8 中出现在约束表达式:

```plaintext
constraint %% 保证任务之间没有重叠
   forall(j in 1..tasks) ( 
      forall(i,k in 1..jobs where i < k) ( 
         s[i,j] + d[i,j] <= s[k,j] \/
         s[k,j] + d[k,j] <= s[i,j] 
      ));
```

对于数据文件

```plaintext
jobs = 2;
tasks = 3;
d = [ | 5, 3, 4 | 2, 6, 3 |]
```

然后展开过程生成

```plaintext
constraint s[1,1] + 5 <= s[2,1] \/ s[2,1] + 2 <= s[1,1];
constraint s[1,2] + 3 <= s[2,2] \/ s[2,2] + 6 <= s[1,2];
constraint s[1,3] + 4 <= s[2,3] \/ s[2,3] + 3 <= s[1,3];
```

具体化在析取式中出现的约束创建新的布尔变量来定义每个表达式的数值。

Chapter 2.8. FlatZinc 和展平
每个基础的约束现在会映射到 FlatZinc 形式下。注意到二维数组 s 是如何映射到一维数组里面的。

```
array[1..6] of var 0..23: s;
constraint int_lin_le_reif([-1, 1], [s[1], s[4]], -5, BOOL01);
constraint int_lin_le_reif([-1, 1], [s[1], s[4]], -2, BOOL02);
constraint int_lin_le_reif([-1, 1], [s[2], s[5]], -3, BOOL03);
constraint int_lin_le_reif([-1, 1], [s[2], s[5]], -6, BOOL04);
constraint int_lin_le_reif([-1, 1], [s[3], s[6]], -4, BOOL05);
constraint int_lin_le_reif([-1, 1], [s[3], s[6]], -3, BOOL06);
constraint array_bool_or([BOOL01, BOOL02], true);
constraint array_bool_or([BOOL03, BOOL04], true);
constraint array_bool_or([BOOL05, BOOL06], true);
```

`int_lin_le_reif` 是线性约束 `int_lin_le` 的具体化形式。

大多数 FlatZinc 基本约束 `p(\bar{x})` 有一个具体化形式 `p_reif(\bar{x}, b)`，它利用最后额外的参数 `b` 来定义一个约束 `b \leftrightarrow p(\bar{x})`。定义像 `int_plus` 和 `int_plus` 的函数式关系的 FlatZinc 基本约束不需要支持具体化。反而，有结果的函数的等式被具体化了。

具体化的另外一个重要作用出现在当我们使用强制转换函数 `bool2int`（可能是显式地或者隐式地把布尔表达式使用成整数表达式使用）。平整过程将创建一个布尔变量来保存一个布尔表达式参数，以及一个整型变量（限制到 `0..1`）来保存这个数值。

考虑 Listing 2.2.12 中的魔术序列问题。

```
int: n;
array[0..n-1] of var 0..n: s;
```
constraint forall(i in 0..n-1) (s[i] = (sum(j in 0..n-1)(bool2int(s[j]=i))));

给定 n = 2，展开创造了

costraint s[0] = bool2int(s[0] = 0) + bool2int(s[1] = 0);
costraint s[1] = bool2int(s[0] = 1) + bool2int(s[1] = 1);

和展平创造了

costraint BOOL01 <-> s[0] = 0;
costraint BOOL03 <-> s[1] = 0;
costraint BOOL05 <-> s[0] = 1;
costraint BOOL07 <-> s[1] = 1;
costraint INT02 = bool2int(BOOL01);
costraint INT04 = bool2int(BOOL03);
costraint INT06 = bool2int(BOOL05);
costraint INT08 = bool2int(BOOL07);
costraint s[0] = INT02 + INT04;
costraint s[1] = INT06 + INT08;

最后的 FlatZinc 形式是

var bool: BOOL01;
var bool: BOOL03;
var bool: BOOL05;
var bool: BOOL07;
var 0..1: INT02;
var 0..1: INT04;
var 0..1: INT06;
var 0..1: INT08;
array [1..2] of var 0..2: s;
costraint int_eq_reif(s[1], 0, BOOL01);
costraint int_eq_reif(s[2], 0, BOOL03);
costraint int_eq_reif(s[1], 1, BOOL05);
costraint int_eq_reif(s[2], 1, BOOL07);
costraint bool2int(BOOL01, INT02);
2.8.6 谓词

MiniZinc 支持许多不同求解器的的一个重要的因素是全局约束 (还有真正的 FlatZinc 约束) 可以根据不同的求解器专业化。

每一个求解器生命一个谓词有时会, 但有时并不会提供具体的定义. 举个例子一个求解器有一个内建的全局 alldifferent 谓词, 会包含定义

```mini
predicate alldifferent(array[int] of var int:x);
```

在全局约束库中, 同时一个求解器预设的分解会有定义

```mini
predicate alldifferent(array[int] of var int:x) =
  forall(i,j in index_set(x) where i < j)(x[i] != x[j]);
```

谓词调用 \( p(t) \) 在平整时, 首先为每个参数项 \( t_i \) 创建对应变量 \( v_i \). 如果谓词没有定义我们只需要使用创建的参数 \( p(v) \) 来调用谓词. 如果一个谓词有一个定义 \( p(\bar{x}) = \phi(\bar{x}) \) 然后我们将用谓词的定义来替换这个谓词调用 \( p(\bar{x}) \) 当中形式参数被替换为对应的参数变量, 即 \( \phi(\bar{v}) \). 注意到如果一个谓词调用 \( p(\bar{x}) \) 出现在具体化位置而且它没有定义, 我们则检查我们适用这个谓词的具体化版本 \( p_{reif}(\bar{x}, b) \).

考虑在 SEND-MORE-MONEY 例子 Listing 2.2.4 中 alldifferent 约束的:

```mini
constraint alldifferent([S,E,N,D,M,O,R,Y]);
```

如果这个求解器有一个内建的 alldifferent 我们只需要为这个参数创建一个新的变量, 然后再调用时替换它:

```mini
array[1..8] of var 0..9: v = [S,E,N,D,M,O,R,Y];
constraint alldifferent(v);
```
注意到边界分析尝试在新的数组变量上找到一个紧的边界．建造这个数组参数的理由就是，如果我们中使用相同的数组两次，FlatZinc 求解器不会创建它两次．在这种情况下因为它不是使用两次，后面的转换会把 v 替换成它的定义．

如果求解器使用预设的定义 alldifferent 呢？然后变量 v 会正常地定义，谓词调用会被替换为一个当中变量被重命名的版本，其中 v 替换了形式参数 x．结果的程序是：

```plaintext
array[1..8] of var 0..9: v = [S,E,N,D,M,O,R,Y];
constraint forall(i, j in 1..8 where i < j)(v[i] != v[j])
```

我们可以在展开表达式 (page 130) 中看到．

考虑到以下约束，其中 alldifferent 在一个具体化位置出现．

```plaintext
constraint alldifferent([A,B,C]) \ alldifferent([B,C,D]);
```

如果求解器有 alldifferent 的具体化形式，这将会被展平为

```plaintext
constraint alldifferent_reif([A,B,C],BOOL01);
constraint alldifferent_reif([B,C,D],BOOL02);
constraint array_bool_or([BOOL01,BOOL02],true);
```

适用这个预设的分解，谓词替换会首先创建

```plaintext
array[1..3] of var int: v1 = [A,B,C];
array[1..3] of var int: v2 = [B,C,D];
constraint forall(i, j in 1..3 where i<j)(v1[i] != v1[j]) \ forall(i, j in 1..3 where i<j)(v2[i] != v2[j]);
```

它最终会展平成 FlatZinc 形式

```plaintext
constraint int_neq_reif(A,B,BOOL01);
constraint int_neq_reif(A,C,BOOL02);
constraint int_neq_reif(B,C,BOOL03);
constraint array_bool_and([BOOL01,BOOL02,BOOL03],BOOL04);
constraint int_neq_reif(B,D,BOOL05);
constraint int_neq_reif(C,D,BOOL06);
constraint array_bool_and([BOOL03,BOOL05,BOOL06],BOOL07);
constraint array_bool_or([BOOL04,BOOL07],true);
```
注意到共同子表达式消除是如何利用具体化不等式 B != C 的。虽然有一个更好的转换把共同约束提升到最顶层的合取式中

### 2.8.7 Let 表达式

Let 表达式是 MiniZinc 中可用于引入新的变量的非常强大的工具。在展平时，let 表达式被转换成变量和约束声明。这个 MiniZinc 的关系语义意味着这些约束必须像在第一个包含的布尔表达式中出现。

let 表达式的一个重要特征是每一次它们被使用时它们都创建新的变量。

考虑一下展平的代码

```plaintext
constraint even(u) \/
  even(v);

predicate even(var int: x) =
  let { var int: y } in x = 2 * y;
```

首先谓词调用被他们的定义取代。

```plaintext
constraint (let { var int: y } in u = 2 * y) \/
  (let { var int: y } in v = 2 * y);
```

然后 let 变量会另外被重命名

```plaintext
constraint (let { var int: y1 } in u = 2 * y1) \/
  (let { var int: y2 } in v = 2 * y2);
```

最后变量声明会被抽取到第一层

```plaintext
var int: y1;
var int: y2;
constraint u = 2 * y1 \/
  v = 2 * y2;
```

一旦 let 表达式被消除我们可以像之前那样展平。

记住 let 表达式可以定义新引人的变量 (对某些参数的确需要这样做)。这些隐式地定义了必须满足的约束。

考虑婚礼座位问题 Listing 2.3.10 的复杂的目标函数.

2.8.7 Let 表达式 139
solve maximize sum(h in Hatreds)(
    let { var Seats: p1 = pos[h1[h]],
         var Seats: p2 = pos[h2[h]],
         var 0..1: same = bool2int(p1 <= 6 <-> p2 <= 6) } in
    same * abs(p1 - p2) + (1-same) * (abs(13 - p1 - p2) + 1));

为了简介我们假设只使用前两个相互敌视的人，所以

set of int: Hatreds = 1..2;
array[Hatreds] of Guests: h1 = [groom, carol];
array[Hatreds] of Guests: h2 = [clara, bestman];

展平的第一步是展开 sum 表达式，给定 (为了简洁我们保留客人名字和参数 Seats，在实际中他们会被他们的定义取代):

solve maximize
    (let { var Seats: p1 = pos[groom],
           var Seats: p2 = pos[clara],
           var 0..1: same = bool2int(p1 <= 6 <-> p2 <= 6) } in
    same * abs(p1 - p2) + (1-same) * (abs(13 - p1 - p2) + 1))
    +
    (let { var Seats: p1 = pos[carol],
           var Seats: p2 = pos[bestman],
           var 0..1: same = bool2int(p1 <= 6 <-> p2 <= 6) } in
    same * abs(p1 - p2) + (1-same) * (abs(13 - p1 - p2) + 1));

然后每一个在 let 表达式的新变量会被分别命名为

solve maximize
    (let { var Seats: p11 = pos[groom],
           var Seats: p21 = pos[clara],
           var 0..1: same1 = bool2int(p11 <= 6 <-> p21 <= 6) } in
    same1 * abs(p11 - p21) + (1-same1) * (abs(13 - p11 - p21) + 1))
    +
    (let { var Seats: p12 = pos[carol],
           var Seats: p22 = pos[bestman],
           var 0..1: same2 = bool2int(p12 <= 6 <-> p22 <= 6) } in
    same2 * abs(p12 - p22) + (1-same2) * (abs(13 - p12 - p22) + 1));
在 `let` 表达式的变量会被抽取到第一层，并且定义约束会被抽取到正确的层（在这里是最顶层）。

```plaintext
var Seats: p11;
var Seats: p21;
var 0..1: same1;
constraint p12 = pos[clara];
constraint p11 = pos[groom];
constraint same1 = bool2int(p11 <= 6 <> p21 <= 6);
var Seats p12;
var Seats p22;
var 0..1: same2;
constraint p12 = pos[carol];
constraint p22 = pos[bestman];
constraint same2 = bool2int(p12 <= 6 <> p22 <= 6) in
solve maximize
  same1 * abs(p11 - p21) + (1-same1) * (abs(13 - p11 - p21) + 1))
  +
  same2 * abs(p12 - p22) + (1-same2) * (abs(13 - p12 - p22) + 1));
```

现在我们已经构成不需要使用 `let` 表达式的等价的 MiniZinc 代码，和展平可以正常进行。

为了说明没有出现在最顶层的 `let` 表达式的情况，看看以下模型

```plaintext
var 0..9: x;
constraint x >= 1 -> let { var 2..9: y = x - 1 } in
  y + (let { var int: z = x * y } in z * z) < 14;
```

我们抽取变量定义到最顶层，约束到第一个围住的布尔语境，这里是蕴含的右手边。

```plaintext
var 0..9: x;
var 2..9: y;
var int: z;
constraint x >= 1 -> (y = x - 1 \ z = x * y \ y + z * z < 14);
```

注意到如果我们知道定义一个变量的等式的真值不会为假，我们可以抽取它到最顶层。这通常可以是求解大幅加快。

对于上面的例子，因为 `y` 的值域对于 `x - 1` 并不够大，所以约束 `y = x - 1` 可能失败。不过约束 `z = x * y` 不可以（实际上边界分析会给予 `z` 足够大的边界来包含 `x * y` 所有的可能值）一个更好的展平可以给出

### 2.8.7. Let 表达式
现在 MiniZinc 编译器通过总是使引入变量声明的边界足够大，它应该可以包含所有它定义的表达式的值。然后在正确的语境中为 let 表达式加入边界约束。在上面的例子中这个结果是

```plaintext
var 0..9: x;
var -1..8: y;
var -9..72: z;
constraint y = x - 1;
constraint z = x * y;
constraint (y >= 2 \ y + z * z > 14) -> x >= 5;
```

这个转换可以使求解更加高效，因为 let 变量的所有可能的复杂计算并没有被具体化。

这种方法的另外一个原因是在引入变量出现在取反语境的时候它也可以被使用（只要它们有一个定义）。考虑一下与之前相似的这个例子:

```plaintext
var 0..9: x;
constraint (let { var 2..9: y = x - 1 } in
    y + (let { var int: z = x * y } in z * z) > 14) -> x >= 5;
```

这个 let 表达式出现在否定语境中，不过每个引入变量都被定义了。展平后的代码是

```plaintext
var 0..9: x;
var -1..8: y;
var -9..72: z;
constraint y = x - 1;
constraint z = x * y;
constraint (y >= 2 \ y + z * z > 14) -> x >= 5;
```

注意到作为对比的 let 消除方法不能给出一个正确的转换:

```plaintext
var 0..9: x;
var 2..9: y;
```
```plaintext
var int: z;
constraint (y = x - 1 \ z = x * y \ y + z * z > 14) -> x >= 5;
```

以上转换对于所有 x 的可能值给出结果，而原来的约束除掉了 x = 4 的可能性。

对于在 let 表达式中的处理跟对定义的变量的处理是相似的。你可以认为一个约束等价于定义一个新的布尔变量。新的布尔变量定义可以从最顶层中抽取出来，而布尔保存在正确的语境下。

```plaintext
constraint z > 1 -> let { var int: y,
    constraint (x >= 0) -> y = x,
    constraint (x < 0) -> y = -x
} in y * (y - 2) >= z;
```

可以处理成

```plaintext
constraint z > 1 -> let { var int: y,
    var bool: b1 = ((x >= 0) -> y = x),
    var bool: b2 = ((x < 0) -> y = -x),
    constraint b1 \ b2
} in y * (y - 2) >= z;
```

然后展平成

```plaintext
constraint b1 = ((x >= 0) -> y = x);
constraint b2 = ((x < 0) -> y = -x);
constraint z > 1 -> (b1 \ b2 \ y * (y - 2) >= z);
```

2.8.7. Let 表达式
Part 3

User Manual
CHAPTER 3.1

The MiniZinc Command Line Tool

The core of the MiniZinc constraint modelling system is the miniZinc tool. You can use it directly from the command line, through the MiniZinc IDE, or through a programmatic interface (API). This chapter summarises the options accepted by the tool, and explains how it interacts with target solvers.

3.1.1 Basic Usage

The miniZinc tool performs three basic functions: it compiles a MiniZinc model (plus instance data), it runs an external solver, and it translates solver output into the form specified in the model. Most users would use all three functions at the same time. For example, let us assume that we want to solve the following simple problem, given as two files (model.mzn and data.dzn):

```plaintext
int: n;
array[1..n] of var 1..2*n: x;
include "alldifferent.mzn";
constraint alldifferent(x);
solve maximize sum(x);
output ["The resulting values are \(x)\n"];
```

```plaintext
n = 5;
```
To run the model file `model.mzn` with data file `data.dzn` using the Gecode solver, you can use the following command line:

```
$ minizinc --solver Gecode model.mzn data.dzn
```

This would result in the output

```
The resulting values are [10, 9, 8, 7, 6].
```

However, each of the three functions can also be accessed individually. For example, to compile the same model and data, use the `-c` option:

```
$ minizinc -c --solver Gecode model.mzn data.dzn
```

This will result in two new files, `model.fzn` and `model.ozn`, being output in the same directory as `model.mzn`. You could then run a target solver directly on the `model.fzn` file, or use `minizinc`:

```
$ minizinc --solver Gecode model.fzn
```

You will see that the solver produces output in a standardised form, but not the output prescribed by the output item in the model:

```
x = array1d(1..5 ,[10, 9, 8, 7, 6]);
```

The translation from this output to the form specified in the model is encoded in the `model.ozn` file. You can use `minizinc` to execute the `.ozn` file. In this mode, it reads a stream of solutions from standard input, so we need to pipe the solver output into `minizinc`:

```
$ minizinc --solver Gecode model.fzn | minizinc --ozn-file model.ozn
```

These are the most basic command line options that you need in order to compile and run models and translate their output. The next section lists all available command line options in detail. Section 4.3.5 explains how new solvers can be added to the system.
3.1.2 Adding Solvers

Solvers that support MiniZinc typically consist of two parts: a solver executable, which can be run on the FlatZinc output of the MiniZinc compiler, and a solver library, which consists of a set of MiniZinc files that define the constraints that the solver supports natively. This section deals with making existing solvers available to the MiniZinc tool chain. For information on how to add FlatZinc support to a solver, refer to Section 4.3.

3.1.2.1 Configuration files

In order for MiniZinc to be able to find both the solver library and the executable, the solver needs to be described in a solver configuration file (see Section 4.3.5 for details). If the solver you want to install comes with a configuration file (which has the file extension .msc for MiniZinc Solver Configuration), it has to be in one of the following locations:

- In the minizinc/solvers/ directory of the MiniZinc installation. If you install MiniZinc from the binary distribution, this directory can be found at /usr/share/minizinc/solvers on Linux systems, inside the MiniZincIDE application on macOS system, and in the Program Files\MiniZinc IDE (bundled) folder on Windows.
- In the directory $HOME/.minizinc/solvers on Linux and macOS systems, and the Application Data directory on Windows systems.
- In any directory listed on the MZN_SOLVER_PATH environment variable (directories are separated by : on Linux and macOS, and by ; on Windows systems).
- In any directory listed in the mzn_solver_path option of the global or user-specific configuration file (see Section 3.1.4)
- Alternatively, you can use the MiniZinc IDE to create solver configuration files, see Section 3.2.5.2 for details.

After adding a solver, it will be listed in the output of the minizinc --solvers command.

3.1.2.2 Configuration for MIP solvers

Some solvers require additional configuration flags before they can be used. For example, the binary bundle of MiniZinc comes with interfaces to the CPLEX and Gurobi Mixed Integer Programming solvers. However, due to licensing restrictions, the solvers themselves are not part of the bundled release. Depending on where CPLEX or Gurobi is installed on your system, MiniZinc
may be able to find the solvers automatically, or it may require an additional option to point it to the shared library.

In case the libraries cannot be found automatically, you can use one of the following:

- **CPLEX**: Specify the location of the shared library using the `--cplex-dll` command line option. On Windows, the library is called `cplexXXXX.dll` and typically found in the same directory as the `cplex` executable. On Linux it is `libcplexXXX.so`, and on macOS `libcplexXXXX.jnilib`, where XXX and XXXX stand for the version number.

- **Gurobi**: The command line option for Gurobi is `--gurobi-dll`. On Windows, the library is called `gurobiXX.dll` (in the same directory as the `gurobi` executable), and on Linux and macOS is it `libgurobiXX.so` (in the `lib` directory of your Gurobi installation).

- You can define these paths as defaults in your user configuration file, see Section 3.1.4.

### 3.1.3 Options

You can get a list of all the options supported by the `minizinc` tool using the `--help` flag.

#### 3.1.3.1 General options

These options control the general behaviour of the `minizinc` tool.

- `--help, -h`
  
  Print a help message.

- `--version`

  Print version information.

- `--solvers`

  Print list of available solvers.

- `--solver <id>, --solver <solver configuration file>.msc`

  Select solver to use. The first form of the command selects one of the solvers known to MiniZinc (that appear in the list of the `--solvers` command). You can select a solver by name, id, or tag, and add a specific version. For example, to select a mixed-integer programming solver, identified by the `mip` tag, you can use `--solver mip`. To select a specific version of Gurobi (in case you have two versions installed), use `--solver Gurobi@7.5.2`. Instead of the name you can also use the solver’s identifier, e.g. `--solver org.gecode.gecode`.
The second form of the command selects the solver from the given configuration file (see Section 4.3.5).

```
--help <id>
    Print help for a particular solver. The scheme for selecting a solver is the same as for the --solver option.
```

```
-v, -l, --verbose
    Print progress/log statements (for both compilation and solver). Note that some solvers may log to stdout.
```

```
--verbose-compilation
    Print progress/log statements for compilation only.
```

```
-s, --statistics
    Print statistics (for both compilation and solving).
```

```
--compiler-statistics
    Print statistics for compilation.
```

```
-c, --compile
    Compile only (do not run solver).
```

```
--config-dirs
    Output configuration directories.
```

```
--solvers-json
    Print configurations of available solvers as a JSON array.
```

### 3.1.3.2 Solving options

Each solver may support specific command line options for controlling its behaviour. These can be queried using the `--help <id>` flag, where `<id>` is the name or identifier of a particular solver. Most solvers will support some or all of the following options.

```
-a, --all-solutions
    Report all solutions in the case of satisfaction problems, or print intermediate solutions of increasing quality in the case of optimisation problems.
```

```
-n <i>, --num-solutions <i>
    Stop after reporting i solutions (only used with satisfaction problems).
```

```
-f, --free-search
    Instructs the solver to conduct a “free search”, i.e., ignore any search annotations. The solver is not required to ignore the annotations, but it is allowed to do so.
```
--solver-statistics
    Print statistics during and/or after the search for solutions.

--verbose-solving
    Print log messages (verbose solving) to the standard error stream.

-p <i>, --parallel <i>
    Run with i parallel threads (for multi-threaded solvers).

-r <i>, --random-seed <i>
    Use i as the random seed (for any random number generators the solver may be using).

3.1.3.3 Flattener input options

These options control aspects of the MiniZinc compiler.

--ignore-stdlib
    Ignore the standard libraries stdlib.mzn and builtins.mzn

--instance-check-only
    Check the model instance (including data) for errors, but do not convert to FlatZinc.

-e, --model-check-only
    Check the model (without requiring data) for errors, but do not convert to FlatZinc.

--model-interface-only
    Only extract parameters and output variables.

--model-types-only
    Only output variable (enum) type information.

--no-optimize
    Do not optimize the FlatZinc

-d <file>, --data <file>
    File named <file> contains data used by the model.

-D <data>, --cmdline-data <data>
    Include the given data assignment in the model.

--stdlib-dir <dir>
    Path to MiniZinc standard library directory

-G --globals-dir --mzn-globals-dir <dir>
    Search for included globals in <stdlib>/<dir>. 

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- **--input-from-stdin**
  Read problem from standard input

- **--search-dir**
  Additionally search for included files in <dir>.

- **-D “fMIPdomains=false”**
  No domain unification for MIP

- **--MIPDMaxIntvEE <n>**
  Max integer domain subinterval length to enforce equality encoding, default 0

- **--MIPDMaxDensEE <n>**
  Max domain cardinality to N subintervals ratio to enforce equality encoding, default 0, either condition triggers

- **--only-range-domains**
  When no MIPdomains: all domains contiguous, holes replaced by inequalities

- **--allow-multiple-assignments**
  Allow multiple assignments to the same variable (e.g. in dzn)

- **--compile-solution-checker <file>.mzc.mzn**
  Compile solution checker model.

**Flattener two-pass options**

Two-pass compilation means that the MiniZinc compiler will first compile the model in order to collect some global information about it, which it can then use in a second pass to improve the resulting FlatZinc. For some combinations of model and target solver, this can lead to substantial improvements in solving time. However, the additional time spent on the first compilation pass does not always pay off.

- **--two-pass**
  Flatten twice to make better flattening decisions for the target

- **--use-gecode**
  Perform root-node-propagation with Gecode (adds –two-pass)

- **--shave**
  Probe bounds of all variables at the root node (adds –use-gecode)

- **--sac**
  Probe values of all variables at the root node (adds –use-gecode)
--pre-passes <n>
   Number of times to apply shave/sac pass (0 = fixed-point, 1 = default)

-O<n>
   Two-pass optimisation levels:
   -O0: Disable optimize (–no-optimize) -O1: Single pass (default) -O2: Same as: –two-pass
   -O3: Same as: –use-gecode -O4: Same as: –shave -O5: Same as: –sac

Flattener output options

These options control how the MiniZinc compiler produces the resulting FlatZinc output. If you run the solver directly through the minizinc command or the MiniZinc IDE, you do not need to use any of these options.

--no-output-ozn, -O-
   Do not output ozn file

--output-base <name>
   Base name for output files

--fzn <file>, --output-fzn-to-file <file>
   Filename for generated FlatZinc output

-O, --ozn, --output-ozn-to-file <file>
   Filename for model output specification (-O- for none)

--keep-paths
   Don’t remove path annotations from FlatZinc

--output-paths
   Output a symbol table (.paths file)

--output-paths-to-file <file>
   Output a symbol table (.paths file) to <file>

--output-to-stdout, --output-fzn-to-stdout
   Print generated FlatZinc to standard output

--output-ozn-to-stdout
   Print model output specification to standard output

--output-paths-to-stdout
   Output symbol table to standard output
3.1.3.4 Solution output options

These options control how solutions are output. Some of these options only apply if `minizinc` is used to translate a stream of solutions coming from a solver into readable output (using a .ozn file generated by the compiler).

`--ozn-file <file>`
Read output specification from ozn file.

`-o <file>, --output-to-file <file>`
Filename for generated output.

`-i <n>, --ignore-lines <n>, --ignore-leading-lines <n>`
Ignore the first <n> lines in the FlatZinc solution stream.

`--soln-sep <s>, --soln-separator <s>, --solution-separator <s>`
Specify the string printed after each solution (as a separate line). The default is to use the same as FlatZinc, "-".

`--soln-comma <s>, --solution-comma <s>`
Specify the string used to separate solutions. The default is the empty string.

`--unsat-msg (--unsatisfiable-msg)`
Specify status message for unsatisfiable problems (default: "=====UNSATISFIABLE=====")

`--unbounded-msg`
Specify status message for unbounded problems (default: "=====UNBOUNDED=====")

`--unsatorunbnd-msg`
Specify status message for unsatisfiable or unbounded problems (default: "=====UNSATorUNBOUNDED=====")

`--unknown-msg`
Specify status message if search finished before determining status (default: "=====UNKNOWN=====")
--error-msg
   Specify status message if search resulted in an error (default: "=====ERROR====")

--search-complete-msg <msg>
   Specify status message if when search exhausted the entire search space (default: "==========")

--non-unique
   Allow duplicate solutions.

-c, --canonicalize
   Canonicalize the output solution stream (i.e., buffer and sort).

--output-non-canonical <file>
   Non-buffered solution output file in case of canonicalization.

--output-raw <file>
   File to dump the solver’s raw output (not for hard-linked solvers)

--no-output-comments
   Do not print comments in the FlatZinc solution stream.

--output-time
   Print timing information in the FlatZinc solution stream.

--no-flush-output
   Don’t flush output stream after every line.

### 3.1.4 User Configuration Files

The minizinc tool reads a system-wide and a user-specific configuration file to determine default paths, solvers and solver options. The files are called Preferences.json, and you can find out the locations for your platform using the option --config-dirs:

```bash
$ minizinc --config-dirs
{
   "globalConfigFile" : "/Applications/MiniZincIDE.app/Contents/Resources/
                      share/minizinc/Preferences.json",
   "userConfigFile" : "/Users/Joe/.minizinc/Preferences.json",
   "userSolverConfigDir" : "/Users/Joe/.minizinc/solvers",
   "mznStdlibDir" : "/Applications/MiniZincIDE.app/Contents/Resources/share/
                    minizinc"
}```
The configuration files are simple JSON files that can contain the following configuration options:

- **mzn_solver_path** (list of strings): Additional directories to scan for solver configuration files.
- **mzn_lib_dir** (string): Location of the MiniZinc standard library.
- **tagDefaults** (list of lists of strings): Each entry maps a tag to the default solver for that tag. For example, `[["cp","org.chuffed.chuffed"],["mip","org.minizinc.gurobi"]]` would declare that whenever a solver with tag "cp" is requested, Chuffed should be used, and for the "mip" tag, Gurobi is the default. The empty tag ("") can be used to define the system-wide default solver (i.e., the solver that is chosen when running minizinc without the --solver argument).
- **solverDefaults** (list of lists of strings): Each entry consists of a list of three strings: a solver identifier, a command line option, and a value for that command line option. For example, `[["org.minizinc.gurobi","--gurobi-dll","/Library/gurobi752/mac64/lib/libgurobi75.so"]]]` would specify the Gurobi shared library to use (on a macOS system with Gurobi 7.5.2). For command line options that don’t take a value, you have to specify an empty string, e.g. `[["org.minizinc.gurobi","--uniform-search",""]].`

Here is a sample configuration file:

```json
{
  "mzn_solver_path": ["/usr/share/choco"],
  "tagDefaults": [["cp","org.choco-solver.choco"],["mip","org.minizinc.cplex"], [""],["","org.gecode.gecode"]],
  "solverDefaults": [["org.minizinc.cplex","--cplex-dll","/opt/CPLEX_ Studio128/cplex/bin/x86-64_sles10_4.1/libcplex128.so"]]
}
```

Configuration values in the user-specific configuration file override the global values, except for solver default arguments, which are only overridden if the name of the option is the same, and otherwise get added to the command line.

Note: Due to current limitations in MiniZinc’s JSON parser, we use lists of strings rather than objects for the default mappings. This may change in a future release, but the current syntax will remain valid. The location of the global configuration is currently the share/minizinc directory.
of the MiniZinc installation. This may change in future versions to be more consistent with file
system standards (e.g., to use /etc on Linux and /Library/Preferences on macOS).
The MiniZinc IDE lets you edit and run MiniZinc models. It requires a working installation of the MiniZinc tool chain (which is included when you download the *bundled version* of the IDE). For installation instructions, see Section 1.2. This document assumes that you have installed the IDE from the bundled binary distribution, which means that it will already be configured correctly when you start it for the first time. The configuration options are described in Section 3.2.5.

Section 1.3 contains an introduction to the absolute basics of working with the IDE. This document goes into a bit more detail.

### 3.2.1 Editing files

The basic editor provides the usual functionality of a simple text editor. You can edit MiniZinc models (file extension `.mzn`) and data files (`.dzn`). When you first open the MiniZinc IDE, you are presented with an empty *Playground* editor, which lets you quickly try out simple MiniZinc models (it does not have to be saved to a file before running a solver on it).

Each file will be opened in a separate tab. To switch between files, click on the tab, select the file from the *Window* menu, or use the *Previous tab*/*Next tab* options from the *View* menu.

When saving a file, make sure that you select the correct file type (represented by the file extension). Models should be saved as `.mzn` files, while data files should have a `.dzn` extension. This is important because the IDE will only let you invoke the solver on model files.
3.2.1.1 Editing functions

The Edit menu contains the usual functions for editing text files, such as undo/redo, copy/cut/paste, and find/replace. It also allows you to jump to a particular line number (Go to line), and to shift the currently selected text (or the current line if nothing is selected) right or left by two spaces. The (Un)comment option will turn the current selection (or current line) into comments, or remove the comment symbols if it is already commented.

3.2.1.2 Fonts and dark mode

You can select the font and font size in the View menu. We recommend to use a fixed-width font (the IDE should pick such a font by default).

Depending on your platform, the View menu also lets you activate “dark mode”, which switches the colour scheme to a dark background. On macOS 10.14 and later, the operating systems supports a global dark mode, and the MiniZinc IDE will follow that setting.

3.2.2 Configuring and Running a Solver

The MiniZinc IDE automatically detects which solvers are available to MiniZinc. You can select the solver to use from the solver selection drop-down menu next to the Run icon in the tool bar:

3.2.2.1 Running a model

MiniZinc models can be compiled and run by clicking the Run icon, selecting Run from the MiniZinc menu, or using the keyboard shortcut Ctr1+R (Cmd+R on macOS). The IDE will use the currently selected solver for compiling and running the model.

Running a model will open the Output window, usually located at the bottom of the IDE’s main window. MiniZinc displays progress messages as well as any output received from the solver there. If compilation resulted in an error message, clicking on the error will jump to the corresponding location in the model or data file.

The current run can be aborted by clicking the Stop icon, selecting Stop from the MiniZinc menu, or using the keyboard shortcut Ctr1+E (Cmd+E on macOS).
If the selected model requires some parameters to be set before it can be run, the MiniZinc IDE will open a parameter dialog. It has two tabs. The left tab lets you enter the parameters manually:

The second tab lets you select one or several of the data files that are currently open:

### 3.2.2.2 Solver configurations

Selecting one of the built-in solvers from the drop-down menu activates its default configuration. In order to change the solver’s behaviour, open the solver configuration editor by clicking on the icon in the tool bar, selecting Show configuration editor from the MiniZinc/Solver configurations menu, or using the keyboard shortcut Ctrl+Shift+C (Cmd+Shift+C on macOS).

Fig. 3.2.1 shows the configuration window. The first section (marked with a 1 in a red circle) contains a drop-down menu to select the solver configuration. In this case, a built-in configuration for the COIN-BC solver was selected. You can make this configuration the default (the MiniZinc IDE will remember this setting), you can reset all values to the defaults, and you can make a clone of the configuration. Cloning a configuration is useful if you want to be able to quickly switch between different sets of options.

Note that any changes to the built-in configurations will be lost when you close the IDE. Any changes to a cloned configuration are saved as part of the project (see Section 3.2.3).

The Solving section below contains a number of general options. First of all, it shows the concrete solver used in this configuration. Below that, you can set a time limit, after which the execution
Fig. 3.2.1: The solver configuration window
will be stopped. The built-in configurations all use the “default behaviour” (marked with a 2), which is to print all intermediate solutions for optimisation problems, and stop after the first found solution for satisfaction problems. To change this, you can select User-defined behavior instead (marked with a 3).

The next section, Compiler options (marked with a 4), controls different aspects of the compilation from MiniZinc to FlatZinc for the selected solver. The first two checkboxes control verbosity and statistics output of the compiler. The drop-down below controls the optimisation level of the compiler, i.e., how much effort it should spend on creating the best possible FlatZinc representation for the given model. The two input fields below allow you to specify additional data (passed into the compilation as if it was part of a .dzn file) and additional command line options for the compiler.

The Solver options section (marked with a 5) contains configuration options for the selected solver. Only options that are supported by the solver will be available, others will be grayed out (e.g., the selected solver in Fig. 3.2.1 does not support setting a random seed, or free search).

Finally, the Output options section gives you control over the output behaviour. The first tick box enables you to clear the Output window automatically every time you run a model. The second option inserts timing information into the stream of solutions. The third check box (Check solutions) is described in Section 3.2.2.3 below. The Compress solution output option is useful for problems that produce a lot (read: thousands) of solutions, which can slow down and clutter the output window. The compression works by printing just the number of solutions rather than the solutions themselves. For example, the following model would produce 1000 solutions when run with User-defined behavior and solution limit set to 0:

```plaintext
var 1..1000: x;
solve satisfy;
```

When running with compression set to 100, MiniZinc will output the first 100 solutions, and then a sequence of output like this:

```plaintext
[ 100 more solutions ]
[ 200 more solutions ]
[ 400 more solutions ]
[ 199 more solutions ]
x = 1000;
---------
--------
```

The number of solutions captured by one of the ... more solutions lines is doubled each time,
in order to keep the overall output low. The last solution produced by the solver will always be printed (since, in the case of optimisation problems, the last solution is the best one found).

3.2.2.3 Automatic Solution Checking

MiniZinc can automatically run the output of a model through a solution checker, another MiniZinc model that verifies that the solution satisfies a given set of rules. This can be useful for teaching constraint modelling, if the solution checker is given to students. Another use case is to use a simple checker while working on a more complex model, to ensure that the complex model still meets the specification.

The default behaviour of the MiniZinc IDE is to run a solution checker if one is present. For a model abc.mzn, a solution checker must be called abc.mzc or abc.mzc.mzn. If a checker is present, the Run icon will turn into a Run + check icon instead. The output of the solution checker is displayed together with the normal solution output in the Output window.

You can disable solution checkers by deselecting the Check solutions option in the solver configuration window.

You can find a detailed description of MiniZinc solution checkers in ch-solution-checkers.

3.2.2.4 Compiling a model

It can sometimes be useful to look at the FlatZinc code generated by the MiniZinc compiler for a particular model. You can use the Compile option from the MiniZinc menu to compile the model without solving. The generated FlatZinc will be opened in a new tab. You can edit and save the FlatZinc to a file, and run it directly (without recompiling).

3.2.2.5 Profiling a model

MiniZinc can produce statistics about the translation of a model to FlatZinc, which can be helpful in order to better understand the behaviour of the model. In the IDE, this feature can be accessed from the MiniZinc menu: Select Profile compilation, which will run the compiler with the current settings (such as selected solver etc.) and output compilation statistics next to each line of code.

For the Magic Series puzzle from Listing 2.6.2, profiling using the Gurobi solver backend could produce the statistics shown in Fig. 3.2.2. The first column ( “Cons” ) represents the number of FlatZinc constraints generated from each line of code, the second column ( “Vars” ) the number of variables, and the third column ( “Time” ) the time it took to compile the line. In this example, for n=10, the constraint in lines 4 and 5 creates 100 variables (one 0/1 variable for each of the...
bool2int calls), and the constraints in lines 7 and 8 each result in a single FlatZinc constraint and no additional variables.

![Screenshot of MiniZinc IDE]

Fig. 3.2.2: Profiling results for the Magic Series problem (n=10)

### 3.2.3 Working With Projects

Each main window of the MiniZinc IDE corresponds to a project, a collection of files and settings that belong together. A project can be saved to and loaded from a file.

You can open a new project by selecting the New project option from the File menu, or using the Ctrl+Shift+N keyboard shortcut (Cmd+Shift+N on macOS).

All the files that belong to the current project are shown in the Project explorer (see Fig. 3.2.3), which can be opened using the tool bar icon, or using the Show project explorer option in the View menu. The project explorer lets you run the model in the currently active tab with any of the data files by right-clicking on a .dzn file and selecting Run model with this data. Right-clicking any file presents a number of options: opening it, removing it from the project, renaming it, running it, and adding new files to the project.

A saved project contains the following pieces of information:

- The names of all files in the project. These are stored as relative paths (relative to the project file).
Fig. 3.2.3: Project explorer
• Which files were open in tabs (and in which order) at the time the project was saved.
• The active solver configuration.
• The state of any cloned solver configuration.

The following will not be saved as part of the project:
• The contents of the Output window.
• The state of the built-in solver configurations.

### 3.2.4 Submitting Solutions to Online Courses

The MiniZinc IDE has built-in support for submitting solutions and models to online courses, including the Coursera courses that introduce modelling in MiniZinc:

• Basic Modeling for Discrete Optimization[^6]
• Advanced Modeling for Discrete Optimization[^7]

The submission system is controlled by a file called _mooc, which is typically part of the projects that you can download for workshops and assignments. When a project contains this file, a new submission icon will appear in the tool bar, together with an option in the MiniZinc menu.

Clicking the icon (or selecting the menu option) opens the submission dialog (see Fig. 3.2.4). It lets you select the problems that you would like to run on your machine, after which the solutions will be sent to the online course auto-grading system. Some projects may also contain model submissions, which are not run on your machine, but are evaluated by the online auto-grader on new data that was not available to you for testing.

You will have to enter the assignment-specific login details. By clicking the Run and submit button, you start the solving process. When it finishes, the MiniZinc IDE will upload the solutions to the auto-grading platform.

### 3.2.5 Configuration Options

The MiniZinc IDE can be configured through the Preferences dialog in the MiniZinc menu (on Windows and Linux) or the MiniZincIDE menu (on macOS), as shown in Fig. ??.

[^6]: https://www.coursera.org/learn/basic-modeling
[^7]: https://www.coursera.org/learn/advanced-modeling
Fig. 3.2.4: Submitting to Coursera
3.2.5. Configuration Options

MiniZinc Handbook, Release 2.4.3

### Preferences

**MiniZinc**

- **MiniZinc path**: 
  - **Select**
  - **Check**

- **Check for updates to MiniZinc on startup once a day**

**Found MiniZinc installation:**

- **MiniZinc to FlatZinc converter, version 2.2.0**
- **Copyright (C) 2014-2018 Monash University, NICTA, Data61**

**Solver**

**Gurobi <unknown version>**

- **Name**: Gurobi
- **Solver ID**: org.minizinc.mip.gurobi
- **Version**: <unknown version>
- **Executable**: 
  - **Select**

- **Run solver detached from IDE**
- **Run with mzn2fzn and solns2out**

**MiniZinc library path**: -Glinear

- **Supported standard command line flags**:
  - ✔️ -a
  - ✔️ -n
  - ✗ -s
  - ✗ -v
  - ✔️ -p
  - ✗ -r
  - ✗ -f

- **Required solver flags**:
  - --gurobi-dll
  - /Library/gurobi752/mac64/lib/libgurobi75.so

- **Delete**
- **Update**

**Close**
### 3.2.5.1 Locating the MiniZinc installation

The most important configuration option is the path to the minizinc executable. In the example in Fig. ??, this field has been left empty, in which case minizinc is assumed to be on the standard search path (usually the PATH environment variable). Typically, in a bundled binary installation of MiniZinc, this field can therefore be left empty.

If you installed MiniZinc from sources, or want to switch between different versions of the compiler, you can add the path to the directory that contains the minizinc executable here. You can select a directory from a file dialog using the Select button, or enter it manually. Clicking the Check button will check that minizinc can in fact be run, and has the right version. The version of minizinc that was found is displayed below the path input field. Fig. ?? below shows an example where MiniZinc is located at /home/me/minizinc-2.2.0/bin.

You can have the MiniZinc IDE check once a day whether a new version of MiniZinc is available.

### 3.2.5.2 Adding Third-Party Solvers

The Solver section of the configuration dialog can be used to inspect the solvers that are currently available to MiniZinc, and to add new solvers to the system.

#### Configuring existing solvers

You can use the configuration dialog to set up defaults for the installed solvers. In the current version of the MiniZinc IDE, this is limited to configuring the CPLEX and Gurobi backends. The bundled binary version of MiniZinc comes with support for loading CPLEX and Gurobi as plugins, i.e., MiniZinc does not ship with the code for these solvers but can load them dynamically if they are installed.

For example, Fig. ?? shows a potential configuration for Gurobi. On Windows, the library is called gurobiXX.d11 (in the same directory as the gurobi executable), and on Linux and macOS is it libgurobiXX.so (in the lib directory of your Gurobi installation), where XX stands for the version number of Gurobi.

If you select the CPLEX solver, a similar option appears (--cplex-dll). On Windows, the CPLEX library is called cplexXXXX.d11 and typically found in same directory as the cplex executable. On Linux it is libcplexXXX.so, and on macOS libcplexXXXX.jnilib, where XXX and XXXX stand for the version number of CPLEX.
Adding new solvers

The example in Fig. ?? shows a potential configuration for Gecode, which was installed in /home/me/gecode.

Each solver needs to be given

- a name;
- a unique identifier (usually in reverse domain name notation);
- a version string; and
- the executable that can run FlatZinc.

In addition, you can specify the location of a solver-specific MiniZinc library (see Section 4.3.3 for details). If you leave this field empty, the MiniZinc standard library will be used. The path entered into this field should be an absolute path in the file system, without extra quoting, and without any command line arguments (such as -I).

Most solvers will require compilation and output processing, since they only deal with FlatZinc files. For these solvers, the Run with mzn2fzn and solns2out option must be selected. For solvers that can deal with MiniZinc models natively, this option can be deselected.

Some solvers open an independent application with its own graphical user interface. One such example is the Gecode (Gist) solver that comes with the bundled version of the IDE. For these solvers, select the Run solver detached from IDE option, so that the IDE does not wait for solver output.

Finally, you can select which command line flags are supported by the solver. This controls which options will be available in the solver configuration window.

Solver configurations that are edited or created through the IDE are saved in a configuration file in a standard location. These solvers are therefore available the next time the IDE is started, as well as through the minizinc command line tool.
Globalizer\textsuperscript{1} analyses a model and set of data to provide suggestions of global constraints that can replace or enhance a user’s model.

### 3.3.1 Basic Usage

To use Globalizer simply execute it on a model and set of data files:

```bash
minizinc --solver org.minizinc.globalizer model.mzn data-1.dzn data-2.dzn
```

Note: Globalizer may also be executed on a non-parameterised model with no data files.

The following demonstrates the basic usage of Globalizer. Below, we see a simple model for the car sequencing problem\textsuperscript{2}, `cars.mzn`.

```
% cars.mzn
include "globals.mzn";

int: n_cars; int: n_options; int: n_classes;
```

\textsuperscript{1} Leo, K. et al., “Globalizing Constraint Models”, 2013.

\textsuperscript{2} http://www.csplib.org/Problems/prob001/
set of int: steps = 1..n_cars;
set of int: options = 1..n_options;
set of int: classes = 1..n_classes;
array [options] of int: max_per_block;
array [options] of int: block_size;
array [classes] of int: cars_in_class;
array [classes, options] of 0..1: need;

% The class of car being started at each step.
array [steps] of var classes: class;

% Which options are required by the car started at each step.
array [steps, options] of var 0..1: used;

% Block p must be used at step s if the class of the car to be
% produced at step s needs it.
constraint forall (s in steps, p in options) (used[s, p]=need[class[s], p]);

% For each option p with block size b and maximum limit m, no consecutive
% sequence of b cars contains more than m that require option p.
constraint forall (p in options, i in 1..(n_cars - (block_size[p] - 1))) ( 
    sum (j in 0..(block_size[p] - 1)) (used[i + j, p]) <= max_per_block[p]);

% Require that the right number of cars in each class are produced.
constraint forall (c in classes) (count(class, c, cars_in_class[c]));
solve satisfy; % Find any solution.

And here we have a data file cars_data.dzn.

<table>
<thead>
<tr>
<th>Listing 3.3.2: Data for the car sequencing problem (cars_data.dzn).</th>
</tr>
</thead>
<tbody>
<tr>
<td>% cars_data.dzn</td>
</tr>
<tr>
<td>n_cars = 10;</td>
</tr>
<tr>
<td>n_options = 5;</td>
</tr>
<tr>
<td>n_classes = 6;</td>
</tr>
</tbody>
</table>
max_per_block = [1, 2, 1, 2, 1];
block_size = [2, 3, 3, 5, 5];
cars_in_class = [1, 1, 2, 2, 2, 2];

need = array2d(1..6, 1..5, [
    1, 0, 1, 1, 0,
    0, 0, 0, 1, 0,
    0, 1, 0, 0, 1,
    0, 1, 0, 1, 0,
    1, 0, 1, 0, 0,
    1, 1, 0, 0, 0
]);

Executing Globalizer on this model and data file we get the following output:

```
% minizinc --solver globalizer cars.mzn cars_data.dzn

cars.mzn|33|12|33|69 [ ] gcc(class,cars_in_class)
cars.mzn|33|35|33|68 [ ] count(class,cars_in_class[c])
cars.mzn|28|27|28|65;cars.mzn|29|30|32 [ ] sliding_sum(0,max_per_block[p],
   block_size[p],used[_,p])
```

Each line of output is comprised of three elements.

1. Expression locations (separated by semicolon ‘;’ ) that the constraint might be able to replace.
2. (Between square brackets ‘[ ‘]’ ) the context under which the constraint might replace the expressions of 1.
3. The replacement constraint.

From the example above we see that a gcc constraint and a sliding_sum constraint can replace some constraints in the model. Taking the sliding_sum case, we see that it replaces the expression cars.mzn|28|27|28|65 which corresponds to i in 1..(n_cars - (block_size[p] - 1)) and cars.mzn|29|27|28|65 which corresponds to the <= expression including the sum. The original constraint can be replaced with:

```
constraint forall (p in options) (sliding_sum(0, max_per_block[p], block_size[p], used[_,p]));
```
3.3.2 Caveats

**MiniZinc syntax support.** Globalizer was implemented with support for a subset of an early version of the MiniZinc 2.0 language. As a result there are some limitations. First, Globalizer does not support set variables or *enum* types. The array slicing syntax supported in MiniZinc was not decided upon when Globalizer was implemented so it uses an incompatible syntax. Globalizer uses _ instead of ... This can be seen above in the translation of used[_,p] to used[..,p].

**New constraint.** A special two argument version of the global_cardinality constraint called gcc has been added which is not in the standard library. It is defined as follows:

```plaintext
predicate gcc(array[int] of var int: x, array[int] of var int: counts) =
    global_cardinality(x,
        [ i | i in index_set(counts) ],
        array1d(counts));
```

3.3.3 Supported Constraints

Globalizer currently supports detection of the following constraints:

<table>
<thead>
<tr>
<th>alldifferent</th>
<th>alldifferent_except_0</th>
<th>all_equal_int</th>
</tr>
</thead>
<tbody>
<tr>
<td>bin_packing</td>
<td>bin_packing_capa</td>
<td>bin_packing_load</td>
</tr>
<tr>
<td>binaries_represent_int</td>
<td>binaries_represent_int_3A</td>
<td>binaries_represent_int_3B</td>
</tr>
<tr>
<td>count_geq</td>
<td>cumulative Assert</td>
<td>decreasing</td>
</tr>
<tr>
<td>gcc</td>
<td>global_cardinality</td>
<td>inverse</td>
</tr>
<tr>
<td>lex_lesseq_int_checking</td>
<td>lex2_checking</td>
<td>maximum_int_checking</td>
</tr>
<tr>
<td>minimum_int_checking</td>
<td>member</td>
<td>nvalue</td>
</tr>
<tr>
<td>strict_lex2_checking</td>
<td>subcircuit_checking</td>
<td>true</td>
</tr>
<tr>
<td>atleast</td>
<td>atmost</td>
<td>bin_packing_load_ub</td>
</tr>
<tr>
<td>circuit_checking</td>
<td>count</td>
<td>diffn</td>
</tr>
<tr>
<td>distribute</td>
<td>element</td>
<td>increasing</td>
</tr>
<tr>
<td>lex_less_int_checking</td>
<td>sliding_sum</td>
<td>sort_checking</td>
</tr>
<tr>
<td>unary</td>
<td>value_precede_checking</td>
<td></td>
</tr>
</tbody>
</table>
3.3.4 Using Globalizer in the MiniZinc IDE

To use the Globalizer in the MiniZinc IDE, open a model, and several data files. Select Globalizer from the solver configuration dropdown menu. Click the solve button (play symbol). If you did not select any data file, a dialog should pop up that allows you to select data files. To select multiple data files hold the Ctrl or Cmd key while clicking on each data file.

![Dialog box selecting data files]

Click run. While processing your model and data files the MiniZinc IDE will display a progress bar on the toolbar.
Any discovered global constraints will be listed in the output pane of the IDE. Clicking on one of these constraints will highlight in the model the expressions that might be replaceable or strengthened by the constraint.

### 3.3.5 How it works

A summary of the algorithm is presented here. For a more detailed exploration of the approach see the Globalizing Constraint Models paper\(^1\).

- **Normalize.** In this step the model is transformed to make analysis easier.
  - Conjunctions of constraints are broken into individual constraints. For example: \( C_1 \land C_2 \); becomes \( C_1; C_2 \);
  - Conjunctions under forall constraints are broken into individual forall constraints.
For example: `forall(...) (C1 \ C2);` becomes
\[
\forall i \in 1..n \ (c(i));
\]
\[
\forall i \in 1..n \ (c(i)); \forall i \in 1..n \ (c(i));
\]
• **Generate submodels.** In this step all subsets containing 1 or 2 constraints (this can be configured using `--numConstraints` argument) are enumerated.

• **Instantiate and unroll into groups.** Each submodel is instantiated with the provided data files. These instantiated submodels are further used to produce more submodels by unrolling loops. For example, a constraint such as `forall(i in 1..n) (c(i));` will be used to produce the constraints: `c(1)` and `c(n)` for each instantiation of `n` in the data files. All instantiated submodels are then grouped together.

• **Process groups.** For each submodel in a group a set of 30 random solutions are found. (configurable using the `--randomSolutions` argument). A template model with all of these random solutions is created.

The different variables (including arrays and array accesses) and parameters used by a submodel are collected into a set of potential arguments along with the constant 0 and a special blank symbol representing an argument that can be inferred based on others used.

The list of constraints above is filtered based on the arguments available. For example, the `alldifferent` constraint will be removed from the list of candidates if the submodel does not reference any arrays of variables.

Finally the set of constraints are filtered by adding them to the template and solving it. If the template is satisfiable, the constraint accepts all of the random solutions.

30 sample solutions for each candidate constraint are generated. (configurable using the `--sampleSolutions` argument). The candidate is then ranked based on how many of these solutions are also solutions to the original submodel. If its score is less than some threshold the candidate is removed from the set of candidates for this group and will not be tested on later submodels.

• **Report.** The remaining candidates are presented to the user.

### 3.3.6 Performance tips

Globalizing constraint models can be a time consuming task. If

• **Use small or relatively easy instances.** Globalizer solves many subproblems while processing your model and data files. Using easier instances can speed up the process considerably.
• **Disable the initial pass.** As discussed above, Globalizer performs two passes. The first pass tries to detect alternate viewpoints that can be added to your model. If you are confident that this will not be useful we recommend disabling this first pass using the `--no-initial-pass` argument.

• **Narrow search using filters.** Globalizer attempts to match a large selection of global constraints to subproblems in your model. If you wish to check if only a few specific constraints are present you can focus Globalizer using the `--constraintFilter` or `-f` arguments followed by a comma separated list of strings. Only global constraints where one of the strings is a substring will be included in the search.

• **Disable implies check.** The implication check can also be disabled to improve performance. This results in less accurate results but may still help a user to understand the structure of their model.

• **Free-search.** To improve accuracy and to avoid false positives, subproblems solved by Globalizer are solved using a random heuristic and restarts. If the subproblems are particularly difficult to solve a random heuristic may be prohibitive and the solver may not be able to produce enough samples within the solving timelimit. In these circumstances a user can either increase the solver timeout using the `-t` argument followed by the number of milliseconds a solver should be given to find samples. Alternatively the `--free-search` argument can be used to force Globalizer to use the solver’s free search to find samples. This has the downside of reducing the diversity of samples but allows enough samples to be found to allow suggested globals to be found.

### 3.3.7 Limitations / Future work

• Globalizer supports only a subset of the MiniZinc language and as such cannot be executed on all MiniZinc models.

• There are some relatively cheap syntactic approaches that should be performed before Globalization that currently is not implemented. For example, there are several common formulations of an `alldifferent` constraint that can be detected syntactically. This would be much cheaper than using Globalizer.
Version: 0.5.0

FindMUS\(^1\) lists unsatisfiable subsets of constraints in your MiniZinc model. These subsets, called Minimal Unsatisfiable Subsets can help you to find faults in your unsatisfiable constraint model. FindMUS uses the hierarchical structure provided by the model to guide its search.

### 3.4.1 Basic Usage

To use FindMUS on the command line simply execute it on a model and set of data files by typing:

`findMUS model.mzn data-1.dzn`

This will perform a simple search for a single generalized MUS. To acquire a more accurate MUS with correct index variable assignments the `--paramset mzn` argument should be used. To search for more than one MUS use the `-a` or `-n <count>` arguments. By default FindMUS will use gecode for checking the satisfiability of a subset of constraints. This can be changed using the `--solver` argument. For example, given a model that is better suited to a Mixed Integer Programming solver, the argument `--solver cbc` could be used to instruct FindMUS to flatten the model with the linear library and use the CBC solver for satisfiability checking.

Note: FindMUS requires a fully instantiated constraint model.

3.4.1.1 Commandline arguments

The FindMUS tool supports the following arguments:

Driver options

The driver creates the map solver and sub-solver and requests MUSes from FindMUS’s enumeration algorithm HierMUS.

- `n &lt;count&gt;` Stop after the n th MUS is found (Default: 1)
- `a` Find all MUSes
- `--timeout &lt;s&gt;` Stop search after s seconds (Default: 1800)
- `--paramset hint,mzn,fzn` Use a preset collection of parameters. The default behaviour of the tool is to use the hint set which should quickly produce an MUS. The mzn and fzn options can acquire MUSes in more detail.
  
  hint –structure gen –depth mzn –shrink-alg map_lin
  mzn –structure normal –depth mzn –shrink-alg map_qx
  fzn –structure normal –depth fzn –shrink-alg map_qx

Enumeration options

The enumeration algorithm (HierMUS) explores the constraint hierarchy provided in the user’s model and proposes potential MUSes to the sub-solver. A shrink algorithm is used to reduce found unsat subsets to MUSes. There are currently four available shrink algorithms available with FindMUS.

- `--shrink-alg lin,map_lin,qx,map_qx` Select shrink algorithm (Default: map_lin)
  
  lin simple linear shrink algorithm.
  map_lin alternative implementation of the linear algorithm that updates a global map of explored subsets while shrinking the subset.
  qx uses the QuickXplain⁴ algorithm to shrink the subset to a MUS.
  map_qx is an alternative implementation of QuickXplain that uses the global map to avoid repeating any SAT checks while shrinking.

- `--depth mzn,fzn,&lt;n&gt;` How deep in the tree should search explore. (Default: mzn)

mzn expands the search as far as the point when the compiler leaves the MiniZinc model.

fzn expands search as far as the FlatZinc constraints.

&lt;n&gt expand search to the n level of the hierarchy.

**Subsolver options**

FindMUS can be used in conjunction with any FlatZinc solver. These options mimic the minizinc arguments --solver and --fzn-flags. The behavior of these arguments is likely to change in later versions of the tool.

--solver &lt;s&gt; Use solver s for SAT checking. (Default: “gecode”)

--solver-flags &lt;f&gt; Pass flags f to sub-solver. (Default: empty)

--solver-timelimit &lt;ms&gt; Set hard time limit for solver in milliseconds. (Default: 1100)

--soft-defines Consider function constraints as part of MUSes

**Filtering options**

FindMUS can include or exclude constraints from its search based on the expression and constraint name annotations as well as properties of their paths (for example, line numbers). These filters are currently based on substrings but in future may support text spans and regular expressions.

--named-only Only consider constraints annotated with string annotations

--filter-named &lt;names&gt; --filter-named-exclude &lt;names&gt; Include/exclude constraints with names that match sep separated names

--filter-path &lt;paths&gt; --filter-path-exclude &lt;paths&gt; Include/exclude based on paths

--filter-sep &lt;sep&gt; Separator used for named and path filters

**Structure options**

The structure coming from a user’s model can significantly impact the performance of a MUS enumeration algorithm. The hierarchy is constructed in two steps. The first takes the existing model structure and can modify it in several ways. The second step adds extra binary structure.

--structure flat, gen, normal, mix

    Alters initial structure: (Default: gen)

    flat Remove all structure
gen Remove instance specific structure (merges constraints)

normal No change

mix Apply gen before normal

idx Remove all location information

idxmix Apply idx before normal

--no-binarize

By default will introduce extra binary structure to the hierarchy. This option disables this behaviour.

Verbosity options

--verbose-{map,enum,subsolve} &lt;n&gt; Set verbosity level for different components

--verbose Set verbosity level of all components to 1

Misc options

--dump-dot &lt;dot&gt; Write tree in GraphViz format to file &lt;dot&gt;

3.4.1.2 Example

The following demonstrates the basic usage of FindMUS on a simple example. Below, we see a model for the latin squares puzzle\(^2\) with some incompatible symmetry breaking constraints added.

```
Listing 3.4.1: Faulty model for Latin Squares (latin_squares.mzn).

% latin_squares.mzn
include "globals.mzn";

int: n = 3;
set of int: N = 1..n;
array[N, N] of var N: X;

constraint :: "ADRRows"
forall (i in N)
   (alldifferent(row(X, i)) :: "AD(row \(\backslash(i)\))");
constraint :: "ADCols"
```

\(^2\)https://en.wikipedia.org/wiki/Latin_square
forall (j in N)
  (alldifferent(col(X, j)) :: "AD(col \(\backslash\)j)")

constraint :: "LLRows"
forall (i in 1..n-1)
  (lex_less(row(X, i), row(X, i+1)) :: "LL(rows \(\backslash\)i \(\backslash\)(i+1))")

constraint :: "LGCols"
forall (j in 1..n-1)
  (lex_greater(col(X, j), col(X, j+1)) :: "LG(cols \(\backslash\)j \(\backslash\)(j+1))")

solve satisfy;

output [ show2d(X) ];

Here we have used the new constraint and expression annotations added in MiniZinc 2.2.0. Note that these annotations are not necessary for FindMUS to work but may help with interpreting the output. The first two constraints: ADRows and ADCols define the alldifferent constraints on the respective rows and columns of the latin square. The next two constraints LLRows and LGCols post lex constraints that order the rows to be increasing and the columns to be increasing. Certain combinations of these constraints are going to be in conflict.

Executing the command minizinc --solver -a latin_squares.mzn returns the following output. Note that the -a argument requests all MUSes that can be found with the default settings (more detail below).
The first two lines, starting with FznSubProblem: and SubsetMap provide some useful information for debugging the findMUS tool. Next we have the list of MUSes separated by a series of equals = signs. Each MUS is described with three sections:

1. MUS: lists the indices of FlatZinc constraints involved in this MUS.

2. Brief: lists the FlatZinc constraint name, the expression name, and the constraint name for each involved FlatZinc constraint.

3. Traces: lists the MiniZinc paths corresponding to the constraints of the MUS. Each path typically contains a list of path elements separated by semi-colons ;. Each element includes a file path, a start line, start column, end line and end column denoting a span of text from the listed file. And finally, a description of the element. In the examples above all paths
point to calls to a \texttt{forall} on different lines of the model. (\texttt{ca}\mid \texttt{forall})

The final line of output lists the Total Time, the number of MUSes found, and some statistics about the number of times the internal map solver \texttt{map} was called, and the number of times the subproblem solver was called \texttt{sat}.

Interpreting the two MUSes listed here we see that the \texttt{lex} constraints from lines 16 and 19 were included in both and only one of the \texttt{allDifferent} constraints from line 9 and 12 are required for the model to be unsatisfiable. The \texttt{lex} constraints being involved in every MUS make them a strong candidate for being the source of unsatisfiability in the user’s model.

\subsection*{3.4.2 Using FindMUS in the MiniZinc IDE}

To use FindMUS in the MiniZinc IDE, upon discovering that a model is unsatisfiable. Select \texttt{FindMUS} from the solver configuration dropdown menu and click the solve button (play symbol). By default FindMUS is configured to return a single MUS using the \texttt{hint} parameter set. This should be relatively fast and help locate the relevant constraint items. The following shows the result of running FindMUS with the default options.
Selecting the returned MUS highlights three top level constraints as shown: ADRows, LLRows and LGCols. To get a more specific MUS we can instruct FindMUS to go deeper than the top level constraints by clicking the “Show configuration editor” button in the top right hand corner of the MiniZinc IDE window, and selecting the “Extra solver options” and selecting the mzn option under “Preset parameters”. The following shows a more specific MUS in this model.
In this case we can see that the output pane lists more specific information about the constraints involved in the MUS. After each listed constraint name we see what any loop variables were assigned to when the constraint was added to the FlatZinc. For example (j=2).
3.4.3 How it works

A simple explanation of the algorithm is presented here. For a more detailed exploration of an earlier version of the approach see the Debugging Unsatisfiable Constraint Models paper\(^1\).

The approach takes the full FlatZinc program and partitions the constraints into groups based on the hierarchy provided in the user’s model. To begin with (at depth ‘1’) we search for MUSes in the set of top level constraint items using the MARCO\(^3\) MUS enumeration algorithm. If we are not at the target depth we recursively select a found MUS, split its constituent constraints into lower level constraints based on the hierarchy and begin another search for MUSes underneath this high-level MUS. If any MUSes are found we know that the high-level MUS is not minimal and so it should not be reported. This process is repeated on any found unsatisfiable subsets until we reach the required depth at which point we will start to report MUSes. If in the recursive search we return to a high-level MUS without finding any sub-MUSes we can report this MUS as a real MUS. This recursive process is referred to as HierMUS. At each stage when we request the enumeration of a set of MUSes underneath a high-level MUS we can use one of several MUS enumeration algorithms.

3.4.4 Performance tips

If you are trying to find MUSes in a very large instance it is advised to make use of the filtering tools available. Use the default settings to find a very high-level MUS and then use the --depth option to find lower-level, more specific MUSes in conjunction with the --filter-name and --filter-path options to focus on finding specific sub-MUSes of a found high-level MUS.

3.4.5 Limitations / Future work

There are several features that we aim to include quite soon:

- **Regular expression based filtering** This will allow more complex filtering to be used.

- **Text span based filtering** This will allow a user to simply click-and-drag a selection around the parts of a constraint model they wish to analyse.

---


CHAPTER 3.5

Using MiniZinc in Jupyter Notebooks

You can use MiniZinc inside a Jupyter / IPython notebook using the iminizinc Python module. The module provides a “cell magic” extension that lets you solve MiniZinc models.

The module requires an existing installation of MiniZinc.

3.5.1 Installation

You can install or upgrade this module via pip:

```bash
pip install -U iminizinc
```

Consult your Python documentation to find out if you need any extra options (e.g. you may want to use the --user flag to install only for the current user, or you may want to use virtual environments).

Make sure that the minizinc binary are on the PATH environment variable when you start the notebook server. The easiest way to do that is to get the “bundled installation” that includes the MiniZinc IDE and a few solvers, available from GitHub here: https://github.com/MiniZinc/MiniZincIDE/releases/latest You then need to change your PATH environment variable to include the MiniZinc installation.
3.5.2 Basic usage

After installing the module, you have to load the extension using %load_ext iminizinc. This will enable the cell magic %%minizinc, which lets you solve MiniZinc models. Here is a simple example:

In[1]: %load_ext iminizinc

In[2]: n=8

In[3]: %%minizinc

    include "globals.mzn";
    int: n;
    array[1..n] of var 1..n: queens;
    constraint all_different(queens);
    constraint all_different([queens[i]+i | i in 1..n]);
    constraint all_different([queens[i]-i | i in 1..n]);
    solve satisfy;

Out[3]: {u'queens': [4, 2, 7, 3, 6, 8, 5, 1]}

As you can see, the model binds variables in the environment (in this case, n) to MiniZinc parameters, and returns an object with fields for all declared decision variables.

Alternatively, you can bind the decision variables to Python variables:

In[1]: %load_ext iminizinc

In[2]: n=8

In[3]: %%minizinc -m bind

    include "globals.mzn";
    int: n;
    array[1..n] of var 1..n: queens;
    constraint all_different(queens);
    constraint all_different([queens[i]+i | i in 1..n]);
    constraint all_different([queens[i]-i | i in 1..n]);
    solve satisfy;
If you want to find all solutions of a satisfaction problem, or all intermediate solutions of an optimisation problem, you can use the -a flag:

In[1]: %load_ext iminizinc

In[2]: n=6

In[3]: %minizinc -a

    include "globals.mzn";
    int: n;
    array[1..n] of var 1..n: queens;
    constraint all_different(queens);
    constraint all_different([queens[i]+i | i in 1..n]);
    constraint all_different([queens[i]-i | i in 1..n]);
    solve satisfy;

Out[3]: [{u'queens': [5, 3, 1, 6, 4, 2]},
         {u'queens': [4, 1, 5, 2, 6, 3]},
         {u'queens': [3, 6, 2, 5, 1, 4]},
         {u'queens': [2, 4, 6, 1, 3, 5]}]

The magic supports a number of additional options, in particular loading MiniZinc models and data from files. Some of these may only work with the development version of MiniZinc (i.e., not the one that comes with the bundled binary releases). You can take a look at the help using

In[1]: %minizinc?
MiniZinc can be directly used from Python using the MiniZinc Python package. This package provides an native python interface that interacts directly with the MiniZinc bundle installed on the machine. For installation instructions and more information about the package that provides the MiniZinc python interface please visit https://minizinc-python.readthedocs.io/.

**Warning:**

The development of MiniZinc Python is still in the very early stages. We appreciate any feedback for the package, but we would recommend refraining from using the package in production software. The package can drastically change and backwards compatible changes are not guaranteed.
Part 4

Reference Manual
CHAPTER 4.1

Specification of MiniZinc

4.1.1 Introduction

This document defines MiniZinc, a language for modelling constraint satisfaction and optimisation problems.

MiniZinc is a high-level, typed, mostly first-order, functional, modelling language. It provides:

- mathematical notation-like syntax (automatic coercions, overloading, iteration, sets, arrays);
- expressive constraints (finite domain, set, linear arithmetic, integer);
- support for different kinds of problems (satisfaction, explicit optimisation);
- separation of data from model;
- high-level data structures and data encapsulation (sets, arrays, enumerated types, constrained type-insts);
- extensibility (user-defined functions and predicates);
- reliability (type checking, instantiation checking, assertions);
- solver-independent modelling;
- simple, declarative semantics.
MiniZinc is similar to OPL and moves closer to CLP languages such as ECLiPSe.

This document has the following structure. Notation (page 200) introduces the syntax notation used throughout the specification. Overview of a Model (page 201) provides a high-level overview of MiniZinc models. Syntax Overview (page 204) covers syntax basics. High-level Model Structure (page 205) covers high-level structure: items, multi-file models, namespaces, and scopes. Types and Type-insts (page 207) introduces types and type-insts. Expressions (page 218) covers expressions. Items (page 235) describes the top-level items in detail. Annotations (page 244) describes annotations. Partiality (page 245) describes how partiality is handled in various cases. Built-in Operations (page 248) describes the language built-ins. Full grammar (page 260) gives the MiniZinc grammar. Content-types (page 258) defines content-types used in this specification.

This document also provides some explanation of why certain design decisions were made. Such explanations are marked by the word Rationale and written in italics, and do not constitute part of the specification as such. Rationale: These explanations are present because they are useful to both the designers and the users of MiniZinc.

4.1.1 Original authors.

The original version of this document was prepared by Nicholas Nethercote, Kim Marriott, Reza Rafeh, Mark Wallace and Maria Garcia de la Banda. MiniZinc is evolving, however, and so is this document.

For a formally published paper on the MiniZinc language and the superset Zinc language, please see:


4.1.2 Notation

The basics of the EBNF used in this specification are as follows.

- Non-terminals are written between angle brackets, `<item>`.
• Terminals are written in double quotes, e.g. "constraint". A double quote terminal is written as a sequence of three double quotes: """.

• Optional items are written in square brackets, e.g. [ "var" ].

• Sequences of zero or more items are written with parentheses and a star, e.g. ("","<ident>\")*.

• Sequences of one or more items are written with parentheses and a plus, e.g. (<msg>)+.

• Non-empty lists are written with an item, a separator/terminator terminal, and three dots. For example, this:

```
<expr> "","... 
```

is short for this:

```
<expr> ( "","<expr>\")* [ ""," ]
```

The final terminal is always optional in non-empty lists.

• Regular expressions are used in some productions, e.g. [-+][0-9]+.

MiniZinc’s grammar is presented piece-by-piece throughout this document. It is also available as a whole in Full grammar (page 260). The output grammar also includes some details of the use of whitespace. The following conventions are used:

• A newline character or CRLF sequence is written \n.

### 4.1.3 Overview of a Model

Conceptually, a MiniZinc problem specification has two parts.

1. The model: the main part of the problem specification, which describes the structure of a particular class of problems.

2. The data: the input data for the model, which specifies one particular problem within this class of problems.

The pairing of a model with a particular data set is a model instance (sometimes abbreviated to instance).

The model and data may be separated, or the data may be “hard-wired” into the model. Model Instance Files (page 206) specifies how the model and data can be structured within files in a
model instance.

There are two broad classes of problems: satisfaction and optimisation. In satisfaction problems all solutions are considered equally good, whereas in optimisation problems the solutions are ordered according to an objective and the aim is to find a solution whose objective is optimal. *Solve Items* (page 238) specifies how the class of problem is chosen.

### 4.1.3.1 Evaluation Phases

A MiniZinc model instance is evaluated in two distinct phases.

1. Instance-time: static checking of the model instance.
2. Run-time: evaluation of the instance (i.e., constraint solving).

The model instance may not compile due to a problem with the model and/or data, detected at instance-time. This could be caused by a syntax error, a type-inst error, the use of an unsupported feature or operation, etc. In this case the outcome of evaluation is a static error; this must be reported prior to run-time. The form of output for static errors is implementation-dependent, although such output should be easily recognisable as a static error.

An implementation may produce warnings during all evaluation phases. For example, an implementation may be able to determine that unsatisfiable constraints exist prior to run-time, and the resulting warning given to the user may be more helpful than if the unsatisfiability is detected at run-time.

An implementation must produce a warning if the objective for an optimisation problem is unbounded.

### 4.1.3.2 Run-time Outcomes

Assuming there are no static errors, the output from the run-time phase has the following abstract form:

\[
\text{<output> ::= <no-solutions> [ <warnings> ] <free-text>}
\]

\[
\text{ | ( <solution> )* [ <complete> ] [ <warnings> ] <free-text>}
\]

If a solution occurs in the output then it must be feasible. For optimisation problems, each solution must be strictly better than any preceding solution.

If there are no solutions in the output, the outcome must indicate that there are no solutions.
If the search is complete the output may state this after the solutions. The absence of the completeness message indicates that the search is incomplete.

Any warnings produced during run-time must be summarised after the statement of completeness. In particular, if there were any warnings at all during run-time then the summary must indicate this fact.

The implementation may produce text in any format after the warnings. For example, it may print a summary of benchmarking statistics or resources used.

### 4.1.3.3 Output

Implementations must be capable of producing output of content type application/x-zinc-output, which is described below and also in Content-types (page 258). Implementations may also produce output in alternative formats. Any output should conform to the abstract format from the previous section and must have the semantics described there.

Content type application/x-zinc-output extends the syntax from the previous section as follows:

```
<solution>::= <solution-text> [ \n ] "---------" \n
The solution text for each solution must be as described in Output Items (page 239). A newline must be appended if the solution text does not end with a newline. Rationale: This allows solutions to be extracted from output without necessarily knowing how the solutions are formatted. Solutions end with a sequence of ten dashes followed by a newline.

<no-solutions>::= "=====UNSATISFIABLE=====" \n
The completeness result is printed on a separate line. Rationale: The strings are designed to clearly indicate the end of the solutions.

<complete>::= "---------" \n
If the search is complete, a statement corresponding to the outcome is printed. For an outcome of no solutions the statement is that the model instance is unsatisfiable, for an outcome of no more solutions the statement is that the solution set is complete, and for an outcome of no better solutions the statement is that the last solution is optimal. Rationale: These are the logical implications of a search being complete.

4.1.3. Overview of a Model
If the search is incomplete, one or more messages describing reasons for incompleteness may be printed. Likewise, if any warnings occurred during search they are repeated after the completeness message. Both kinds of message should have lines that start with % so they are recognized as comments by post-processing. Rationale: This allows individual messages to be easily recognised.

For example, the following may be output for an optimisation problem:

```
UNSATISFIABLE
% trentin.fzn:4: warning: model inconsistency detected before search.
```

Note that, as in this case, an unbounded objective is not regarded as a source of incompleteness.

### 4.1.4 Syntax Overview

#### 4.1.4.1 Character Set

The input files to MiniZinc must be encoded as UTF-8.

MiniZinc is case sensitive. There are no places where upper-case or lower-case letters must be used.

MiniZinc has no layout restrictions, i.e., any single piece of whitespace (containing spaces, tabs and newlines) is equivalent to any other.

#### 4.1.4.2 Comments

A % indicates that the rest of the line is a comment. MiniZinc also has block comments, using symbols /* and */ to mark the beginning and end of a comment.

#### 4.1.4.3 Identifiers

Identifiers have the following syntax:
A number of keywords are reserved and cannot be used as identifiers. The keywords are: `ann`, `annotation`, `any`, `array`, `bool`, `case`, `constraint`, `diff`, `div`, `else`, `elseif`, `endif`, `enum`, `false`, `float`, `function`, `if`, `in`, `include`, `int`, `intersect`, `let`, `list`, `maximize`, `minimize`, `mod`, `not`, `of`, `op`, `opt`, `output`, `par`, `predicate`, `record`, `satisfy`, `set`, `solve`, `string`, `subset`, `superset`, `symdiff`, `test`, `then`, `true`, `tuple`, `type`, `union`, `var`, `where`, `xor`.

A number of identifiers are used for built-ins; see *Built-in Operations* (page 248) for details.

### 4.1.5 High-level Model Structure

A MiniZinc model consists of multiple items:

\[
\text{<model>} ::= \left[ \text{<item>} ; ; \text{...} \right]
\]

Items can occur in any order; identifiers need not be declared before they are used. Items have the following top-level syntax:

\[
\text{<item>} ::= \text{<include-item>}
\| \text{<var-decl-item>}
\| \text{<enum-item>}
\| \text{<assign-item>}
\| \text{<constraint-item>}
\| \text{<solve-item>}
\| \text{<output-item>}
\| \text{<predicate-item>}
\| \text{<test-item>}
\| \text{<function-item>}
\| \text{<annotation-item>}
\]

\[
\text{<ti-expr-and-id>} ::= \text{<ti-expr>} ':' \text{<ident>}
\]
Include items provide a way of combining multiple files into a single instance. This allows a model to be split into multiple files (Include Items (page 235)).

Variable declaration items introduce new global variables and possibly bind them to a value (Variable Declaration Items (page 235)).

Assignment items bind values to global variables (Assignment Items (page 238)).

Constraint items describe model constraints (Constraint Items (page 238)).

Solve items are the “starting point” of a model, and specify exactly what kind of solution is being looked for: plain satisfaction, or the minimization/maximization of an expression. Each model must have exactly one solve item (Solve Items (page 238)).

Output items are used for nicely presenting the result of a model execution (Output Items (page 239)).

Predicate items, test items (which are just a special type of predicate) and function items introduce new user-defined predicates and functions which can be called in expressions (User-defined Operations (page 240)). Predicates, functions, and built-in operators are described collectively as operations.

Annotation items augment the ann type, values of which can specify non-declarative and/or solver-specific information in a model.

4.1.5.1 Model Instance Files

MiniZinc models can be constructed from multiple files using include items (see Include Items (page 235)). MiniZinc has no module system as such; all the included files are simply concatenated and processed as a whole, exactly as if they had all been part of a single file. Rationale: We have not found much need for one so far. If bigger models become common and the single global namespace becomes a problem, this should be reconsidered.

Each model may be paired with one or more data files. Data files are more restricted than model files. They may only contain variable assignments (see Assignment Items (page 238)).

Data files may not include calls to user-defined operations.

Models do not contain the names of data files; doing so would fix the data file used by the model and defeat the purpose of allowing separate data files. Instead, an implementation must allow one or more data files to be combined with a model for evaluation via a mechanism such as the command-line.

When checking a model with data, all global variables with fixed type-insts must be assigned, unless they are not used (in which case they can be removed from the model without effect).
A data file can only be checked for static errors in conjunction with a model, since the model contains the declarations that include the types of the variables assigned in the data file.

A single data file may be shared between multiple models, so long as the definitions are compatible with all the models.

### 4.1.5.2 Namespaces

All names declared at the top-level belong to a single namespace. It includes the following names.

1. All global variable names.
2. All function and predicate names, both built-in and user-defined.
3. All enumerated type names and enum case names.
4. All annotation names.

Because multi-file MiniZinc models are composed via concatenation (Model Instance Files (page 206)), all files share this top-level namespace. Therefore a variable x declared in one model file could not be declared with a different type in a different file, for example.

MiniZinc supports overloading of built-in and user-defined operations.

### 4.1.5.3 Scopes

Within the top-level namespace, there are several kinds of local scope that introduce local names:

- Comprehension expressions (Set Comprehensions (page 226)).
- Let expressions (Let Expressions (page 232)).
- Function and predicate argument lists and bodies (User-defined Operations (page 240)).

The listed sections specify these scopes in more detail. In each case, any names declared in the local scope overshadow identical global names.

### 4.1.6 Types and Type-insts

MiniZinc provides four scalar built-in types: Booleans, integers, floats, and strings; enumerated types; two compound built-in types: sets and multi-dimensional arrays; and the user extensible annotation type `ann`. 

---

4.1.6. Types and Type-insts
Each type has one or more possible instantiations. The instantiation of a variable or value indicates if it is fixed to a known value or not. A pairing of a type and instantiation is called a type-inst.

We begin by discussing some properties that apply to every type. We then introduce instantiations in more detail. We then cover each type individually, giving: an overview of the type and its possible instantiations, the syntax for its type-insts, whether it is a finite type (and if so, its domain), whether it is varifiable, the ordering and equality operations, whether its variables must be initialised at instance-time, and whether it can be involved in automatic coercions.

**4.1.6.1 Properties of Types**

The following list introduces some general properties of MiniZinc types.

- Currently all types are monotypes. In the future we may allow types which are polymorphic in other types and also the associated constraints.

- We distinguish types which are finite types. In MiniZinc, finite types include Booleans, enums, types defined via set expression type-insts such as range types (see Set Expression Type-insts (page 217)), as well as sets and arrays, composed of finite types. Types that are not finite types are unconstrained integers, unconstrained floats, unconstrained strings, and ann. Finite types are relevant to sets (spec-sets) and array indices (spec-arrays). Every finite type has a domain, which is a set value that holds all the possible values represented by the type.

- Every first-order type (this excludes ann) has a built-in total order and a built-in equality; >, <, ==/=, !=, <= and >= comparison operators can be applied to any pair of values of the same type. Rationale: This facilitates the specification of symmetry breaking and of polymorphic predicates and functions. Note that, as in most languages, using equality on floats or types that contain floats is generally not reliable due to their inexact representation. An implementation may choose to warn about the use of equality with floats or types that contain floats.

**4.1.6.2 Instantiations**

When a MiniZinc model is evaluated, the value of each variable may initially be unknown. As it runs, each variable’s domain (the set of values it may take) may be reduced, possibly to a single value.

An instantiation (sometimes abbreviated to inst) describes how fixed or unfixed a variable is at instance-time. At the most basic level, the instantiation system distinguishes between two kinds
of variables:

1. Parameters, whose values are fixed at instance-time (usually written just as “fixed”).

2. Decision variables (often abbreviated to variables), whose values may be completely unfixed at instance-time, but may become fixed at run-time (indeed, the fixing of decision variables is the whole aim of constraint solving).

In MiniZinc decision variables can have the following types: Booleans, integers, floats, and sets of integers, and enums. Arrays and ann can contain decision variables.

### 4.1.6.3 Type-insts

Because each variable has both a type and an inst, they are often combined into a single type-inst. Type-insts are primarily what we deal with when writing models, rather than types.

A variable’s type-inst never changes. This means a decision variable whose value becomes fixed during model evaluation still has its original type-inst (e.g. var int), because that was its type-inst at instance-time.

Some type-insts can be automatically coerced to another type-inst. For example, if a par int value is used in a context where a var int is expected, it is automatically coerced to a var int. We write this par int \(\rightarrow\) var int. Also, any type-inst can be considered coercible to itself. MiniZinc allows coercions between some types as well.

Some type-insts can be varified, i.e., made unfixed at the top-level. For example, par int is varified to var int. We write this par int \(\leftarrow\) var int.

Type-insts that are varifiable include the type-insts of the types that can be decision variables (Booleans, integers, floats, sets, enumerated types). Varification is relevant to type-inst synonyms and array accesses.

### 4.1.6.4 Type-inst expression overview

This section partly describes how to write type-insts in MiniZinc models. Further details are given for each type as they are described in the following sections.

A type-inst expression specifies a type-inst. Type-inst expressions may include type-inst constraints. Type-inst expressions appear in variable declarations (Variable Declaration Items (page 235)) and user-defined operation items (User-defined Operations (page 240)).

Type-inst expressions have this syntax:
The final alternative, for range types, uses the numeric-specific `<num-expr>` non-terminal, defined in Expressions Overview (page 218), rather than the `<expr>` non-terminal. If this were not the case, the rule would never match because the `..` operator would always be matched by the first `<expr>`.)

This fully covers the type-inst expressions for scalar types. The compound type-inst expression syntax is covered in more detail in Built-in Compound Types and Type-insts (page 213).

The `par` and `var` keywords (or lack of them) determine the instantiation. The `par` annotation can be omitted; the following two type-inst expressions are equivalent:

```
par int
int
```

*Rationale: The use of the explicit `var` keyword allows an implementation to check that all parameters are initialised in the model or the instance. It also clearly documents which variables are parameters,*
and allows more precise type-inst checking.

A type-inst is fixed if it does not contain `var`, with the exception of `ann`.

Note that several type-inst expressions that are syntactically expressible represent illegal type-insts. For example, although the grammar allows `var` in front of all these base type-inst expression tails, it is a type-inst error to have `var` in the front of a string or array expression.

### 4.1.6.5 Built-in Scalar Types and Type-insts

#### Booleans

*Overview.* Booleans represent truthhood or falsity. *Rationale:* Boolean values are not represented by integers. *Booleans can be explicit converted to integers with the `bool2int` function, which makes the user’s intent clear.*

*Allowed Insts.* Booleans can be fixed or unfixed.

*Syntax.* Fixed Booleans are written `bool` or `par bool`. Unfixed Booleans are written as `var bool`.

*Finite?* Yes. The domain of a Boolean is `false`, `true`.

*Varifiable?* `par bool ↦ var bool, var bool ↦ var bool`.

*Ordering.* The value `false` is considered smaller than `true`.

*Initialisation.* A fixed Boolean variable must be initialised at instance-time; an unfixed Boolean variable need not be.

*Coercions.* `par bool ↦ var bool`.

Also Booleans can be automatically coerced to integers; see *Integers* (page 211).

#### Integers

*Overview.* Integers represent integral numbers. Integer representations are implementation-defined. This means that the representable range of integers is implementation-defined. However, an implementation should abort at run-time if an integer operation overflows.

*Allowed Insts.* Integers can be fixed or unfixed.

*Syntax.* Fixed integers are written `int` or `par int`. Unfixed integers are written as `var int`.

*Finite?* Not unless constrained by a set expression (see *Set Expression Type-insts* (page 217)).

*Varifiable?* `par int ↦ var int, var int ↦ var int`.
Ordering. The ordering on integers is the standard one.

Initialisation. A fixed integer variable must be initialised at instance-time; an unfixed integer variable need not be.

Coercions. \( \text{par int} \rightarrow \text{var int}, \text{par bool} \rightarrow \text{par int}, \text{par bool} \rightarrow \text{var int}, \text{var bool} \rightarrow \text{var int} \).

Also, integers can be automatically coerced to floats; see Floats (page 212).

Floats

Overview. Floats represent real numbers. Float representations are implementation-defined. This means that the representable range and precision of floats is implementation-defined. However, an implementation should abort at run-time on exceptional float operations (e.g., those that produce \( \text{NaN} \), if using IEEE754 floats).

Allowed Insts. Floats can be fixed or unfixed.

Syntax. Fixed floats are written \text{float} or \text{par float}. Unfixed floats are written as \text{var float}.

Finite? Not unless constrained by a set expression (see Set Expression Type-insts (page 217)).

Varifiable? \( \text{par float} \rightarrow \text{var float}, \text{var float} \rightarrow \text{var float} \).

Ordering. The ordering on floats is the standard one.

Initialisation. A fixed float variable must be initialised at instance-time; an unfixed float variable need not be.

Coercions. \( \text{par int} \rightarrow \text{par float}, \text{par int} \rightarrow \text{var float}, \text{var int} \rightarrow \text{var float}, \text{par float} \rightarrow \text{var float} \).

Enumerated Types

Overview. Enumerated types (or enums for short) provide a set of named alternatives. Each alternative is identified by its case name. Enumerated types, like in many other languages, can be used in the place of integer types to achieve stricter type checking.

Allowed Insts. Enums can be fixed or unfixed.

Syntax. Variables of an enumerated type named \( X \) are represented by the term \( X \) or \text{par } X \) if fixed, and \text{var } X \) if unfixed.

Finite? Yes.

The domain of an enum is the set containing all of its case names.
Varifiable? \( \text{par} \ X \rightarrow \text{var} \ X, \text{var} \ X \rightarrow \text{var} \ X. \)

Ordering. When two enum values with different case names are compared, the value with the case name that is declared first is considered smaller than the value with the case name that is declared second.

Initialisation. A fixed enum variable must be initialised at instance-time; an unfixed enum variable need not be.

Coercions. \( \text{par} \ X \rightarrow \text{par} \ \text{int}, \text{var} \ X \rightarrow \text{var} \ \text{int}. \)

Strings

Overview. Strings are primitive, i.e., they are not lists of characters.

String expressions are used in assertions, output items and annotations, and string literals are used in include items.

Allowed Inits. Strings must be fixed.

Syntax. Fixed strings are written \text{string} or \text{par string}.

Finite? Not unless constrained by a set expression (see \text{Set Expression Type-insts} (page 217)).

Varifiable? No.

Ordering. Strings are ordered lexicographically using the underlying character codes.

Initialisation. A string variable (which can only be fixed) must be initialised at instance-time.

Coercions. None automatic. However, any non-string value can be manually converted to a string using the built-in \text{show} function or using string interpolation (see \text{String Literals and String Interpolation} (page 224)).

4.1.6.6 Built-in Compound Types and Type-insts

Sets

Overview. A set is a collection with no duplicates.

Allowed Inits. The type-inst of a set’s elements must be fixed. Rationale: This is because current solvers are not powerful enough to handle sets containing decision variables. Sets may contain any type, and may be fixed or unfixed. If a set is unfixed, its elements must be finite, unless it occurs in one of the following contexts:
• the argument of a predicate, function or annotation.

• the declaration of a variable or let local variable with an assigned value.

Syntax. A set base type-inst expression is a special case of the base type-inst rule:

\[
<\text{base-ti-expr}> ::= \text{<var-par>} \text{<opt-ti>} "\text{set}" "\text{of}" <\text{base-ti-expr-tail}>
\]

Some example set type-inst expressions:

<table>
<thead>
<tr>
<th>set of int</th>
</tr>
</thead>
<tbody>
<tr>
<td>var set of bool</td>
</tr>
</tbody>
</table>

Finite? Yes, if the set elements are finite types. Otherwise, no.

The domain of a set type that is a finite type is the powerset of the domain of its element type. For example, the domain of set of 1..2 is \(\text{powerset}(1..2)\), which is \(\{\}, \{1\}, \{1,2\}, \{2\}\).

Varifiable? \(\text{par set of TI} \rightarrow \text{var set of TI}\), \(\text{var set of TI} \rightarrow \text{var set of TI}\).

Ordering. The pre-defined ordering on sets is a lexicographic ordering of the sorted set form, where \(\{1,2\}\) is in sorted set form, for example, but \(\{2,1\}\) is not. This means, for instance, \(\{\} < \{1,3\} < \{2\}\).

Initialisation. A fixed set variable must be initialised at instance-time; an unfixed set variable need not be.

Coercions. \(\text{par set of TI} \rightarrowpar \text{set of UI}\) and \(\text{par set of TI} \rightarrowvar \text{set of UI}\) and \(\text{var set of TI} \rightarrowvar \text{set of UI}\), if \(\text{TI} \rightarrowvari\).

Arrays

Overview. MiniZinc arrays are maps from fixed integers to values. Values can be of any type. The values can only have base type-insts. Arrays-of-arrays are not allowed. Arrays can be multi-dimensional.

MiniZinc arrays can be declared in two different ways.

• Explicitly-indexed arrays have index types in the declaration that are finite types. For example:

\[
\begin{align*}
\text{array[0..3] of int: a1;} \\
\text{array[1..5, 1..10] of var float: a5;}
\end{align*}
\]
For such arrays, the index type specifies exactly the indices that will be in the array - the array’s index set is the domain of the index type - and if the indices of the value assigned do not match then it is a run-time error.

For example, the following assignments cause run-time errors:

```
a1 = [4,6,4,3,2]; % too many elements
a5 = []; % too few elements
```

- *Implicitly-indexed* arrays have index types in the declaration that are not finite types. For example:

```
array[int,int] of int: a6;
```

No checking of indices occurs when these variables are assigned.

In MiniZinc all index sets of an array must be contiguous ranges of integers, or enumerated types. The expression used for initialisation of an array must have matching index sets. An array expression with an enum index set can be assigned to an array declared with an integer index set, but not the other way around. The exception are array literals, which can be assigned to arrays declared with enum index sets.

For example:

```
enum X = {A,B,C};
enum Y = {D,E,F};
array[X] of int: x = array1d(X, [5,6,7]); % correct
array[Y] of int: y = x; % index set mismatch: Y != X
array[int] of int: z = x; % correct: assign X index set to...
\downarrow\text{int}
array[X] of int: x2 = [10,11,12]; % correct: automatic coercion for...
\downarrow\text{array literals}
```

The initialisation of an array can be done in a separate assignment statement, which may be present in the model or a separate data file.

Arrays can be accessed. See *Array Access Expressions* (page 229) for details.

*Allowed Inests.* An array’s size must be fixed. Its indices must also have fixed type-insts. Its elements may be fixed or unfixed.

*Syntax.* An array base type-inst expression tail has this syntax:
Some example array type-inst expressions:

```
array[1..10] of int
list of var int
```

Note that `list of <T>` is just syntactic sugar for `array[int] of <T>`. Rationale: Integer-indexed arrays of this form are very common, and so worthy of special support to make things easier for modellers. Implementing it using syntactic sugar avoids adding an extra type to the language, which keeps things simple for implementers.

Because arrays must be fixed-size it is a type-inst error to precede an array type-inst expression with `var`.

**Finite?** Yes, if the index types and element type are all finite types. Otherwise, no.

The domain of an array type that is a finite array is the set of all distinct arrays whose index set equals the domain of the index type and whose elements are of the array element type.

**Varifiable?** No.

**Ordering.** Arrays are ordered lexicographically, taking absence of a value for a given key to be before any value for that key. For example, `[1, 1]` is less than `[1, 2]`, which is less than `[1, 2, 3]` and `array1d(2..4, [0, 0, 0])` is less than `[1, 2, 3].`

**Initialisation.** An explicitly-indexed array variable must be initialised at instance-time only if its elements must be initialised at instance time. An implicitly-indexed array variable must be initialised at instance-time so that its length and index set is known.

**Coercions.** `array[TI0] of TI ⇝ array[UI0] of UI` if `TI0 ⇝ UI0` and `TI ⇝ UI`.

## Option Types

**Overview.** Option types defined using the `opt` type constructor, define types that may or may not be there. They are similar to Maybe types of Haskell implicitly adding a new value `<>` to the type.

**Allowed Insts.** The argument of an option type must be one of the base types `bool`, `int` or `float`.

**Syntax.** The option type is written `opt <T>` where `<T>` if one of the three base types, or one of their constrained instances.
Finite? Yes if the underlying type is finite, otherwise no.

Varifiable? Yes.

Ordering. \(<\) is always less than any other value in the type. But beware that overloading of operators like \(<\) is different for option types.

Initialisation. An opt type variable does not need to be initialised at instance-time. An uninitialised opt type variable is automatically initialised to \(<\).

Coercions. \(TI \rightarrow opt \rightarrow UI\) if \(TI \rightarrow UI\).

The Annotation Type

Overview. The annotation type, \(ann\), can be used to represent arbitrary term structures. It is augmented by annotation items (Annotation Items (page 239)).

Allowed Insts. \(ann\) is always considered unfixed, because it may contain unfixed elements. It cannot be preceded by \(var\).

Syntax. The annotation type is written \(ann\).

Finite? No.

Varifiable? No.

Ordering. N/A. Annotation types do not have an ordering defined on them.

Initialisation. An \(ann\) variable must be initialised at instance-time.

Coercions. None.

4.1.6.7 Constrained Type-insts

One powerful feature of MiniZinc is constrained type-insts. A constrained type-inst is a restricted version of a base type-inst, i.e., a type-inst with fewer values in its domain.

Set Expression Type-insts

Three kinds of expressions can be used in type-insts.

1. Integer ranges: e.g. \(1..3\).

2. Set literals: e.g. \(var\ \{1,3,5\}\).
3. Identifiers: the name of a set parameter (which can be global, let-local, the argument of a predicate or function, or a generator value) can serve as a type-inst.

In each case the base type is that of the set’s elements, and the values within the set serve as the domain. For example, whereas a variable with type-inst `var int` can take any integer value, a variable with type-inst `var 1..3` can only take the value 1, 2 or 3.

All set expression type-lnsts are finite types. Their domain is equal to the set itself.

**Float Range Type-lnsts**

Float ranges can be used as type-lnsts, e.g. `1.0 .. 3.0`. These are treated similarly to integer range type-lnsts, although `1.0 .. 3.0` is not a valid expression whereas `1 .. 3` is.

Float ranges are not finite types.

### 4.1.7 Expressions

#### 4.1.7.1 Expressions Overview

Expressions represent values. They occur in various kinds of items. They have the following syntax:

```plaintext
<expr> ::= <expr-atom> <expr-binop-tail>

<expr-atom> ::= <expr-atom-head> <expr-atom-tail> <annotations>

<expr-binop-tail> ::= [ <bin-op> <expr> ]

<expr-atom-head> ::= <builtin-un-op> <expr-atom>
    | "(" <expr> ")"
    | <ident-or-quoted-op>
    | "_"
    | <bool-literal>
    | <int-literal>
    | <float-literal>
    | <string-literal>
    | <set-literal>
    | <set-comp>
```
Expressions can be composed from sub-expressions combined with operators. All operators (binary and unary) are described in Operators (page 220), including the precedences of the binary operators. All unary operators bind more tightly than all binary operators.

Expressions can have one or more annotations. Annotations bind more tightly than unary and binary operator applications, but less tightly than access operations and non-operator applications. In some cases this binding is non-intuitive. For example, in the first three of the following lines, the annotation a binds to the identifier expression x rather than the operator application. However, the fourth line features a non-operator application (due to the single quotes around the not) and so the annotation binds to the whole application.

\[
\text{not } x::a; \\
\text{not } (x)::a; \\
\text{not}(x)::a; \\
\text{‘not’}(x)::a;
\]

Annotations (page 244) has more on annotations.

Expressions can be contained within parentheses.

The array access operations all bind more tightly than unary and binary operators and annotations. They are described in more detail in Array Access Expressions (page 229).

The remaining kinds of expression atoms (from <ident> to <gen-call-expr>) are described in Identifier Expressions and Quoted Operator Expressions (page 223) to Generator Call Expressions (page 234).

We also distinguish syntactically valid numeric expressions. This allows range types to be parsed correctly.

4.1.7. Expressions
4.1.7.2 Operators

Operators are functions that are distinguished by their syntax in one or two ways. First, some of them contain non-alphanumeric characters that normal functions do not (e.g. +). Second, their application is written in a manner different to normal functions.

We distinguish between binary operators, which can be applied in an infix manner (e.g. 3 + 4), and unary operators, which can be applied in a prefix manner without parentheses (e.g. not x). We also distinguish between built-in operators and user-defined operators. The syntax is the following:
Again, we syntactically distinguish numeric operators.

\[
\begin{align*}
<\text{num-bin-op}> & ::= <\text{builtin-num-bin-op}> \mid '<\text{ident}>' \\
<\text{builtin-num-bin-op}> & ::= '+' \mid '-' \mid '*' \mid '/' \mid 'div' \mid 'mod' \\
<\text{builtin-num-un-op}> & ::= '+' \mid '-'
\end{align*}
\]

Some operators can be written using their unicode symbols, which are listed in Table 4.1.7.2 (recall that MiniZinc input is UTF-8).

Table 1.1: Unicode equivalents of binary operators

<table>
<thead>
<tr>
<th>Operator</th>
<th>Unicode symbol</th>
<th>UTF-8 code</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;-&gt;</td>
<td>(\leftrightarrow)</td>
<td>E2 86 94</td>
</tr>
<tr>
<td>-&gt;</td>
<td>(\rightarrow)</td>
<td>E2 86 92</td>
</tr>
<tr>
<td>&lt;=</td>
<td>(\leftarrow)</td>
<td>E2 86 90</td>
</tr>
<tr>
<td>not</td>
<td>(\neg)</td>
<td>C2 AC</td>
</tr>
<tr>
<td>(\lor)</td>
<td>(\lor)</td>
<td>E2 88 A8</td>
</tr>
<tr>
<td>(\land)</td>
<td>(\land)</td>
<td>E2 88 A7</td>
</tr>
<tr>
<td>!=</td>
<td>(\neq)</td>
<td>E2 89 A0</td>
</tr>
<tr>
<td>&lt;=</td>
<td>(\leq)</td>
<td>E2 89 A4</td>
</tr>
<tr>
<td>&gt;=</td>
<td>(\geq)</td>
<td>E2 89 A5</td>
</tr>
<tr>
<td>in</td>
<td>(\in)</td>
<td>E2 88 88</td>
</tr>
<tr>
<td>subset</td>
<td>(\subset)</td>
<td>E2 A8 86</td>
</tr>
<tr>
<td>superset</td>
<td>(\supset)</td>
<td>E2 A8 87</td>
</tr>
<tr>
<td>union</td>
<td>(\cup)</td>
<td>E2 88 AA</td>
</tr>
<tr>
<td>intersect</td>
<td>(\cap)</td>
<td>E2 88 A9</td>
</tr>
</tbody>
</table>

The binary operators are listed in Table 4.1.7.2. A lower precedence number means tighter binding; for example, \(1+2*3\) is parsed as \(1+(2*3)\) because \(*\) binds tighter than \(+\). Associativity indicates how chains of operators with equal precedences are handled; for example, \(1+2+3\) is parsed as \((1+2)+3\) because \(+\) is left-associative, \(a+b+c\) is parsed as \(a+(b+c)\) because \(++\) is right-associative, and \(1<x<2\) is a syntax error because \(<\) is non-associative.
Table 1.2: Binary infix operators

<table>
<thead>
<tr>
<th>Symbol(s)</th>
<th>Assoc.</th>
<th>Prec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt;&gt;</td>
<td>left</td>
<td>1200</td>
</tr>
<tr>
<td>-&gt;</td>
<td>left</td>
<td>1100</td>
</tr>
<tr>
<td>&lt;=</td>
<td>left</td>
<td>1100</td>
</tr>
<tr>
<td>/</td>
<td>left</td>
<td>1000</td>
</tr>
<tr>
<td>xor</td>
<td>left</td>
<td>1000</td>
</tr>
<tr>
<td>\</td>
<td>left</td>
<td>900</td>
</tr>
<tr>
<td>&lt;</td>
<td>none</td>
<td>800</td>
</tr>
<tr>
<td>&gt;</td>
<td>none</td>
<td>800</td>
</tr>
<tr>
<td>&lt;=</td>
<td>none</td>
<td>800</td>
</tr>
<tr>
<td>&gt;=</td>
<td>none</td>
<td>800</td>
</tr>
<tr>
<td>==,</td>
<td>none</td>
<td>800</td>
</tr>
<tr>
<td>=</td>
<td>none</td>
<td>800</td>
</tr>
<tr>
<td>!=</td>
<td>none</td>
<td>800</td>
</tr>
<tr>
<td>in</td>
<td>none</td>
<td>700</td>
</tr>
<tr>
<td>subset</td>
<td>none</td>
<td>700</td>
</tr>
<tr>
<td>superset</td>
<td>none</td>
<td>700</td>
</tr>
<tr>
<td>union</td>
<td>left</td>
<td>600</td>
</tr>
<tr>
<td>diff</td>
<td>left</td>
<td>600</td>
</tr>
<tr>
<td>symdiff</td>
<td>left</td>
<td>600</td>
</tr>
<tr>
<td>..</td>
<td>none</td>
<td>500</td>
</tr>
<tr>
<td>+</td>
<td>left</td>
<td>400</td>
</tr>
<tr>
<td>-</td>
<td>left</td>
<td>400</td>
</tr>
<tr>
<td>*</td>
<td>left</td>
<td>300</td>
</tr>
<tr>
<td>div</td>
<td>left</td>
<td>300</td>
</tr>
<tr>
<td>mod</td>
<td>left</td>
<td>300</td>
</tr>
<tr>
<td>/</td>
<td>left</td>
<td>300</td>
</tr>
<tr>
<td>intersect</td>
<td>left</td>
<td>300</td>
</tr>
<tr>
<td>++</td>
<td>right</td>
<td>200</td>
</tr>
<tr>
<td><code>&lt;ident&gt;</code></td>
<td>left</td>
<td>100</td>
</tr>
</tbody>
</table>

A user-defined binary operator is created by backquoting a normal identifier, for example:

```
A `min2` B
```

This is a static error if the identifier is not the name of a binary function or predicate.
The unary operators are: +, - and not. User-defined unary operators are not possible.

As Identifiers (page 204) explains, any built-in operator can be used as a normal function identifier by quoting it, e.g: "+(3, 4) is equivalent to 3 + 4.

The meaning of each operator is given in Built-in Operations (page 248).

4.1.7.3 Expression Atoms

Identifier Expressions and Quoted Operator Expressions

Identifier expressions and quoted operator expressions have the following syntax:

\[
\text{<ident-or-quoted-op> ::= <ident> | ' <builtin-op>'}
\]

Examples of identifiers were given in Identifiers (page 204). The following are examples of quoted operators:

'+'
'union'

In quoted operators, whitespace is not permitted between either quote and the operator. Operators (page 220) lists MiniZinc’s built-in operators.

Syntactically, any identifier or quoted operator can serve as an expression. However, in a valid model any identifier or quoted operator serving as an expression must be the name of a variable.

Anonymous Decision Variables

There is a special identifier, _, that represents an unfixed, anonymous decision variable. It can take on any type that can be a decision variable. It is particularly useful for initialising decision variables within compound types. For example, in the following array the first and third elements are fixed to 1 and 3 respectively and the second and fourth elements are unfixed:

\[
\text{array[1..4] of var int: xs = [1, _, 3, _];}
\]

Any expression that does not contain _ and does not involve decision variables is fixed.
Boolean Literals

Boolean literals have this syntax:

```
<bool-literal> ::= "false" | "true"
```

Integer and Float Literals

There are three forms of integer literals - decimal, hexadecimal, and octal - with these respective forms:

```
<int-literal> ::= [0-9]+ | 0x[0-9A-Fa-f]+ | 0o[0-7]+
```

For example: 0, 005, 123, 0x1b7, 0o777; but not -1.

Float literals have the following form:

```
<float-literal> ::= [0-9]+\.[0-9]+ | [0-9]+\.[0-9]+[Ee][-+]?[0-9]+ | 0[xX][0-9a-fA-F]+\.[0-9]+[Ee][-+]?[0-9]+ | 0[xX][0-9a-fA-F]+(pP)[+-]?[0-9]+ | (0[xX][0-9a-fA-F]+[pP][+-]?[0-9]+)
```

For example: 1.05, 1.3e-5, 1.3+e5; but not 1., .5, 1.e5, .1e5, -1.0, -1E05. A - symbol preceding an integer or float literal is parsed as a unary minus (regardless of intervening whitespace), not as part of the literal. This is because it is not possible in general to distinguish a - for a negative integer or float literal from a binary minus when lexing.

String Literals and String Interpolation

String literals are written as in C:

```
<string-contents> ::= (\[\] | \^\[\])*
<string-literal> ::= """ <string-contents> ""
```
This includes C-style escape sequences, such as \" for double quotes, \ for backslash, and \n for newline.

For example: "Hello, world!\n".

String literals must fit on a single line.

Long string literals can be split across multiple lines using string concatenation. For example:

```mini
string: s = "This is a string literal \\
++ "split across two lines."
```

A string expression can contain an arbitrary MiniZinc expression, which will be converted to a string similar to the builtin show function and inserted into the string.

For example:

```mini
var set of 1..10: q;
solve satisfy;
output [show("The value of q is \(q), and it has \(card(q)) elements.");
```

### Set Literals

Set literals have this syntax:

```mini
<set-literal> ::= "{ [ <expr> "," ... ] "}" 
```

For example:

```mini
{ 1, 3, 5 }
{ }  
{ 1, 2.0 }
```
The type-insts of all elements in a literal set must be the same, or coercible to the same type-inst (as in the last example above, where the integer 1 will be coerced to a float).

**Set Comprehensions**

Set comprehensions have this syntax:

```
<set-comp> ::= "{" <expr> "|" <comp-tail> "}"
<comp-tail> ::= <generator> [ "where" <expr> ] "," ...
<generator> ::= <ident> "," ... "in" <expr>
```

For example (with the literal equivalent on the right):

```
{ 2*i | i in 1..5 }   %  { 2, 4, 6, 8, 10 }
{ 1  | i in 1..5 }   %  { 1 }  (no duplicates in sets)
```

The expression before the | is the **head expression**. The expression after the in is a **generator expression**. Generators can be restricted by a where-expression. For example:

```
{ i  | i in 1..10 where (i mod 2 = 0) }   %  { 2, 4, 6, 8, 10 }
```

When multiple generators are present, the right-most generator acts as the inner-most one. For example:

```
{ 3*i+j | i in 0..2, j in {0, 1} }   %  { 0, 1, 3, 4, 6, 7 }
```

The scope of local generator variables is given by the following rules:

- They are visible within the head expression (before the |).
- They are visible within the where-expression of their own generator.
- They are visible within generator expressions and where-expressions in any subsequent generators.

The last of these rules means that the following set comprehension is allowed:

```
{ i+j | i in 1..3, j in 1..i } %  { 1+1, 2+1, 2+2, 3+1, 3+2, 3+3 }
```
Multiple where-expressions are allowed, as in the following example:

\[
[f(i, j) \mid i \in A1 \text{ where } p(i), j \in A2 \text{ where } q(i, j)]
\]

A generator expression must be an array or a fixed set.

*Rationale:* For set comprehensions, set generators would suffice, but for array comprehensions, array generators are required for full expressivity (e.g., to provide control over the order of the elements in the resulting array). Set comprehensions have array generators for consistency with array comprehensions, which makes implementations simpler.

The where-expression (if present) must be Boolean. It can be var, in which case the type of the comprehension is lifted to an optional type.

**Array Literals**

Array literals have this syntax:

\[
<\text{array-literal}> ::= "[" [ <expr> "," \ldots ] "]"
\]

For example:

\[
[1, 2, 3, 4]  
[]  
[1, _]
\]

In a array literal all elements must have the same type-inst, or be coercible to the same type-inst (as in the last example above, where the fixed integer 1 will be coerced to a *var int*).

The indices of a array literal are implicitly \(1..n\), where \(n\) is the length of the literal.

**2d Array Literals**

Simple 2d array literals have this syntax:

\[
<\text{array-literal-2d}> ::= "[|" [ ( <expr> "," \ldots ) |" \ldots ] "|]"
\]

For example:
In a 2d array literal, every sub-array must have the same length.

In a 2d array literal all elements must have the same type-inst, or be coercible to the same type-inst (as in the last example above, where the fixed integer 1 will be coerced to a var int).

The indices of a 2d array literal are implicitly \((1, 1)\) \((m, n)\), where \(m\) and \(n\) are determined by the shape of the literal.

### Array Comprehensions

Array comprehensions have this syntax:

\[
<array-comp> ::= \"[\" <expr> \"|\" <comp-tail> \"]\"
\]

For example (with the literal equivalents on the right):

\[
[2*i | i in 1..5] \quad \% [2, 4, 6, 8, 10]
\]

Array comprehensions have more flexible type and inst requirements than set comprehensions (see Set Comprehensions (page 226)).

Array comprehensions are allowed over a variable set with finite type, the result is an array of optional type, with length equal to the cardinality of the upper bound of the variable set. For example:

```mini
var set of 1..5: x;
array[int] of var opt int: y = [ i * i | i in x ];
```

The length of array will be 5.

Array comprehensions are allowed where the where-expression is a var bool. Again the resulting array is of optional type, and of length equal to that given by the generator expressions. For example:
**MiniZinc Handbook, Release 2.4.3**

```plaintext
var int x;
array[int] of var opt int: y = [ i | i in 1..10 where i != x ];
```

The length of the array will be 10.

The indices of an evaluated simple array comprehension are implicitly 1..n, where n is the length of the evaluated comprehension.

**Array Access Expressions**

Array elements are accessed using square brackets after an expression:

```plaintext
<array-access-tail> ::= "[" <expr> "," ... "]"
```

For example:

```plaintext
int: x = a1[1];
```

If all the indices used in an array access are fixed, the type-inst of the result is the same as the element type-inst. However, if any indices are not fixed, the type-inst of the result is the varified element type-inst. For example, if we have:

```plaintext
array[1..2] of int: a2 = [1, 2];
var int: i;
```

then the type-inst of `a2[i]` is `var int`. If the element type-inst is not varifiable, such an access causes a static error.

Multi dimensional arrays are accessed using comma separated indices.

```plaintext
array[1..3,1..3] of int: a3;
int: y = a3[1, 2];
```

Indices must match the index set type of the array. For example, an array declared with an enum index set can only be accessed using indices from that enum.

```plaintext
enum X = {A,B,C};
array[X] of int: a4 = [1,2,3];
```
MiniZinc Handbook, Release 2.4.3

int: y = a4[1]; % wrong index type
int: z = a4[8]; % correct

Array Slice Expressions

Arrays can be sliced in order to extract individual rows, columns or blocks. The syntax is that of an array access expression (see above), but where one or more of the expressions inside the square brackets are set-valued.

For example, the following extracts row 2 from a two-dimensional array:

```
array[1..n,4..8] of int: x;
array[int] of int: row_2_of_x = x[2,4..8];
```

Note that the resulting array `row_2_of_x` will have index set 4..8.

A short-hand for all indices of a particular dimension is to use just dots:

```
array[1..n,4..8] of int: x;
array[int] of int: row_2_of_x = x[2,..];
```

You can also restrict the index set by giving a sub-set of the original index set as the slice:

```
array[1..n,4..8] of int: x;
array[int] of int: row_2_of_x = x[2,5..6];
```

The resulting array `row_2_of_x` will now have length 2 and index set 5..6.

The dots notation also allows for partial bounds, for example:

```
array[1..n,4..8] of int: x;
array[int] of int: row_2_of_x = x[2,..6];
```

The resulting array will have length 3 and index set 4..6. Of course 6.. would also be allowed and result in an array with index set 6..8.
Annotation Literals

Literals of the `ann` type have this syntax:

\[
<\text{ann-literal}> ::= <\text{ident}> [ "(" <\text{expr}> "," ... ")"
\]

For example:

```
foo
    cons(1, cons(2, cons(3, nil)))
```

There is no way to inspect or deconstruct annotation literals in a MiniZinc model; they are intended to be inspected only by an implementation, e.g., to direct compilation.

If-then-else Expressions

MiniZinc provides if-then-else expressions, which provide selection from two alternatives based on a condition. They have this syntax:

\[
<\text{if-then-else-expr}> ::= "if" <\text{expr}> "then" <\text{expr}> [ "elseif" <\text{expr}> "then" ... <\text{expr}> ]* "else" <\text{expr}> "endif"
\]

For example:

```
if x <= y then x else y endif
if x < 0 then -1 elseif x > 0 then 1 else 0 endif
```

The presence of the `endif` avoids possible ambiguity when an if-then-else expression is part of a larger expression.

The type-inst of the `if` expression must be `par bool` or `var bool`. The `then` and `else` expressions must have the same type-inst, or be coercible to the same type-inst, which is also the type-inst of the whole expression.

If the `if` expression is `var bool` then the type-inst of the `then` and `else` expressions must be varifiable.

If the `if` expression is `par bool` then evaluation of if-then-else expressions is lazy: the condition is evaluated, and then only one of the `then` and `else` branches are evaluated, depending on whether the condition succeeded or failed. This is not the case if it is `var bool`. 

4.1.7. Expressions
Let Expressions

Let expressions provide a way of introducing local names for one or more expressions and local constraints that can be used within another expression. They are particularly useful in user-defined operations.

Let expressions have this syntax:

```plaintext
<let-expr> ::= "let" "{" <let-item> ";" ... "}" "in" <expr>

<let-item> ::= <var-decl-item>
  | <constraint-item>
```

For example:

```plaintext
let { int: x = 3; int: y = 4; } in x + y;
let { var int: x;
    constraint x >= y /\ x >= -y /\ (x = y \lor x = -y); }
in x
```

The scope of a let local variable covers:

- The type-inst and initialisation expressions of any subsequent variables within the let expression (but not the variable’s own initialisation expression).
- The expression after the `in`, which is parsed as greedily as possible.

A variable can only be declared once in a let expression.

Thus in the following examples the first is acceptable but the rest are not:

```plaintext
let { int: x = 3; int: y = x; } in x + y;  % ok
let { int: y = x; int: x = 3; } in x + y;   % x not visible in y's defn.
let { int: x = x; } in x;                  % x not visible in x's defn.
let { int: x = 3; int: x = 4; } in x;      % x declared twice
```

The initialiser for a let local variable can be omitted only if the variable is a decision variable. For example:

```plaintext
let { var int: x; } in ...;  % ok
let { int: x; } in ...;      % illegal
```
The type-inst of the entire let expression is the type-inst of the expression after the \texttt{in} keyword.

There is a complication involving let expressions in negative contexts. A let expression occurs in a negative context if it occurs in an expression of the form \texttt{not X}, \texttt{X <-> Y}} or in the sub-expression \texttt{X in X -> Y} or \texttt{Y <-> X}, or in a subexpression \texttt{bool2int(X)}.

If a let expression is used in a negative context, then any let-local decision variables must be defined only in terms of non-local variables and parameters. This is because local variables are implicitly existentially quantified, and if the let expression occurred in a negative context then the local variables would be effectively universally quantified which is not supported by MiniZinc.

Constraints in let expressions float to the nearest enclosing Boolean context. For example

\begin{verbatim}
constraint b -> x + let { var 0..2: y; constraint y != -1;} in y >= 4;
\end{verbatim}

is equivalent to

\begin{verbatim}
var 0..2: y;
constraint b -> (x + y >= 4 \&\& y != 1);
\end{verbatim}

For backwards compatibility with older versions of MiniZinc, items inside the let can also be separated by commas instead of semicolons.

\section*{Call Expressions}

Call expressions are used to call predicates and functions.

Call expressions have this syntax:

\begin{verbatim}
<call-expr> ::= <ident-or-quoted-op> [ "(" <expr> "," ... ")"]
\end{verbatim}

For example:

\begin{verbatim}
x = min(3, 5);
\end{verbatim}

The type-insts of the expressions passed as arguments must match the argument types of the called predicate/function. The return type of the predicate/function must also be appropriate for the calling context.
Note that a call to a function or predicate with no arguments is syntactically indistinguishable from the use of a variable, and so must be determined during type-inst checking.

Evaluation of the arguments in call expressions is strict: all arguments are evaluated before the call itself is evaluated. Note that this includes Boolean operations such as \( \text{\textbackslash\textbackslash} \), \( \text{\textbackslash} \), \( \text{\textbackslash} \), \( \rightarrow \) and \( \leftarrow \) which could be lazy in one argument. The one exception is \( \text{assert} \), which is lazy in its third argument (Other Operations (page 257)).

**Rationale:** Boolean operations are strict because: (a) this minimises exceptional cases; (b) in an expression like \( A \rightarrow B \) where \( A \) is not fixed and \( B \) causes an abort, the appropriate behaviour is unclear if laziness is present; and (c) if a user needs laziness, an if-then-else can be used.

The order of argument evaluation is not specified. **Rationale:** Because MiniZinc is declarative, there is no obvious need to specify an evaluation order, and leaving it unspecified gives implementors some freedom.

**Generator Call Expressions**

MiniZinc has special syntax for certain kinds of call expressions which makes models much more readable.

Generator call expressions have this syntax:

\[
\text{<gen-call-expr> ::= <ident-or-quoted-op> "(" <comp-tail> ")" "(" <expr> ")"}
\]

A generator call expression \( P(Gs)(E) \) is equivalent to the call expression \( P([E \mid Gs]) \). For example, the expression:

\[
\text{forall(i,j in Domain where i<j)}
\]

\[
\text{} \quad \text{(noattack(i, j, queens[i], queens[j]));}
\]

(in a model specifying the N-queens problem) is equivalent to:

\[
\text{forall([noattack(i, j, queens[i], queens[j])}
\]

\[
\text{} \quad \text{| i,j in Domain where i<j ];}
\]

The parentheses around the latter expression are mandatory; this avoids possible confusion when the generator call expression is part of a larger expression.

The identifier must be the name of a unary predicate or function that takes an array argument.
The generators and where-expression (if present) have the same requirements as those in array comprehensions (*Array Comprehensions* (page 228)).

**4.1.8 Items**

This section describes the top-level program items.

**4.1.8.1 Include Items**

Include items allow a model to be split across multiple files. They have this syntax:

```
<include-item> ::= "include" <string-literal>
```

For example:

```
include "foo.mzn";
```

includes the file foo.mzn.

Include items are particularly useful for accessing libraries or breaking up large models into small pieces. They are not, as *Model Instance Files* (page 206) explains, used for specifying data files.

If the given name is not a complete path then the file is searched for in an implementation-defined set of directories. The search directories must be able to be altered with a command line option.

**4.1.8.2 Variable Declaration Items**

Variable declarations have this syntax:

```
<var-decl-item> ::= <ti-expr-and-id> <annotations> [ "=" <expr> ]
```

For example:

```
int: A = 10;
```
It is a type-inst error if a variable is declared and/or defined more than once in a model.

A variable whose declaration does not include an assignment can be initialised by a separate assignment item (Assignment Items (page 238)). For example, the above item can be separated into the following two items:

\[
\text{int: } A; \\
\ldots \\
A = 10;
\]

All variables that contain a parameter component must be defined at instance-time.

Variables can have one or more annotations. Annotations (page 244) has more on annotations.

### 4.1.8.3 Enum Items

Enumerated type items have this syntax:

\[
<\text{enum-item}> ::= "\text{enum}\" <\text{ident}> <\text{annotations}> \ [ "\text{=}\" <\text{enum-cases}> \ ] \\
<\text{enum-cases}> ::= "\{\" <\text{ident}> \",\ldots\" \}"
\]

An example of an enum:

\[
\text{enum country} = \{\text{Australia, Canada, China, England, USA}\};
\]

Each alternative is called an *enum case*. The identifier used to name each case (e.g. Australia) is called the *enum case name*.

Because enum case names all reside in the top-level namespace (Namespaces (page 207)), case names in different enums must be distinct.

An enum can be declared but not defined, in which case it must be defined elsewhere within the model, or in a data file. For example, a model file could contain this:

\[
\text{enum Workers;} \\
\text{enum Shifts;}
\]

and the data file could contain this:
Workers = { welder, driller, stamper };  
Shifts = { idle, day, night };

Sometimes it is useful to be able to refer to one of the enum case names within the model. This can be achieved by using a variable. The model would read:

```
enum Shifts;  
Shifts: idle; % Variable representing the idle constant.
```

and the data file:

```
enum Shifts = { idle_const, day, night };  
idle = idle_const; % Assignment to the variable.
```

Although the constant `idle_const` cannot be mentioned in the model, the variable `idle` can be.

All enums must be defined at instance-time.

Enum items can be annotated. `Annotations` (page 244) has more details on annotations.

Each case name can be coerced automatically to the integer corresponding to its index in the type.

```
int: oz = Australia; % oz = 1
```

For each enumerated type `T`, the following functions exist:

```
% Return next greater enum value of x in enum type X  
function T: enum_next(set of T: X, T: x);  
function var T: enum_next(set of T: X, var T: x);  

% Return next smaller enum value of x in enum type X  
function T: enum_prev(set of T: X, T: x);  
function var T: enum_prev(set of T: X, var T: x);  

% Convert x to enum type X  
function T: to_enum(set of T: X, int: x);  
function var T: to_enum(set of T: X, var int: x);
```
4.1.8.4 Assignment Items

Assignments have this syntax:

```
<assign-item> ::= <ident> "=" <expr>
```

For example:

```
A = 10;
```

4.1.8.5 Constraint Items

Constraint items form the heart of a model. Any solutions found for a model will satisfy all of its constraints.

Constraint items have this syntax:

```
<constraint-item> ::= "constraint" <string-annotation> <expr>
```

For example:

```
constraint a*X < b;
```

The expression in a constraint item must have type-inst `par bool` or `var bool`; note however that constraints with fixed expressions are not very useful.

4.1.8.6 Solve Items

Every model must have exactly one or no solve item. Solve items have the following syntax:

```
<solve-item> ::= "solve" <annotations> "satisfy"
               | "solve" <annotations> "minimize" <expr>
               | "solve" <annotations> "maximize" <expr>
```

Example solve items:
The solve item determines whether the model represents a constraint satisfaction problem or an optimisation problem. If there is no solve item, the model is assumed to be a satisfaction problem. For optimisation problems, the given expression is the one to be minimized/maximized.

The expression in a minimize/maximize solve item can have integer or float type.

*Rationale: This is possible because all type-insts have a defined order.* Note that having an expression with a fixed type-inst in a solve item is not very useful as it means that the model requires no optimisation.

Solve items can be annotated. *Annotations* (page 244) has more details on annotations.

### 4.1.8.7 Output Items

Output items are used to present the solutions of a model instance. They have the following syntax:

```plaintext
<output-item> ::= "output" <expr>
```

For example:

```plaintext
output ["The value of x is ", show(x), "!

The expression must have type-inst array[int] of par string. It can be composed using the built-in operator ++ and the built-in functions show, show_int, and show_float (Built-in Operations (page 248)), as well as string interpolations (String Literals and String Interpolation (page 224)). The output is the concatenation of the elements of the array. If multiple output items exist, the output is the concatenation of all of their outputs, in the order in which they appear in the model.

If no output item is present, the implementation should print all the global variables and their values in a readable format.

### 4.1.8.8 Annotation Items

Annotation items are used to augment the ann type. They have the following syntax:
For example:

```
annotation solver(int: kind);
```

It is a type-inst error if an annotation is declared and/or defined more than once in a model.

The use of annotations is described in *Annotations* (page 244).

### 4.1.8.9 User-defined Operations

MiniZinc models can contain user-defined operations. They have this syntax:

```
<predicate-item> ::=
  "predicate" <operation-item-tail>

<test-item> ::=
  "test" <operation-item-tail>

<function-item> ::=
  "function" <ti-expr> ":" <operation-item-tail>

<operation-item-tail> ::=
  <ident> <params> <annotations> [ "=" <expr> ]

<params> ::=[ ( <ti-expr-and-id> "," ... ) ]
```

The type-inst expressions can include type-inst variables in the function and predicate declaration.

For example, predicate `even` checks that its argument is an even number.

```
predicate even(var int: x) =
  x mod 2 = 0;
```

A predicate supported natively by the target solver can be declared as follows:

```
predicate alldifferent(array [int] of var int: xs);
```

Predicate declarations that are natively supported in MiniZinc are restricted to using FlatZinc types (for instance, multi-dimensional and non-1-based arrays are forbidden). .. % pjs{need to
fix this if we allow 2d arrays in FlatZinc!

Declarations for user-defined operations can be annotated. Annotations (page 244) has more details on annotations.

**Basic Properties**

The term “predicate” is generally used to refer to both test items and predicate items. When the two kinds must be distinguished, the terms “test item” and “predicate item” can be used.

The return type-inst of a test item is implicitly `par bool`. The return type-inst of a predicate item is implicitly `var bool`.

Predicates and functions are allowed to be recursive. Termination of a recursive function call depends solely on its fixed arguments, i.e., recursive functions and predicates cannot be used to define recursively constrained variables. .. % Rationale{This ensures that the satisfiability of models is decidable.}

Predicates and functions introduce their own local names, being those of the formal arguments. The scope of these names covers the predicate/function body. Argument names cannot be repeated within a predicate/function declaration.

**Ad-hoc polymorphism**

MiniZinc supports ad-hoc polymorphism via overloading. Functions and predicates (both built-in and user-defined) can be overloaded. A name can be overloaded as both a function and a predicate.

It is a type-inst error if a single version of an overloaded operation with a particular type-inst signature is defined more than once in a model. For example:

```plaintext
predicate p(1..5: x);
predicate p(1..5: x) = false;    % ok: first definition
predicate p(1..5: x) = true;     % error: repeated definition
```

The combination of overloading and coercions can cause problems. Two overloading of an operation are said to overlap if they could match the same arguments. For example, the following overloading of `p` overlap, as they both match the call `p(3)`.

```plaintext
predicate p(par int: x);
predicate p(var int: x);
```
However, the following two predicates do not overlap because they cannot match the same argument:

\[
\text{predicate } q(\text{int: } x);
\]
\[
\text{predicate } q(\text{set of int: } x);
\]

We avoid two potential overloading problems by placing some restrictions on overlapping overloads of operations.

1. The first problem is ambiguity. Different placement of coercions in operation arguments may allow different choices for the overloaded function. For instance, if a MiniZinc function \( f \) is overloaded like this:

\[
\begin{align*}
\text{function int: } f(\text{int: } x, \text{float: } y) &= 0; \\
\text{function int: } f(\text{float: } x, \text{int: } y) &= 1;
\end{align*}
\]

then \( f(3,3) \) could be either 0 or 1 depending on coercion/overloading choices.

To avoid this problem, any overlapping overloads of an operation must be semantically equivalent with respect to coercion. For example, the two overloads of the predicate \( p \) above must have bodies that are semantically equivalent with respect to overloading.

Currently, this requirement is not checked and the modeller must satisfy it manually. In the future, we may require the sharing of bodies among different versions of overloaded operations, which would provide automatic satisfaction of this requirement.

2. The second problem is that certain combinations of overloads could require a MiniZinc implementation to perform combinatorial search in order to explore different choices of coercions and overloading. For example, if function \( g \) is overloaded like this:

\[
\begin{align*}
\text{function float: } g(\text{int: } t1, \text{float: } t2) &= t2; \\
\text{function int : } g(\text{float: } t1, \text{int: } t2) &= t1;
\end{align*}
\]

then how the overloading of \( g(3,4) \) is resolved depends upon its context:

\[
\begin{align*}
\text{float: } s &= g(3,4); \\
\text{int: } t &= g(3,4);
\end{align*}
\]

In the definition of \( s \) the first overloaded definition must be used while in the definition of \( t \) the second must be used.
To avoid this problem, all overlapping overloadingsof an operation must be closed under intersection of their input type-insts. That is, if overloaded versions have input type-inst \((S_1, ..., S_n)\) and \((T_1, ..., T_n)\) then there must be another overloaded version with input type-inst \((R_1, ..., R_n)\) where each \(R_i\) is the greatest lower bound (\(\text{glb}\)) of \(S_i\) and \(T_i\).

Also, all overlapping overloadingsof an operation must be monotonic. That is, if there are overloaded versions with input type-insts \((S_1, ..., S_n)\) and \((T_1, ..., T_n)\) and output type-inst \(S\) and \(T\), respectively, then \(S_i \preceq T_i\) for all \(i\), implies \(S \preceq T\). At call sites, the matching overloading that is lowest on the type-inst lattice is always used.

For \(g\) above, the type-inst intersection (or \(\text{glb}\)) of \((\text{int}, \text{float})\) and \((\text{float}, \text{int})\) is \((\text{int}, \text{int})\). Thus, the overloaded versions are not closed under intersection and the user needs to provide another overloading for \(g\) with input type-inst \((\text{int}, \text{int})\). The natural definition is:

```plaintext
function int: g(int: t1, int: t2) = t1;
```

Once \(g\) has been augmented with the third overloading, it satisfies the monotonicity requirement because the output type-inst of the third overloading is \(\text{int}\) which is less than the output type-inst of the original overloading.

Monotonicity and closure under type-inst conjunction ensure that whenever an overloaded function or predicate is reached during type-inst checking, there is always a unique and safe “minimal” version to choose, and so the complexity of type-inst checking remains linear. Thus in our example \(g(3, 4)\) is always resolved by choosing the new overloaded definition.

### Local Variables

Local variables in operation bodies are introduced using let expressions. For example, the predicate \(\text{have_common_divisor}\) takes two integer values and checks whether they have a common divisor greater than one:

```plaintext
predicate have_common_divisor(int: A, int: B) =
   let {
      var 2..min(A,B): C;
   } in
   A mod C = 0 /
   B mod C = 0;
```

---

4.1.8. Items 243
However, as Let Expressions (page 232) explained, because \( C \) is not defined, this predicate cannot be called in a negative context. The following is a version that could be called in a negative context:

```plaintext
predicate have_common_divisor(int: A, int: B) =
    exists(C in 2..min(A,B))
        (A mod C = 0 \( \land \) B mod C = 0);
```

### 4.1.9 Annotations

Annotations allow a modeller to specify non-declarative and solver-specific information that is beyond the core language. Annotations do not change the meaning of a model, however, only how it is solved.

Annotations can be attached to variables (on their declarations), constraints, expressions, type-inst synonyms, enum items, solve items and on user defined operations. They have the following syntax:

```plaintext
<annotations> ::= [ "::" <annotation> ]*

<annotation> ::= <expr-atom-head> <expr-atom-tail>

<string-annotation> ::= "::" <string-literal>
```

For example:

```plaintext
int: x::foo;
    x = (3 + 4)::bar("a", 9)::baz("b");
solve :: blah(4)
    minimize x;
```

The types of the argument expressions must match the argument types of the declared annotation. Like user-defined predicates and functions, annotations can be overloaded.

Annotation signatures can contain type-inst variables.

The order and nesting of annotations do not matter. For the expression case it can be helpful to view the annotation connector \( :: \) as an overloaded operator:
Both operators are associative, the second is commutative. This means that the following expressions are all equivalent:

\[
\begin{align*}
e &:: a :: b \\
e &:: b :: a \\
(e :: a) &:: b \\
(e :: b) &:: a \\
e &:: (a :: b) \\
e &:: (b :: a)
\end{align*}
\]

This property also applies to annotations on solve items and variable declaration items. **Rationale:** This property make things simple, as it allows all nested combinations of annotations to be treated as if they are flat, thus avoiding the need to determine what is the meaning of an annotated annotation. It also makes the MiniZinc abstract syntax tree simpler by avoiding the need to represent nesting.

Annotations have to be values of the `ann` type or string literals. The latter are used for naming constraints and expressions, for example

```plaintext
constraint :: "first constraint" alldifferent(x);
constraint :: "second constraint" alldifferent(y);
constraint forall (i in 1..n) (my_constraint(x[i],y[i]):="constraint \((i)"));
```

Note that constraint items can only be annotated with string literals.

**Rationale:** Allowing arbitrary annotations on constraint items makes the grammar ambiguous, and seems unnecessary since we can just as well annotate the constraint expression.

### 4.1.10 Partiality

The presence of constrained type-insts in MiniZinc means that various operations are potentially partial, i.e., not clearly defined for all possible inputs. For example, what happens if a function expecting a positive argument is passed a negative argument? What happens if a variable is assigned a value that does not satisfy its type-inst constraints? What happens if an array index is out of bounds? This section describes what happens in all these cases.
In general, cases involving fixed values that do not satisfy constraints lead to run-time aborts. *Rationale:* Our experience shows that if a fixed value fails a constraint, it is almost certainly due to a programming error. Furthermore, these cases are easy for an implementation to check.

But cases involving unfixed values vary, as we will see. *Rationale:* The best thing to do for unfixed values varies from case to case. Also, it is difficult to check constraints on unfixed values, particularly because during search a decision variable might become fixed and then backtracking will cause this value to be reverted, in which case aborting is a bad idea.

### 4.1.10.1 Partial Assignments

The first operation involving partiality is assignment. There are four distinct cases for assignments.

- A value assigned to a fixed, constrained global variable is checked at run-time; if the assigned value does not satisfy its constraints, it is a run-time error. In other words, this:

  ```
  1..5: x = 3;
  ```

  is equivalent to this:

  ```
  int: x = 3;
  constraint assert(x in 1..5,
                     "assignment to global parameter 'x' failed")
  ```

- A value assigned to an unfixed, constrained global variable makes the assignment act like a constraint; if the assigned value does not satisfy the variable’s constraints, it causes a run-time model failure. In other words, this:

  ```
  var 1..5: x = 3;
  ```

  is equivalent to this:

  ```
  var int: x = 3;
  constraint x in 1..5;
  ```

  *Rationale:* This behaviour is easy to understand and easy to implement.

- A value assigned to a fixed, constrained let-local variable is checked at run-time; if the assigned value does not satisfy its constraints, it is a run-time error. In other words, this:
is equivalent to this:

```mini
let { int: x = 3; } in
assert(x in 1..5,
"assignment to let parameter 'x' failed", x+1)
```

- A value assigned to an unfixed, constrained let-local variable makes the assignment act like a constraint; if the constraint fails at run-time, the failure “bubbles up” to the nearest enclosing Boolean scope, where it is interpreted as false.

  Rationale: This behaviour is consistent with assignments to global variables.

Note that in cases where a value is partly fixed and partly unfixed, e.g., some arrays, the different elements are checked according to the different cases, and fixed elements are checked before unfixed elements. For example:

```mini
u = [ let { var 1..5: x = 6 } in x, let { par 1..5: y = 6; } in y ];
```

This causes a run-time abort, because the second, fixed element is checked before the first, unfixed element. This ordering is true for the cases in the following sections as well. Rationale: This ensures that failures cannot mask aborts, which seems desirable.

### 4.1.10.2 Partial Predicate/Function and Annotation Arguments

The second kind of operation involving partiality is calls and annotations.

The semantics is similar to assignments: fixed arguments that fail their constraints will cause aborts, and unfixed arguments that fail their constraints will cause failure, which bubbles up to the nearest enclosing Boolean scope.

### 4.1.10.3 Partial Array Accesses

The third kind of operation involving partiality is array access. There are two distinct cases.

- A fixed value used as an array index is checked at run-time; if the index value is not in the index set of the array, it is a run-time error.
• An unfixed value used as an array index makes the access act like a constraint; if the access fails at run-time, the failure “bubbles up” to the nearest enclosing Boolean scope, where it is interpreted as false. For example:

```plaintext
array[1..3] of int: a = [1,2,3];
var int: i;
constraint (a[i] + 3) > 10 \/
```

Here the array access fails, so the failure bubbles up to the disjunction, and \( i \) is constrained to be 99. Rationale: Unlike predicate/function calls, modellers in practice sometimes do use array accesses that can fail. In such cases, the “bubbling up” behaviour is a reasonable one.

## 4.1.11 Built-in Operations

This appendix lists built-in operators, functions and predicates. They may be implemented as true built-ins, or in libraries that are automatically imported for all models. Many of them are overloaded.

Operator names are written within single quotes when used in type signatures, e.g. `bool: '\\/'(bool, bool).

We use the syntax `\( \text{Tl: } f(\text{Tl1}, \ldots, \text{Tln}) \)` to represent an operation named \( f \) that takes arguments with type-insts \( \text{Tl1}, \ldots, \text{Tln} \) and returns a value with type-inst \( \text{Tl} \). This is slightly more compact than the usual MiniZinc syntax, in that it omits argument names.

### 4.1.11.1 Comparison Operations

Less than. Other comparisons are similar: greater than (>), less than or equal (<=), greater than or equal (>=), equality (==, =), and disequality (!=).

```plaintext
bool: '\<\'( $T, $T)
var bool: '\<\'(var $T, var $T)
```

### 4.1.11.2 Arithmetic Operations

Addition. Other numeric operations are similar: subtraction (-), and multiplication (*).
**Unary minus. Unary plus (+) is similar.**

```plaintext
int:  '+(  int,    int)
var int:  '+(var int, var int)
float:  '+'(  float,   float)
var float:  '+'(var float, var float)
```

**Integer and floating-point division and modulo.**

```plaintext
int:  'div'(  int,    int)
var int:  'div'(var int, var int)
int:  'mod'(  int,    int)
var int:  'mod'(var int, var int)
float:  '/'(  float,   float)
var float:  '/'(var float, var float)
```

The result of the modulo operation, if non-zero, always has the same sign as its first operand. The integer division and modulo operations are connected by the following identity:

\[
x = (x \div y) \times y + (x \mod y)
\]

Some illustrative examples:

<table>
<thead>
<tr>
<th>7 div 4 = 1</th>
<th>7 mod 4 = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7 div 4 = -1</td>
<td>-7 mod 4 = -3</td>
</tr>
<tr>
<td>7 div -4 = -1</td>
<td>7 mod -4 = 3</td>
</tr>
<tr>
<td>-7 div -4 = 1</td>
<td>-7 mod -4 = -3</td>
</tr>
</tbody>
</table>

Sum multiple numbers. Product (product) is similar. Note that the sum of an empty array is 0, and the product of an empty array is 1.
MiniZinc Handbook, Release 2.4.3

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>int</td>
<td><code>sum(array[T] of int)</code></td>
</tr>
<tr>
<td>var int</td>
<td><code>sum(array[T] of var int)</code></td>
</tr>
<tr>
<td>float</td>
<td><code>sum(array[T] of float)</code></td>
</tr>
<tr>
<td>var float</td>
<td><code>sum(array[T] of var float)</code></td>
</tr>
</tbody>
</table>

Minimum of two values; maximum (max) is similar.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td><code>min($T, T)</code></td>
</tr>
<tr>
<td>var $T$</td>
<td><code>min(var $T, var $T)</code></td>
</tr>
</tbody>
</table>

Minimum of an array of values; maximum (max) is similar. Aborts if the array is empty.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$U$</td>
<td><code>min(array[T] of $U)</code></td>
</tr>
<tr>
<td>var $U$</td>
<td><code>min(array[T] of var $U)</code></td>
</tr>
</tbody>
</table>

Minimum of a fixed set; maximum (max) is similar. Aborts if the set is empty.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td><code>min(set of $T$)</code></td>
</tr>
</tbody>
</table>

Absolute value of a number.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>int</td>
<td><code>abs(int)</code></td>
</tr>
<tr>
<td>var int</td>
<td><code>abs(var int)</code></td>
</tr>
<tr>
<td>float</td>
<td><code>abs(float)</code></td>
</tr>
<tr>
<td>var float</td>
<td><code>abs(var float)</code></td>
</tr>
</tbody>
</table>

Square root of a float. Aborts if argument is negative.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>float</td>
<td><code>sqrt(float)</code></td>
</tr>
<tr>
<td>var float</td>
<td><code>sqrt(var float)</code></td>
</tr>
</tbody>
</table>

Power operator. E.g. `pow(2, 5)` gives 32.

<table>
<thead>
<tr>
<th>Type</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>int</td>
<td><code>pow(int, int)</code></td>
</tr>
<tr>
<td>float</td>
<td><code>pow(float, float)</code></td>
</tr>
</tbody>
</table>

Natural exponent.
Natural logarithm. Logarithm to base 10 (\log_{10}) and logarithm to base 2 (\log_2) are similar.

\[
\begin{align*}
\text{float: } & \exp(\text{float}) \\
\text{var float: } & \exp(\text{var float})
\end{align*}
\]

General logarithm; the first argument is the base.

\[
\begin{align*}
\text{float: } & \ln(\text{float}) \\
\text{var float: } & \ln(\text{var float})
\end{align*}
\]

Sine. Cosine (\cos), tangent (\tan), inverse sine (\arcsin), inverse cosine (\arccos), inverse tangent (\arctan), hyperbolic sine (\sinh), hyperbolic cosine (\cosh), hyperbolic tangent (\tanh), inverse hyperbolic sine (\arsinh), inverse hyperbolic cosine (\arcosh) and inverse hyperbolic tangent (\artanh) are similar.

\[
\begin{align*}
\text{float: } & \sin(\text{float}) \\
\text{var float: } & \sin(\text{var float})
\end{align*}
\]

### 4.1.11.3 Logical Operations

Conjunction. Other logical operations are similar: disjunction (\lor) reverse implication (\n\Rightarrow\n), forward implication (\Rightarrow), bi-implication (\iff), exclusive disjunction (\xor), logical negation (\not).

Note that the implication operators are not written using \Rightarrow, \Leftarrow and \iff as is the case in some languages. This allows \Leftarrow to instead represent “less than or equal” .

\[
\begin{align*}
\text{bool: } & '/\ Schüler, Schüler) \\
\text{var bool: } & '/\ Schüler, Schüler)
\end{align*}
\]

Universal quantification. Existential quantification (\exists) is similar. Note that, when applied to an empty list, \forall returns true, and \exists returns false.

\[
\begin{align*}
\text{bool: } & \forall (\Rightarrow Schüler, Schüler) \\
\text{var bool: } & \forall (\Rightarrow Schüler, Schüler)
\end{align*}
\]
N-ary exclusive disjunction. N-ary bi-implication (iffall) is similar, with true instead of false.

```
bool: xorall(array[$T] of bool: bs) = foldl('xor', false, bs)
var bool: xorall(array[$T] of var bool: bs) = foldl('xor', false, bs)
```

### 4.11.4 Set Operations

Set membership.

```
bool: 'in'( $T, set of $T )
var bool: 'in'(var int, var set of int)
```

Non-strict subset. Non-strict superset (superset) is similar.

```
bool: 'subset'( set of $T, set of $T )
var bool: 'subset'(var set of int, var set of int)
```

Set union. Other set operations are similar: intersection (intersect), difference (diff), symmetric difference (symdiff).

```
set of $T: 'union'( set of $T, set of $T )
var set of int: 'union'(var set of int, var set of int)
```

Set range. If the first argument is larger than the second (e.g. 1..0), it returns the empty set.

```
set of int: '..'(int, int)
```

Cardinality of a set.

```
int: card( set of $T)
var int: card(var set of int)
```

Union of an array of sets. Intersection of multiple sets (array_intersect) is similar.

```
set of $U: array_union(array[$T] of set of $U)
var set of int: array_union(array[$T] of var set of int)
```
4.1.11.5 Array Operations

Length of an array.

```plaintext
int: length(array[T] of U)
```

List concatenation. Returns the list (integer-indexed array) containing all elements of the first argument followed by all elements of the second argument, with elements occurring in the same order as in the arguments. The resulting indices are in the range 1..n, where n is the sum of the lengths of the arguments. Rationale: This allows list-like arrays to be concatenated naturally and avoids problems with overlapping indices. The resulting indices are consistent with those of implicitly indexed array literals. Note that '++' also performs string concatenation.

```plaintext
array[int] of T: '++'(array[int] of T, array[int] of T)
```

Index sets of arrays. If the argument is a literal, returns 1..n where n is the (sub-)array length. Otherwise, returns the declared or inferred index set. This list is only partial, it extends in the obvious way, for arrays of higher dimensions.

```plaintext
set of T: index_set (array[T] of V)
set of T: index_set_1of2(array[T, U] of V)
set of U: index_set_2of2(array[T, U] of V)
...
```

Replace the indices of the array given by the last argument with the Cartesian product of the sets given by the previous arguments. Similar versions exist for arrays up to 6 dimensions.

```plaintext
array[T] of V: array1d(set of T1, array[U] of V)
array[T1,T2] of V:
  array2d(set of T1, set of T2, array[U] of V)
array[T1,T2,T3] of V:
  array3d(set of T1, set of T2, set of T3, array[U] of V)
```

4.1.11.6 Coercion Operations

Round a float towards $+\infty$, $-\infty$, and the nearest integer, respectively.
Explicit casts from one type-inst to another.

```plaintext
int: ceil (float)
int: floor(float)
int: round(float)
```

**4.11.7 String Operations**

To-string conversion. Converts any value to a string for output purposes. The exact form of the resulting string is implementation-dependent.

```plaintext
string: show($T)
```

Formatted to-string conversion for integers. Converts the integer given by the second argument into a string right justified by the number of characters given by the first argument, or left justified if that argument is negative. If the second argument is not fixed, the form of the string is implementation-dependent.

```plaintext
string: show_int(int, var int);
```

Formatted to-string conversion for floats. Converts the float given by the third argument into a string right justified by the number of characters given by the first argument, or left justified if that argument is negative. The number of digits to appear after the decimal point is given by the second argument. It is a run-time error for the second argument to be negative. If the third argument is not fixed, the form of the string is implementation-dependent.

```plaintext
string: show_float(int, int, var float)
```

String concatenation. Note that `++` also performs array concatenation.
**MiniZinc Handbook, Release 2.4.3**

**4.11.11.8 Bound and Domain Operations**

The bound operations `lb` and `ub` return fixed, correct lower/upper bounds to the expression. For numeric types, they return a lower/upper bound value, e.g. the lowest/highest value the expression can take. For set types, they return a subset/superset, e.g. the intersection/union of all possible values of the set expression.

The bound operations abort on expressions that have no corresponding finite bound. For example, this would be the case for a variable declared without bounds in an implementation that does not assign default bounds. (Set expressions always have a finite lower bound of course, namely `{}`, the empty set.)

**Numeric lower/upper bound:**

```
int:   lb(var int)
float: lb(var float)
int:   ub(var int)
float: ub(var float)
```

**Set lower/upper bound:**

```
set of int: lb(var set of int)
set of int: ub(var set of int)
```

Versions of the bound operations that operate on arrays are also available, they return a safe lower bound or upper bound for all members of the array - they abort if the array is empty:
Integer domain:

<table>
<thead>
<tr>
<th>Type</th>
<th>Declaration</th>
</tr>
</thead>
<tbody>
<tr>
<td>int</td>
<td>int: lb_array(array[$T] of var int)</td>
</tr>
<tr>
<td>float</td>
<td>float: lb_array(array[$T] of var float)</td>
</tr>
<tr>
<td>set of int</td>
<td>set of int: lb_array(array[$T] of var set of int)</td>
</tr>
<tr>
<td>int</td>
<td>int: ub_array(array[$T] of var int)</td>
</tr>
<tr>
<td>float</td>
<td>float: ub_array(array[$T] of var float)</td>
</tr>
<tr>
<td>set of int</td>
<td>set of int: ub_array(array[$T] of var set of int)</td>
</tr>
</tbody>
</table>

The domain operation `dom` returns a fixed superset of the possible values of the expression.

Integer array domain, returns a superset of all possible values that may appear in the array - this aborts if the array is empty:

<table>
<thead>
<tr>
<th>Type</th>
<th>Declaration</th>
</tr>
</thead>
<tbody>
<tr>
<td>int</td>
<td>set of int: dom_array(array[$T] of var int)</td>
</tr>
</tbody>
</table>

Domain size for integers:

<table>
<thead>
<tr>
<th>Type</th>
<th>Declaration</th>
</tr>
</thead>
<tbody>
<tr>
<td>int</td>
<td>int: dom_size(var int)</td>
</tr>
</tbody>
</table>

The domain size operation `dom_size` is equivalent to `card(dom(x))`.

Note that these operations can produce different results depending on when they are evaluated and what form the argument takes. For example, consider the numeric lower bound operation.

- If the argument is a fixed expression, the result is the argument’s value.
- If the argument is a decision variable, then the result depends on the context.
  - If the implementation can determine a lower bound for the variable, the result is that lower bound. The lower bound may be from the variable’s declaration, or higher than that due to preprocessing, or lower than that if an implementation-defined lower bound is applied (e.g. if the variable was declared with no lower bound, but the implementation imposes a lowest possible bound).
  - If the implementation cannot determine a lower bound for the variable, the operation aborts.
- If the argument is any other kind of unfixed expression, the lower bound depends on the bounds of unfixed subexpressions and the connecting operators.
4.11.9 Option Type Operations

The option type value ($T$) is written

\[
\text{opt}\ T: \ '<>';
\]

One can determine if an option type variable actually occurs or not using \textit{occurs} and \textit{absent}

\[
\begin{align*}
\text{par\ bool: occur}(\text{par\ opt}\ T); \\
\text{var\ bool: occur}(\text{var\ opt}\ T); \\
\text{par\ bool: absent}(\text{par\ opt}\ T); \\
\text{var\ bool: absent}(\text{var\ opt}\ T); \\
\end{align*}
\]

One can return the non-optional value of an option type variable using the function \textit{deopt}

\[
\begin{align*}
\text{par}\ T: \text{deopt}(\text{par\ opt}\ T); \\
\text{var}\ T: \text{deopt}(\text{var\ opt}\ T); \\
\end{align*}
\]

4.11.10 Other Operations

Check a Boolean expression is true, and abort if not, printing the second argument as the error message. The first one returns the third argument, and is particularly useful for sanity-checking arguments to predicates and functions; importantly, its third argument is lazy, i.e. it is only evaluated if the condition succeeds. The second one returns \textit{true} and is useful for global sanity-checks (e.g. of instance data) in constraint items.

\[
\begin{align*}
T: & \quad \text{assert}\ (\text{bool, string, s}T) \\
\text{par\ bool: assert}\ (\text{bool, string})
\end{align*}
\]

Abort evaluation, printing the given string.

\[
T: \text{abort}\ (\text{string})
\]

Return true. As a side-effect, an implementation may print the first argument.

\[
\text{bool: trace}\ (\text{string})
\]
Return the second argument. As a side-effect, an implementation may print the first argument.

\$T: \texttt{trace(string, }$T)\$

Check if the argument’s value is fixed at this point in evaluation. If not, abort; if so, return its value. This is most useful in output items when decision variables should be fixed: it allows them to be used in places where a fixed value is needed, such as if-then-else conditions.

\$T: \texttt{fix(var }$T)\$

As above, but return false if the argument’s value is not fixed.

\texttt{par bool: is\_fixed(var }$T)\$

### 4.1.12 Content-types

The content-type application/x-zinc-output defines a text output format for Zinc. The format extends the abstract syntax and semantics given in \textit{Run-time Outcomes} (page 202), and is discussed in detail in \textit{Output} (page 203).

The full syntax is as follows:

```markdown
% Output
<output> ::= <no-solutions> [ <warnings> ] <free-text>
  | ( <solution> )* [ <complete> ] [ <warnings> ] <free-text>

% Solutions
<solution> ::= <solution-text> [ \n ] "----------" \n
% Unsatisfiable
<no-solutions> ::= "=====UNSATISFIABLE====" \n
% Complete
<complete> ::= "=========" \n
% Messages
<warnings> ::= ( <message> )* +
```
The solution text for each solution must be as described in Output Items (page 239). A newline must be appended if the solution text does not end with a newline.

### 4.1.13 JSON support

MiniZinc can support reading input parameters and providing output formatted as JSON objects. A JSON input file needs to have the following structure:

- Consist of a single top-level object
- The members of the object (the key-value pairs) represent model parameters
- Each member key must be a valid MiniZinc identifier (and it supplies the value for the corresponding parameter of the model)
- Each member value can be one of the following:
  - A string (assigned to a MiniZinc string parameter)
  - A number (assigned to a MiniZinc int or float parameter)
  - The values true or false (assigned to a MiniZinc bool parameter)
  - An array of values. Arrays of arrays are supported only if all inner arrays are of the same length, so that they can be mapped to multi-dimensional MiniZinc arrays.
  - A set of values encoded as an object with a single member with key "set" and a list of values (the elements of the set).
  - A value of an enumerated type encoded as an object with a single member with key "e" and a string value (the identifier of the enumerated value).

Assume a MiniZinc model declaring the following parameters:

```plaintext
int: n;
enum Customers;
array[1..n,Customers] of int: distances;
array[1..n] of set of int: patterns;
```
Here is an example of a JSON parameter file using all of the above features:

```json
{
    "n" : 2,
    "Customers" : [ {"e" : "Customer A"}, {"e" : "Customer B"}, {"e" : "Customer C"} ],
    "distances" : [ [1,2,3], [4,5,6] ],
    "patterns" : [ {"set" : [1,3,5]}, {"set" : [2,4,6]} ]
}
```

The first parameter declares a simple integer \( n \). The `Customers` parameter defines the members of an enumerated type. The `distances` parameter is a two-dimensional array; note that all inner arrays must be of the same size in order to map to a (rectangular) MiniZinc two-dimensional array. The `patterns` parameter is an array of sets of integers.

### 4.1.14 Full grammar

#### 4.1.14.1 Items

```text
% A MiniZinc model
<model> ::= [ <item> ";" ... ]

% Items
<item> ::= <include-item>
    | <var-decl-item>
    | <enum-item>
    | <assign-item>
    | <constraint-item>
    | <solve-item>
    | <output-item>
    | <predicate-item>
    | <test-item>
    | <function-item>
    | <annotation-item>

<ti-expr-and-id> ::= <ti-expr> ":" <ident>
```
% Include items
<include-item> ::= "include" <string-literal>

% Variable declaration items
<var-decl-item> ::= <ti-expr-and-id> <annotations> [ "=" <expr> ]

% Enum items
<enum-item>  ::= "enum" <ident> <annotations> [ "=" <enum-cases> ]
<enum-cases> ::= {" <ident> "," ... "}

% Assign items
<assign-item> ::= <ident> "=" <expr>

% Constraint items
<constraint-item> ::= "constraint" <string-annotation> <expr>

% Solve item
<solve-item> ::= "solve" <annotations> "satisfy"
| "solve" <annotations> "minimize" <expr>
| "solve" <annotations> "maximize" <expr>

% Output items
<output-item> ::= "output" <expr>

% Annotation items
<annotation-item> ::= "annotation" <ident> <params>

% Predicate, test and function items
<predicate-item> ::= "predicate" <operation-item-tail>
<test-item> ::= "test" <operation-item-tail>
<function-item> ::= "function" <ti-expr> ":" <operation-item-tail>
<operation-item-tail> ::= <ident> <params> <annotations> [ "=" <expr> ]
<params> ::= [ ( <ti-expr-and-id> "," ... ) ]

### 4.1.14.2 Type-Inst Expressions

<ti-expr> ::= <base-ti-expr>
  | <array-ti-expr>

<base-ti-expr> ::= <var-par> <opt-ti> <set-ti> <base-ti-expr-tail>

<var-par> ::= "var" | "par" | ε

<opt-ti> ::= "opt" | ε

<set-ti> ::= "set" "of" | ε

<base-type> ::= "bool"
  | "int"
  | "float"
  | "string"

<base-ti-expr-tail> ::= <ident>
  | <base-type>
  | <ti-variable-expr-tail>
  | "ann"
  | { <expr> "," ... }
  | <num-expr> ".." <num-expr>

% Type-inst variables
<ti-variable-expr-tail> ::= $[A-Za-z][A-Za-z0-9_]*

% Array type-inst expressions
<array-ti-expr> ::= "array" [ <ti-expr> "," ... ] "of" <base-ti-expr>
  | "list" "of" <base-ti-expr>
4.1.14.3 Expressions

<expr> ::= <expr-atom> <expr-binop-tail>
<expr-atom> ::= <expr-atom-head> <expr-atom-tail> <annotations>
<expr-binop-tail> ::= [ <bin-op> <expr> ]
<expr-atom-head> ::= <builtin-un-op> <expr-atom>
| "(" <expr> ")"
| <ident-or-quoted-op>
| "+" 
| <bool-literal>
| <int-literal>
| <float-literal>
| <string-literal>
| <set-literal>
| <set-comp>
| <array-literal>
| <array-literal-2d>
| <array-comp>
| <ann-literal>
| <if-then-else-expr>
| <let-expr>
| <call-expr>
| <gen-call-expr>
<expr-atom-tail> ::= ϵ
| <array-access-tail> <expr-atom-tail>

% Numeric expressions
<num-expr> ::= <num-expr-atom> <num-expr-binop-tail>
<num-expr-atom> ::= <num-expr-atom-head> <expr-atom-tail> <annotations>
<num-expr-binop-tail> ::= [ <num-bin-op> <num-expr> ]
<num-expr-atom-head> ::= <builtin-num-un-op> <num-expr-atom>
% Built-in operators
<builtin-op> ::= <builtin-bin-op> | <builtin-un-op>

<bin-op> ::= <builtin-bin-op> | '<ident>'

<builtin-bin-op> ::= "<->" | "->" | "<-" | ""\"" | "xor" | "\"" | ""<" | "">" | "=" |">=" | "==" | "=" | "!=" | "in" | "subset" | "superset" | "union" | "diff" | "symdiff" | ".." | "intersect" | "++" | <builtin-num-bin-op>

<builtin-un-op> ::= "not" | <builtin-num-un-op>

% Built-in numeric operators
<num-bin-op> ::= <builtin-num-bin-op> | '<ident>'

<builtin-num-bin-op> ::= "+" | "-" | "*" | "/" | "div" | "mod"

<builtin-num-un-op> ::= "+" | "-"

% Boolean literals
<bool-literal> ::= "false" | "true"

% Integer literals
<int-literal> ::= [0-9]+ | 0x[0-9A-Fa-f]+ | 0o[0-7]+

% Float literals
<float-literal> ::= [0-9]+"."[0-9]+ |
       [0-9]+"."[0-9][Ee][+-]?[0-9]+ |
       [0-9]+[Ee][+-][0-9]+ |
       0[Xx][0-9a-fA-F]*"."[0-9a-fA-F]+[0-9a-fA-F]*+.
| (0[Xx][0-9a-fA-F]+[pP][+-]?[0-9]+)

% String literals
<string-contents> ::= (["\n"] | ["\n()]*

<string-literal> ::= ""
     <string-contents> ""
| ""
     <string-contents> "\" <string-interpolate-tail>

<string-interpolate-tail> ::= <expr> ")"<string-contents>""
| <expr> ")"<string-contents>"\"<...

% Set literals
<set-literal> ::= "{ [ expr ]," ... ] "}"

% Set comprehensions
<set-comp> ::= "{ [ expr ]," " <comp-tail> "}"

<comp-tail> ::= <generator> [ "where" <expr> ] "," ...

<generator> ::= <ident> "," ... "in" <expr>

% Array literals
<array-literal> ::= "[ [ expr ]," ... ] "]

% 2D Array literals
<array-literal-2d> ::= "[ [ <expr> "," ... ] "[ "...

% Array comprehensions
<array-comp> ::= "[ [ expr ]," " <comp-tail> "]"

% Array access
<array-access-tail> ::= "[ [ expr ]"," ... "]"
% Annotation literals
<ann-literal> ::= <ident> [ "(" <expr> "," ... ")" ]

% If-then-else expressions
<if-then-else-expr> ::= "if" <expr> "then" <expr> [ "elseif" <expr> "then"...
                      <expr> ]* "else" <expr> "endif"

% Call expressions
<call-expr> ::= <ident-or-quoted-op> [ "(" <expr> "," ... ")" ]

% Let expressions
<let-expr> ::= "let" "{" <let-item> ";" ... "}" "in" <expr>
              <let-item> ::= <var-decl-item>
                      | <constraint-item>

% Generator call expressions
<gen-call-expr> ::= <ident-or-quoted-op> "(" <comp-tail> ")" "(" <expr> ")"

4.1.14.4 Miscellaneous Elements

% Identifiers
<ident> ::= [A-Za-z][A-Za-z0-9_]* | '^[^\xa\xd\x0]*'

% Identifiers and quoted operators
<ident-or-quoted-op> ::= <ident>
                        | 'builtin-op'

% Annotations
<annotations> ::= [ "::" <annotation> ]*

<annotation> ::= <expr-atom-head> <expr-atom-tail>

<string-annotation> ::= "::" <string-literal>
4.2.1 Annotations

These annotations control evaluation and solving behaviour.

4.2.1.1 General annotations

Parameters

- **annotation add_to_output**

  Declare that the annotated variable should be added to the output of the model. This annotation only has an effect when the model does not have an output item.

- **annotation domain_change_constraint**

  Marks a constraint as a recorded domain changing constraint (when mzn2fzn called with -g flag)

- **annotation is_defined_var**
Declare the annotated variable as being functionally defined. This annotation is introduced into FlatZinc code by the compiler.

```markdown
annotation is_reverse_map
```

Declare that the annotated expression is used to map an expression back from FlatZinc to MiniZinc.

```markdown
annotation maybe_partial
```

Declare that expression may have undefined result (to avoid warnings)

```markdown
annotation mzn_break_here
```

With debug build of mzn2fzn, call MiniZinc::mzn_break_here when flattening this expression to make debugging easier. This annotation is ignored by the release build.

```markdown
annotation mzn_check_var
```

Declare that the annotated variable is required for checking solutions.

```markdown
annotation mzn_rhs_from_assignment
```

Used internally by the compiler

```markdown
annotation output_only
```

Declare that the annotated variable should be only used for output. This annotation can be used to define variables that are required for solution checkers, or that are necessary for the output item. The annotated variable must be par.

```markdown
annotation output_var
```

Declare that the annotated variable should be printed by the solver. This annotation is introduced into FlatZinc code by the compiler.
Declare function as total, i.e. it does not put any constraints on its arguments.

Declare a variable as being introduced by the compiler.

**Functions and Predicates**

Used to attach a name `s` to a constraint and its decomposition. String annotations on constraint keywords are re-written as `constraint_name` annotations

Declare variable: `c` as being functionally defined by the annotated constraint. This annotation is introduced into FlatZinc code by the compiler.

Document the function or variable declaration item with the string `s`.

Used to attach a name `s` to an expression, this should also propagate to any sub-expressions or decomposition of the annotated expression. String annotations on expressions are re-written as `expression_name` annotations

Declare that the annotated variable is required for checking solutions and has an enum type.
Declare a name for the annotated constraint.

```
annotation mzn_constraint_name(string)
```

State that a function is deprecated.

```
annotation mzn_deprecated(string: version, string: explanation)
```

Declare a name for the annotated expression.

```
annotation mzn_expression_name(string)
```

Representation of the call-stack when the annotated item was introduced, as a string s. Can be used to uniquely identify variables and constraints across different compilations of a model that may have different names. This annotations is introduced into FlatZinc code by the compiler and is retained if –keep-paths argument is used.

```
annotation mzn_path(string: s)
```

Declare that the annotated array should be printed by the solver with the given index sets a. This annotation is introduced into FlatZinc code by the compiler.

```
annotation output_array(array [$u] of set of int: a)
```

### 4.2.1.2 Propagation strength annotations

```
annotation bounds
```

Annotate a constraint to use bounds propagation

```
annotation domain
```

Annotate a constraint to use domain propagation
4.2.1.3 Search annotations

Variable selection annotations

- **annotation anti_first_fail**
  Choose the variable with the largest domain

- **annotation dom_w_deg**
  Choose the variable with largest domain, divided by the number of attached constraints weighted by how often they have caused failure

- **annotation first_fail**
  Choose the variable with the smallest domain

- **annotation impact**
  Choose the variable with the highest impact so far during the search

- **annotation input_order**
  Search variables in the given order

- **annotation largest**
  Choose the variable with the largest value in its domain

- **annotation max_regret**
  Choose the variable with largest difference between the two smallest values in its domain

- **annotation most_constrained**
Choose the variable with the smallest domain, breaking ties using the number of attached constraints

```
annotation occurrence
```

Choose the variable with the largest number of attached constraints

```
annotation smallest
```

Choose the variable with the smallest value in its domain

**Value choice annotations**

```
annotation indomain
```

Assign values in ascending order

```
annotation indomain_interval
```

If the domain consists of several contiguous intervals, reduce the domain to the first interval. Otherwise bisect the domain.

```
annotation indomain_max
```

Assign the largest value in the domain

```
annotation indomain_median
```

Assign the middle value in the domain

```
annotation indomain_middle
```

Assign the value in the domain closest to the mean of its current bounds

```
annotation indomain_min
```
Assign the smallest value in the domain

```plaintext
annotation indomain_random
```

Assign a random value from the domain

```plaintext
annotation indomain_reverse_split
```

Bisect the domain, excluding the lower half first

```plaintext
annotation indomain_split
```

Bisect the domain, excluding the upper half first

```plaintext
annotation indomain_split_random
```

Bisect the domain, randomly selecting which half to exclude first

```plaintext
annotation outdomain_max
```

Exclude the largest value from the domain

```plaintext
annotation outdomain_median
```

Exclude the middle value from the domain

```plaintext
annotation outdomain_min
```

Exclude the smallest value from the domain

```plaintext
annotation outdomain_random
```

Exclude a random value from the domain

---

**Exploration strategy annotations**

---

4.2.1. Annotations
Perform a complete search

**Restart annotations**

**Parameters**

<table>
<thead>
<tr>
<th>Annotation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>restart_none</td>
<td>Do not restart</td>
</tr>
</tbody>
</table>

**Functions and Predicates**

<table>
<thead>
<tr>
<th>Annotation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>restart_constant&lt;int: scale&gt;</td>
<td>Restart after constant number of nodes scale</td>
</tr>
<tr>
<td>restart_geometric&lt;float: base, int: scale&gt;</td>
<td>Restart with geometric sequence with parameters base and scale</td>
</tr>
<tr>
<td>restart_linear&lt;int: scale&gt;</td>
<td>Restart with linear sequence scaled by scale</td>
</tr>
<tr>
<td>restart_luby&lt;int: scale&gt;</td>
<td>Restart with Luby sequence scaled by scale</td>
</tr>
</tbody>
</table>

**Other declarations**
Specify search on variables $x$, with variable selection strategy select, value choice strategy choice, and exploration strategy explore. If $x$ is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the array1d function).

Specify search on variables $x$, with variable selection strategy select, and value choice strategy choice. If $x$ is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the array1d function).

Specify search on variables $x$, with precision $\text{prec}$, variable selection strategy select, value choice strategy choice, and exploration strategy explore. If $x$ is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the array1d function).

Specify search on variables $x$, with precision $\text{prec}$, variable selection strategy select, and value choice strategy choice. If $x$ is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the array1d function).
Specify search on variables $x$, with variable selection strategy select, value choice strategy choice, and exploration strategy explore. If $x$ is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the array1d function).

```plaintext
annotation int_search(array [X] of var int: x,
    ann: select,
    ann: choice)
```

Specify search on variables $x$, with variable selection strategy select, and value choice strategy choice. If $x$ is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the array1d function).

```plaintext
annotation seq_search(array [int] of ann: s)
```

Sequentially perform the searches specified in array $s$

```plaintext
annotation set_search(array [X] of var set of int: x,
    ann: select,
    ann: choice,
    ann: explore)
```

Specify search on variables $x$, with variable selection strategy select, value choice strategy choice, and exploration strategy explore. If $x$ is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the array1d function).

```plaintext
annotation set_search(array [X] of var set of int: x,
    ann: select,
    ann: choice)
```

Specify search on variables $x$, with variable selection strategy select, and value choice strategy choice. If $x$ is a multi-dimensional array, it is coerced to one-dimensional in row-major order (as with the array1d function).

**4.2.1.4 Warm start annotations**

To be put on the solve item, similar to search annotations. A variable can be mentioned several times and in different annotations but only one of the values is taken
**Warm start annotations with optional values**

The value arrays can contain <> elements (absent values). The following decompositions eliminate those elements because FlatZinc 1.6 does not support optionals.

```plaintext
annotation warm_start(array [int] of var bool: x,
                      array [int] of opt bool: v)
```

Specify warm start values v for an array of booleans x

```plaintext
annotation warm_start(array [int] of var int: x,
                      array [int] of opt int: v)
```

Specify warm start values v for an array of integers x

```plaintext
annotation warm_start(array [int] of var float: x,
                      array [int] of opt float: v)
```

Specify warm start values v for an array of floats x

```plaintext
annotation warm_start(array [int] of var set of int: x,
                      array [int] of opt set of int: v)
```

Specify warm start values v for an array of sets x

**Other declarations**

```plaintext
annotation warm_start(array [int] of var bool: x,
                      array [int] of bool: v)
```

Specify warm start values v for an array of booleans x

```plaintext
annotation warm_start(array [int] of var int: x, array [int] of int: v)
```

Specify warm start values v for an array of integers x
Specify warm start values \( v \) for an array of floats \( x \)

\[
\text{annotation warm_start(array [int] of var float: } x, \\
\text{array [int] of float: } v) 
\]

Specify warm start values \( v \) for an array of sets \( x \)

\[
\text{annotation warm_start(array [int] of var set of int: } x, \\
\text{array [int] of set of int: } v) 
\]

Specify an array \( w \) of warm_start annotations or other warm_start_array annotations. Can be useful to keep the annotation order in FlatZinc for manual updating.

Note: if you have search annotations as well, put warm_starts into seq_search in order to have precedence between both, which may matter.

### 4.2.2 Option type support

These functions and predicates implement the standard library for working with option types. Note that option type support is still incomplete.

#### 4.2.2.1 Option type support for Booleans

\[
\text{predicate 'not'(var opt bool: } x) 
\]

Usage: \( \text{not } x \)

True iff \( x \) is absent or false

\[
\text{predicate absent(var opt bool: } x) 
\]

True iff \( x \) is absent
**4.2.2. Option type support**

```plaintext
predicate bool_eq(var opt bool: b0, var opt bool: b1)
```

True iff both b0 and b1 are absent or both are present and have the same value.

```plaintext
predicate bool_eq(var opt bool: b0, var bool: b1)
```

True iff b0 occurs and is equal to b1

```plaintext
predicate bool_eq(var bool: b0, var opt bool: b1)
```

True iff b1 occurs and is equal to b0

```plaintext
annotation bool_search(array [int] of var opt bool: x,
  ann: a1,
  ann: a2,
  ann: a3)
```

Search annotation for optional Boolean variables

```plaintext
annotation bool_search(array [int] of var opt bool: x,
  ann: a1,
  ann: a2)
```

Search annotation for optional Boolean variables

```plaintext
predicate deopt(var opt bool: x)
```

Return value of x (assumes that x is not absent)

```plaintext
function var opt bool: element(var opt int: idx,
  array [int] of var bool: x)
```

Return absent if idx is absent, otherwise return x [ idx ]
array [int,int] of var bool: x)

Return absent if idx1 or idx2 is absent, otherwise return x [idx1, idx2]

function var opt bool: element(var int: idx,
      array [int] of var opt bool: x)

Return x [idx]

function var opt bool: element(var int: idx1,
      var int: idx2,
      array [int,int] of var opt bool: x)

Return x [idx1, idx2]

function var opt bool: element(var opt int: idx,
      array [int] of var opt bool: x)

Return absent if idx is absent, otherwise return x [idx]

function var opt bool: element(var opt int: idx1,
      var opt int: idx2,
      array [int,int] of var opt bool: x)

Return absent if idx1 or idx2 is absent, otherwise return x [idx1, idx2]

predicate exists(array [int] of var opt bool: x)

True iff for at least one i, x[i] occurs and is true

predicate forall(array [int] of var opt bool: x)

True iff for any i, x[i] is absent or true

predicate occurs(var opt bool: x)

True iff x is not absent
4.2.2.2 Option type support for integers

**function var int: '×'(var opt int: x, var opt int: y)**

Usage: x * y
Optional multiplication. Return product of x and y, with absent replaced by 1.

**function int: '×'(opt int: x, opt int: y)**

Usage: x * y
Optional multiplication. Return product of x and y, with absent replaced by 1.

**function var int: '+'(var opt int: x, var opt int: y)**

Usage: x + y
Optional addition. Return sum of x and y, with absent replaced by 0.

**function int: '+'(opt int: x, opt int: y)**

Usage: x + y
Optional addition. Return sum of x and y, with absent replaced by 0.

**function var opt int: '-'(var opt int: x, var opt int: y)**

Usage: x - y
Optional subtraction. Return absent if x is absent, x if y is absent, difference of x and y if both are present.

**function opt int: '-'(opt int: x, opt int: y)**

Usage: x - y
Optional subtraction. Return absent if x is absent, x if y is absent, difference of x and y if both are present.
**Predicate `<`**

Usage: `x < y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is less than the value of `y`.

**Test `<`**

Usage: `x < y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is less than the value of `y`.

**Predicate `<=`**

Usage: `x <= y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is less than or equal to the value of `y`.

**Test `<=`**

Usage: `x <= y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is less than or equal to the value of `y`.

**Predicate `>`**

Usage: `x > y`

Weak comparison: true iff either `x` or `y` is absent, or both occur and the value of `x` is greater than the value of `y`.

**Test `>`**

Usage: `x > y`
Weak comparison: true iff either \( x \) or \( y \) is absent, or both occur and the value of \( x \) is greater than the value of \( y \).

\[
\text{predicate } \geq (\text{var opt int: } x, \text{ var opt int: } y)
\]

Usage: \( x \geq y \)

Weak comparison: true iff either \( x \) or \( y \) is absent, or both occur and the value of \( x \) is greater than or equal to the value of \( y \).

\[
\text{test } \geq (\text{opt int: } x, \text{ opt int: } y)
\]

Usage: \( x \geq y \)

Weak comparison: true iff either \( x \) or \( y \) is absent, or both occur and the value of \( x \) is greater than or equal to the value of \( y \).

\[
\text{function var opt int: 'div'(var opt int: } x, \text{ var opt int: } y)
\]

Usage: \( x \div y \)

Optional division. Return absent if \( x \) is absent, \( x \) if \( y \) is absent, \( x \) divided by \( y \) if both are present.

\[
\text{function opt int: 'div'(opt int: } x, \text{ opt int: } y)
\]

Usage: \( x \div y \)

Optional division. Return absent if \( x \) is absent, \( x \) if \( y \) is absent, \( x \) divided by \( y \) if both are present.

\[
\text{function var opt int: 'mod'(var opt int: } x, \text{ var opt int: } y)
\]

Usage: \( x \mod y \)

Optional modulo. Return absent if \( x \) or \( y \) is absent, \( x \) modulo \( y \) if both are present.

\[
\text{function opt int: 'mod'(opt int: } x, \text{ opt int: } y)
\]

Usage: \( x \mod y \)

Optional modulo. Return absent if \( x \) or \( y \) is absent, \( x \) modulo \( y \) if both are present.
predicate absent(var opt int: x)

True iff x is absent

function var opt int: bool2int(var opt bool: x)

Return optional 0/1 integer that is absent iff x is absent, and 1 iff x occurs and is true.

function var $E$: deopt(var opt $E$: x)

Return value of x (assumes that x is not absent)

function var opt int: element(var opt int: idx, array [int] of var int: x)

Return absent if idx is absent, otherwise return x [ idx ]

function var opt int: element(var opt int: idx1, var opt int: idx2, array [int,int] of var int: x)

Return absent if idx1 or idx2 is absent, otherwise return x [ idx1, idx2 ]

function var opt int: element(var int: idx, array [int] of var opt int: x)

Return x [ idx ]

function var opt int: element(var int: idx1, var int: idx2, array [int,int] of var opt int: x)

Return x [ idx1, idx2 ]

function var opt int: element(var opt int: idx, array [int] of var opt int: x)
Return absent if \( \text{idx} \) is absent, otherwise return \( x[\text{idx}] \)

\[
\text{function var opt int: element(var opt int: \text{idx}1,}
\text{ var opt int: \text{idx}2,}
\text{ array [int,int] of var opt int: } x)\]

Return absent if \( \text{idx}1 \) or \( \text{idx}2 \) is absent, otherwise return \( x[\text{idx}1, \text{idx}2] \)

\[
\text{function var opt float: int2float(var opt int: } x)\]

Return optional 0/1 integer that is absent iff \( x \) is absent, and 1 iff \( x \) occurs and is true.

\[
\text{predicate int_eq(var opt int: } x, \text{ var opt int: } y)\]

True iff both \( x \) and \( y \) are absent or both are present and have the same value.

\[
\text{predicate int_ne(var opt int: } x, \text{ var opt int: } y)\]

True iff only one of \( x \) and \( y \) is absent or both are present and have different values.

\[
\text{annotation int_search(array [int] of var opt int: } x, 
\text{ ann: a1,}
\text{ ann: a2,}
\text{ ann: a3)}\]

Search annotation for optional integer variables

\[
\text{annotation int_search(array [int] of var opt int: } x, \text{ ann: a1, ann: a2)}\]

Search annotation for optional integer variables

\[
\text{function var opt int: max(array [int] of var opt int: } x)\]

Return maximum of elements in \( x \) that are not absent, or absent if all elements in \( x \) are absent.

\[
\text{function var opt int: min(array [int] of var opt int: } x)\]
Return minimum of elements in $x$ that are not absent, or absent if all elements in $x$ are absent.

$$\text{predicate occurs}(\text{var opt int: } x)$$

True iff $x$ is not absent

$$\text{function var int: product(\text{array [int] of var opt int: } x)}$$

Return product of non-absent elements of $x$.

$$\text{function int: product(\text{array [int] of opt int: } x)}$$

Return product of non-absent elements of $x$.

$$\text{function var int: sum(\text{array [int] of var opt int: } x)}$$

Return sum of non-absent elements of $x$.

$$\text{function int: sum(\text{array [int] of opt int: } x)}$$

Return sum of non-absent elements of $x$.

$$\text{function var opt int: } \times(\text{var opt int: } x, \text{ var opt int: } y)$$

Weak multiplication. Return product of $x$ and $y$ if both are present, otherwise return absent.

$$\text{function var opt int: } +(\text{var opt int: } x, \text{ var opt int: } y)$$

Weak addition. Return sum of $x$ and $y$ if both are present, otherwise return absent.

$$\text{function var opt int: } -(\text{var opt int: } x, \text{ var opt int: } y)$$

Weak subtraction. Return difference of $x$ and $y$ if both are present, otherwise return absent.

$$\text{predicate } \neq(\text{var opt int: } x, \text{ var opt int: } y)$$

Weak equality. True if either $x$ or $y$ are absent, or present and equal.
### 4.2.2.3 Option type support for floats

**Function:**

```plaintext
function var float: '*'(var opt float: x, var opt float: y)
```

**Usage:** `x * y`

Optional multiplication. Return product of `x` and `y`, with absent replaced by 1.

**Function:**

```plaintext
function float: '*'(opt float: x, opt float: y)
```

**Usage:** `x * y`

Optional multiplication. Return product of `x` and `y`, with absent replaced by 1.

**Function:**

```plaintext
function var float: '+'(var opt float: x, var opt float: y)
```

**Usage:** `x + y`

Optional addition. Return sum of `x` and `y`, with absent replaced by 0.

**Function:**

```plaintext
function float: '+'(opt float: x, opt float: y)
```

**Usage:** `x + y`

Optional addition. Return sum of `x` and `y`, with absent replaced by 0.

**Function:**

```plaintext
function var opt float: '-'(var opt float: x, var opt float: y)
```

**Usage:** `x - y`

Optional subtraction. Return absent if `x` is absent, `x` if `y` is absent, difference of `x` and `y` if both are present.

**Function:**

```plaintext
function opt float: '-'(opt float: x, opt float: y)
```

**Usage:** `x - y`

Optional subtraction. Return absent if `x` is absent, `x` if `y` is absent, difference of `x` and `y` if both are present.
function var opt float: '/'(var opt float: x, var opt float: y)

Usage: x / y
Optional division. Return absent if x is absent, x if y is absent, x divided by y if both are present.

function opt float: '/'(opt float: x, opt float: y)

Usage: x / y
Optional division. Return absent if x is absent, x if y is absent, x divided by y if both are present.

class

predicate '<'(var opt float: x, var opt float: y)

Usage: x < y
Weak comparison: true iff either x or y is absent, or both occur and the value of x is less than the value of y.

test '<'(opt float: x, opt float: y)

Usage: x < y
Weak comparison: true iff either x or y is absent, or both occur and the value of x is less than the value of y.

predicate '<='(var opt float: x, var opt float: y)

Usage: x <= y
Weak comparison: true iff either x or y is absent, or both occur and the value of x is less than or equal to the value of y.

test '<='(opt float: x, opt float: y)

Usage: x <= y
Weak comparison: true iff either x or y is absent, or both occur and the value of x is less than or equal to the value of y.
**predicate '＞'(var opt float: x, var opt float: y)**

Usage: x > y

Weak comparison: true iff either x or y is absent, or both occur and the value of x is greater than the value of y.

**test '＞'(opt float: x, opt float: y)**

Usage: x > y

Weak comparison: true iff either x or y is absent, or both occur and the value of x is greater than the value of y.

**predicate '＞='(var opt float: x, var opt float: y)**

Usage: x >= y

Weak comparison: true iff either x or y is absent, or both occur and the value of x is greater than or equal to the value of y.

**test '＞='(opt float: x, opt float: y)**

Usage: x >= y

Weak comparison: true iff either x or y is absent, or both occur and the value of x is greater than or equal to the value of y.

**predicate absent(var opt float: x)**

True iff x is absent

**function var opt float: bool2float(var opt bool: x)**

Return optional 0/1 float that is absent iff x is absent, and 1 iff x occurs and is true.

**function var float: deopt(var opt float: x)**

Return value of x (assumes that x is not absent)
function var opt float: element(var opt int: idx,
    array [int] of var float: x)

Return absent if idx is absent, otherwise return x [ idx ]

function var opt float: element(var opt int: idx1,
    var opt int: idx2,
    array [int,int] of var float: x)

Return absent if idx1 or idx2 is absent, otherwise return x [ idx1, idx2 ]

function var opt float: element(var int: idx,
    array [int] of var float: x)

Return x [ idx ]

function var opt float: element(var int: idx1,
    var int: idx2,
    array [int,int] of var float: x)

Return x [ idx1, idx2 ]

function var opt float: element(var opt int: idx,
    array [int] of var float: x)

Return absent if idx is absent, otherwise return x [ idx ]

function var opt float: element(var opt int: idx1,
    var opt int: idx2,
    array [int,int] of var float: x)

Return absent if idx1 or idx2 is absent, otherwise return x [ idx1, idx2 ]

predicate float_eq(var opt float: x, var opt float: y)

True iff both x and y are absent or both are present and have the same value.
**predicate float_ne(var opt float: x, var opt float: y)**

True iff only one of x and y is absent or both are present and have different values.

**annotation float_search(array [int] of var opt float: x,**
  
  ann: a1,
  ann: a2,
  ann: a3)

Search annotation for optional float variables

**annotation float_search(array [int] of var opt float: x,**
  
  ann: a1,
  ann: a2)

Search annotation for optional float variables

**function var opt float: max(array [int] of var opt float: x)**

Return maximum of elements in x that are not absent, or absent if all elements in x are absent.

**function var opt float: min(array [int] of var opt float: x)**

Return minimum of elements in x that are not absent, or absent if all elements in x are absent.

**predicate occurs(var opt float: x)**

True iff x is not absent

**function var float: product(array [int] of var opt float: x)**

Return product of non-absent elements of x.

**function float: product(array [int] of opt float: x)**

Return product of non-absent elements of x.
function var float: sum(array [int] of var opt float: x)

Return sum of non-absent elements of x.

function float: sum(array [int] of opt float: x)

Return sum of non-absent elements of x.

function var opt float: ~*(var opt float: x, var opt float: y)

Weak multiplication. Return product of x and y if both are present, otherwise return absent.

function var opt float: ~+(var opt float: x, var opt float: y)

Weak addition. Return sum of x and y if both are present, otherwise return absent.

function var opt float: ~-(var opt float: x, var opt float: y)

Weak subtraction. Return difference of x and y if both are present, otherwise return absent.

predicate ~(var opt float: x, var opt float: y)

Weak equality. True if either x or y are absent, or present and equal.

4.2.2.4 Other declarations

test absent(var $T: x)

Test if x is absent (always returns false)

test absent(opt $T: x)

Test if x is absent
function $T: deopt(opt $T: x)

Return value of $x$ if $x$ is not absent. Aborts when evaluated on absent value.

function var $T: deopt(var $T: x)

Return value $x$ unchanged (since $x$ is guaranteed to be non-optional).

test occurs(var $T: x)

Test if $x$ is not absent (always returns true)

test occurs(opt $T: x)

Test if $x$ is not absent

### 4.2.3 Compiler options

#### 4.2.3.1 Parameters

**opt bool: mzn_opt_only_range_domains**

Whether to only generate domains that are contiguous ranges

**opt int: mzn_min_version_required**

If defined, this can be used to check that the MiniZinc compiler supports all the features used in the model.

#### 4.2.3.2 Functions and Predicates

**test mzn_check_only_range_domains()**

Check whether to only generate domains that are contiguous ranges
4.2.4 Builtins

These functions and predicates define built-in operations of the MiniZinc language.

4.2.4.1 Comparison Builtins

These builtins implement comparison operations.

<table>
<thead>
<tr>
<th>Syntax</th>
<th>Description</th>
</tr>
</thead>
</table>
| `test $'\ne'$ ($T$: x, $T$: y)` | Usage: `x != y`  
Return if x is not equal to y |
| `predicate $'\ne'$ (var $T$: x, var $T$: y)` | Usage: `x != y`  
Return if x is not equal to y |
| `test $'\ne'$ (array [$T$] of int: x, array [$T$] of int: y)` | Usage: `x != y`  
Return if array x is not equal to array y |
| `predicate $'\ne'$ (array [$T$] of var int: x, array [$T$] of var int: y)` | Usage: `x != y`  
Return if array x is not equal to array y |
| `test $'\ne'$ (array [$T$] of bool: x, array [$T$] of bool: y)` | Usage: `x != y`  
Return if array x is not equal to array y |
predicate '!='(array [$T] of var bool: x, array [$T] of var bool: y)

Usage: x != y
Return if array x is not equal to array y

test '!='(array [$T] of set of int: x, array [$T] of set of int: y)

Usage: x != y
Return if array x is not equal to array y

predicate '!='(array [$T] of var set of int: x, array [$T] of var set of int: y)

Usage: x != y
Return if array x is not equal to array y

test '!='(array [$T] of float: x, array [$T] of float: y)

Usage: x != y
Return if array x is not equal to array y

predicate '!='(array [$T] of var float: x, array [$T] of var float: y)

Usage: x != y
Return if array x is not equal to array y

test '<'($T: x, $T: y)

Usage: x < y
Return if x is less than y

predicate '<'(var $T: x, var $T: y)
Usage: $x < y$

Return if $x$ is less than $y$

```plaintext
test '<'([U] of $T$: x, array [U] of $T$: y)
```

Usage: $x < y$

Return if array $x$ is lexicographically smaller than array $y$

```plaintext
predicate '<'(array [U] of var $T$: x, array [U] of var $T$: y)
```

Usage: $x <= y$

Return if $x$ is less than or equal to $y$

```plaintext
test '<='($T$: x, $T$: y)
```

Usage: $x <= y$

Return if array $x$ is lexicographically smaller than or equal to array $y$

```plaintext
predicate '<='(array [U] of var $T$: x, array [U] of var $T$: y)
```

Usage: $x <= y$

Return if array $x$ is lexicographically smaller than or equal to array $y$
test '!='($T: x, $T: y)

Usage: x = y
Return if x is equal to y

test '!='(opt $T: x, opt $T: y)

Usage: x = y
Return if x is equal to y

predicate '!='(var $T: x, var $T: y)

Usage: x = y
Return if x is equal to y

predicate '!='(var opt $T: x, var opt $T: y)

Usage: x = y
Return if x is equal to y

test '!='(array [$T] of int: x, array [$T] of int: y)

Usage: x = y
Return if array x is equal to array y

predicate '!='(array [$T] of var int: x, array [$T] of var int: y)

Usage: x = y
Return if array x is equal to array y

test '!='(array [$T] of bool: x, array [$T] of bool: y)

Usage: x = y
Return if array $x$ is equal to array $y$

\[
\text{predicate } '='(\text{array } [\mathcal{T}] \text{ of var bool}: x, \text{array } [\mathcal{T}] \text{ of var bool}: y)
\]

Usage: $x = y$

Return if array $x$ is equal to array $y$

\[
\text{test } '='(\text{array } [\mathcal{T}] \text{ of set of int}: x, \text{array } [\mathcal{T}] \text{ of set of int}: y)
\]

Usage: $x = y$

Return if array $x$ is equal to array $y$

\[
\text{predicate } '='(\text{array } [\mathcal{T}] \text{ of var set of int}: x, \text{array } [\mathcal{T}] \text{ of var set of int}: y)
\]

Usage: $x = y$

Return if array $x$ is equal to array $y$

\[
\text{test } '='(\text{array } [\mathcal{T}] \text{ of float}: x, \text{array } [\mathcal{T}] \text{ of float}: y)
\]

Usage: $x = y$

Return if array $x$ is equal to array $y$

\[
\text{predicate } '='(\text{array } [\mathcal{T}] \text{ of var float}: x, \text{array } [\mathcal{T}] \text{ of var float}: y)
\]

Usage: $x = y$

Return if array $x$ is equal to array $y$

\[
\text{test } '>'(\mathcal{T}: x, \mathcal{T}: y)
\]

Usage: $x > y$

Return if $x$ is greater than $y$
predicate '>(var $T$: $x$, var $T$: $y$)

Usage: $x > y$
Return if $x$ is greater than $y$

test '>(array [$U]$ of $T$: $x$, array [$U]$ of $T$: $y$)

Usage: $x > y$
Return if array $x$ is lexicographically greater than array $y$

predicate '>='(var $T$: $x$, var $T$: $y$)

Usage: $x >= y$
Return if $x$ is greater than or equal to $y$

predicate '>='(array [$U]$ of $T$: $x$, array [$U]$ of $T$: $y$)

Usage: $x >= y$
Return if $x$ is greater than or equal to $y$

test '>='(array [$U]$ of $T$: $x$, array [$U]$ of $T$: $y$)

Usage: $x >= y$
Return if array $x$ is lexicographically greater than or equal to array $y$

4.2.4.2 Arithmetic Builtins

These builtins implement arithmetic operations.
function int: '*'(int: x, int: y)

Usage: x * y
Return x * y

function var int: '*'(var int: x, var int: y)

Usage: x * y
Return x * y

function float: '*'(float: x, float: y)

Usage: x * y
Return x * y

function var float: '*'(var float: x, var float: y)

Usage: x * y
Return x * y

function int: '+'(int: x, int: y)

Usage: x + y
Return x + y

function var int: '+'(var int: x, var int: y)

Usage: x + y
Return x + y

function float: '+'(float: x, float: y)

Usage: x + y
Return \( x + y \)

\[
\text{function \( \text{var \ float} \): } '+'(\text{var \ float} : x, \text{var \ float} : y)
\]

Usage: \( x + y \)

Return \( x + y \)

\[
\text{function \( \text{int} \): } '-'(\text{int} : x, \text{int} : y)
\]

Usage: \( x - y \)

Return \( x - y \)

\[
\text{function \( \text{var \ int} \): } '-'(\text{var \ int} : x, \text{var \ int} : y)
\]

Usage: \( x - y \)

Return \( x - y \)

\[
\text{function \( \text{float} \): } '-'(\text{float} : x, \text{float} : y)
\]

Usage: \( x - y \)

Return \( x - y \)

\[
\text{function \( \text{var \ float} \): } '-'(\text{var \ float} : x, \text{var \ float} : y)
\]

Usage: \( x - y \)

Return \( x - y \)

\[
\text{function \( \text{int} \): } '-'(\text{int} : x)
\]

Usage: \( -x \)

Return negative \( x \)

\[
\text{function \( \text{var \ int} \): } '-'(\text{var \ int} : x)
\]
Usage: \(- x\)
Return negative \(x\)

\[
\text{function float: } '(- (\text{float: } x)
\]

Usage: \(- x\)
Return negative \(x\)

\[
\text{function var float: } '(- (\text{var float: } x)
\]

Usage: \(- x\)
Return negative \(x\)

\[
\text{function float: } '/ (\text{float: } x, \text{float: } y)
\]

Usage: \(x / y\)
Return result of floating point division \(x / y\)

\[
\text{function var float: } '/ (\text{var float: } x, \text{var float: } y)
\]

Usage: \(x / y\)
Return result of floating point division \(x / y\)

\[
\text{function int: } '^' (\text{int: } x, \text{int: } y)
\]

Usage: \(x ^ y\)
Return \(x^y\)

\[
\text{function var int: } '^' (\text{var int: } x, \text{var int: } y)
\]

Usage: \(x ^ y\)
Return \(x^y\)
function float: '^'(float: x, float: y)

Usage: x ^ y

Return x^y

function var float: '^'(var float: x, var float: y)

Usage: x ^ y

Return x^y

function int: 'div'(int: x, int: y)

Usage: x div y

Return result of integer division x / y

function var int: 'div'(var int: x, var int: y)

Usage: x div y

Return result of integer division x / y

function int: 'mod'(int: x, int: y)

Usage: x mod y

Return remainder of integer division x % y

function var int: 'mod'(var int: x, var int: y)

Usage: x mod y

Return remainder of integer division x % y

function int: abs(int: x)

Return absolute value of x
function var int: abs(var int: x)

Return absolute value of x

function float: abs(float: x)

Return absolute value of x

function var float: abs(var float: x)

Return absolute value of x

function $E$: arg_max(array [$E$] of int: x)

Returns the index of the maximum value in the array x. When breaking ties the least index is returned.

function $E$: arg_max(array [$E$] of float: x)

Returns the index of the maximum value in the array x. When breaking ties the least index is returned.

function $E$: arg_min(array [$E$] of int: x)

Returns the index of the minimum value in the array x. When breaking ties the least index is returned.

function $E$: arg_min(array [$E$] of float: x)

Returns the index of the minimum value in the array x. When breaking ties the least index is returned.

function int: count(array [$T$] of bool: x)

Return number of true elements in array x
<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>function var int: count(array [T] of var bool: x)</code></td>
<td>Return number of true elements in array x</td>
</tr>
<tr>
<td><code>function T: max(T: x, T: y)</code></td>
<td>Return maximum of x and y</td>
</tr>
<tr>
<td><code>function T: max(array [U] of T: x)</code></td>
<td>Return maximum of elements in array x</td>
</tr>
<tr>
<td><code>function E: max(set of E: x)</code></td>
<td>Return maximum of elements in set x</td>
</tr>
<tr>
<td><code>function var int: max(var int: x, var int: y)</code></td>
<td>Return maximum of x and y</td>
</tr>
<tr>
<td><code>function var int: max(array [U] of var int: x)</code></td>
<td>Return maximum of elements in array x</td>
</tr>
<tr>
<td><code>function var float: max(var float: x, var float: y)</code></td>
<td>Return maximum of x and y</td>
</tr>
<tr>
<td><code>function var float: max(array [U] of var float: x)</code></td>
<td>Return maximum of elements in array x</td>
</tr>
<tr>
<td><code>function T: min(T: x, T: y)</code></td>
<td>Return minimum of x and y</td>
</tr>
</tbody>
</table>
function $T: \min(\text{array} [\$U] \text{of} \; T; \; x)$

Return minimum of elements in array $x$

function $$E: \min(\text{set} \text{of} \; $$E; \; x)$$

Return minimum of elements in set $x$

function var int: \( \min(\text{var int:} \; x, \; \text{var int:} \; y) \)

Return minimum of $x$ and $y$

function var int: \( \min(\text{array} [\$U] \text{of} \; \text{var int:} \; x) \)

Return minimum of elements in array $x$

function var float: \( \min(\text{var float:} \; x, \; \text{var float:} \; y) \)

Return minimum of $x$ and $y$

function var float: \( \min(\text{array} [\$U] \text{of} \; \text{var float:} \; x) \)

Return minimum of elements in array $x$

function int: \text{pow}(\text{int:} \; x, \; \text{int:} \; y)$

Return $x^y$

function var int: \text{pow}(\text{var int:} \; x, \; \text{var int:} \; y)\)

Return $x^y$

function float: \text{pow}(\text{float:} \; x, \; \text{float:} \; y)\)

Return $x^y$
function var float: pow(var float: x, var float: y)

Return \(x^y\)

function int: product(array [\$T\] of int: x)

Return product of elements in array \(x\)

function var int: product(array [\$T\] of var int: x)

Return product of elements in array \(x\)

function float: product(array [\$T\] of float: x)

Return product of elements in array \(x\)

function var float: product(array [\$T\] of var float: x)

Return product of elements in array \(x\)

function float: sqrt(float: x)

Return \(\sqrt{x}\)

function var float: sqrt(var float: x)

Return \(\sqrt{x}\)

function int: sum(array [\$T\] of bool: x)

Return sum of elements in array \(x\)

function var int: sum(array [\$T\] of var bool: x)

Return sum of elements in array \(x\)
**4.2.4.3 Exponential and logarithmic builtins**

These builtins implement exponential and logarithmic functions.

\[
\text{function } \text{float}: \text{exp}(\text{float}: x) \]

Return \(e^x\)

\[
\text{function } \text{var float}: \text{exp}(\text{var float}: x) \]

Return \(e^x\)

\[
\text{function } \text{float}: \text{ln}(\text{float}: x) \]

Return \(\ln x\)

\[
\text{function } \text{var float}: \text{ln}(\text{var float}: x) \]

Return \(\ln x\)
function float: log(float: x, float: y)

Return \(\log_x y\)

function float: log10(float: x)

Return \(\log_{10} x\)

function var float: log10(var float: x)

Return \(\log_{10} x\)

function float: log2(float: x)

Return \(\log_2 x\)

function var float: log2(var float: x)

Return \(\log_2 x\)

### 4.2.4.4 Trigonometric functions

These builtins implement the standard trigonometric functions.

function float: acos(float: x)

Return \(\arccos x\)

function var float: acos(var float: x)

Return \(\arccos x\)

function float: acosh(float: x)

Return \(\text{arccosh} x\)
function var float: acosh(var float: x)

Return acosh x

function float: asin(float: x)

Return asin x

function var float: asin(var float: x)

Return asin x

function float: asinh(float: x)

Return asinh x

function var float: asinh(var float: x)

Return asinh x

function float: atan(float: x)

Return atan x

function var float: atan(var float: x)

Return atan x

function float: atanh(float: x)

Return tanh x

function var float: atanh(var float: x)

Return tanh x
function float: cos(float: x)  
Return cos x

function var float: cos(var float: x)  
Return cos x

function float: sinh(float: x)  
Return sinh x

function var float: sinh(var float: x)  
Return sinh x

function float: tan(float: x)  
Return tan x


**4.2.4.5 Logical operations**

Logical operations are the standard operators of Boolean logic.

- **test '->'(bool: x, bool: y)**
  - Usage: x -> y
  - Return truth value of x implies y

- **predicate '->'(var bool: x, var bool: y)**
  - Usage: x -> y
  - Return truth value of x implies y

- **test '/'(bool: x, bool: y)**
  - Usage: x /\ y
  - Return truth value of x &and; y

- **predicate '/'(var bool: x, var bool: y)**
  - Usage: x /\ y
Return truth value of \( x \land y \)

\[
\text{test } '<-'(\text{bool}: x, \text{bool}: y)
\]

Usage: \( x <- y \)
Return truth value of \( y \) implies \( x \)

\[
\text{predicate } '<-(\text{var bool}: x, \text{var bool}: y)
\]

Usage: \( x <- y \)
Return truth value of \( y \) implies \( x \)

\[
\text{test } '<->'(\text{bool}: x, \text{bool}: y)
\]

Usage: \( x <-> y \)
Return truth value of \( x \) if-and-only-if \( y \)

\[
\text{predicate } '<->'(\text{var bool}: x, \text{var bool}: y)
\]

Usage: \( x <-> y \)
Return truth value of \( x \) if-and-only-if \( y \)

\[
\text{test } '\lor'(\text{bool}: x, \text{bool}: y)
\]

Usage: \( x \lor y \)
Return truth value of \( x \) \( \lor \) \( y \)

\[
\text{predicate } '\lor'(\text{var bool}: x, \text{var bool}: y)
\]

Usage: \( x \lor y \)
Return truth value of \( x \) \( \lor \) \( y \)

\[
\text{test } 'not'(\text{bool}: x)
\]

Usage: \( \text{not} x \)
Usage: not x
Return truth value of the negation of x

```minizinc
predicate 'not'(var bool: x)
```

Usage: not x
Return truth value of the negation of x

```minizinc
test 'xor'(bool: x, bool: y)
```

Usage: x xor y
Return truth value of x xor y

```minizinc
predicate 'xor'(var bool: x, var bool: y)
```

Usage: x xor y
Return truth value of x xor y

```minizinc
predicate clause(array [T] of var bool: x, array [T] of var bool: y)
```

Return truth value of (∨_i x[i]) ∨ (∨_j ¬y[j])

```minizinc
predicate clause(array [T] of bool: x, array [T] of bool: y)
```

Return truth value of (∨_i x[i]) ∨ (∨_j ¬y[j])

```minizinc
test exists(array [T] of bool: x)
```

Return truth value of ∨_i x[i]

```minizinc
predicate exists(array [T] of var bool: x)
```

Return truth value of ∨_i x[i]
forall(array [T] of bool: x)

Return truth value of $\bigwedge_i x[i]$

define forall(array [T] of var bool: x)

Return truth value of $\bigwedge_i x[i]$

define test forall(array [T] of bool: x)

Return truth value of $\bigwedge_i x[i]$

define predicate forall(array [T] of var bool: x)

Return truth value of $\bigwedge_i x[i]$

define predicate test forall(array [T] of bool: x)

Return truth value of $\bigwedge_i x[i]$

define test iffall(array [T] of bool: x)

Return truth value of true $\oplus \bigvee_i x[i]$

define predicate iffall(array [T] of var bool: x)

Return truth value of true $\oplus \bigvee_i x[i]$

define predicate test iffall(array [T] of bool: x)

Return truth value of true $\oplus \bigvee_i x[i]$

define test xorall(array [T] of bool: x)

Return truth value of $\bigoplus_i x[i]$

define predicate xorall(array [T] of var bool: x)

Return truth value of $\bigoplus_i x[i]$

define predicate test xorall(array [T] of bool: x)

Return truth value of $\bigoplus_i x[i]$

4.2.4.6 Set operations

These functions implement the basic operations on sets.

function set of $E$: '..'(E: a, $E$: b)

Usage: a .. b

Return the set \{a, ..., b\}

function set of float: '..'(float: a, float: b)
Usage: \( a .. b \)
Return the set \( \{a, \ldots, b\} \)

\[
\text{function set of bool: } '\.'(\text{bool: } a, \text{ bool: } b)
\]

Usage: \( a .. b \)
Return the set \( \{a, \ldots, b\} \)

\[
\text{function set of } T: 'diff'(\text{set of } T: x, \text{ set of } T: y)
\]

Usage: \( x \text{ diff } y \)
Return the set difference of sets \( x \setminus y \)

\[
\text{function var set of } T: 'diff'(\text{var set of } T: x, \text{ var set of } T: y)
\]

Usage: \( x \text{ diff } y \)
Return the set difference of sets \( x \setminus y \)

\[
\text{test } 'in'(\text{int: } x, \text{ set of int: } y)
\]

Usage: \( x \text{ in } y \)
Test if \( x \) is an element of the set \( y \)

\[
\text{predicate } 'in'(\text{var int: } x, \text{ var set of int: } y)
\]

Usage: \( x \text{ in } y \)
\( x \) is an element of the set \( y \)

\[
\text{test } 'in'(\text{float: } x, \text{ set of float: } y)
\]

Usage: \( x \text{ in } y \)
Test if \( x \) is an element of the set \( y \)
**predicate 'in'(var float: x, set of float: y)**

Usage: x in y
Test if x is an element of the set y

**function set of $T$: 'intersect'(set of $T$: x, set of $T$: y)**

Usage: x intersect y
Return the intersection of sets x and y

**function var set of $$T$: 'intersect'(var set of $$T$: x, var set of $$T$: y)**

Usage: x intersect y
Return the intersection of sets x and y

**test 'subset'(set of $T$: x, set of $T$: y)**

Usage: x subset y
Test if x is a subset of y

**predicate 'subset'(var set of int: x, var set of int: y)**

Usage: x subset y
x is a subset of y

**test 'superset'(set of $T$: x, set of $T$: y)**

Usage: x superset y
Test if x is a superset of y

**predicate 'superset'(var set of int: x, var set of int: y)**
Usage: x superset y

x is a superset of y

```
function set of $T: 'symdiff'(set of $T: x, set of $T: y)
```

Usage: x symdiff y

Return the symmetric set difference of sets x and y

```
function var set of $$T: 'symdiff'(var set of $$T: x, var set of $$T: y)
```

Usage: x symdiff y

Return the symmetric set difference of sets x and y

```
function set of $T: 'union'(set of $T: x, set of $T: y)
```

Usage: x union y

Return the union of sets x and y

```
function var set of $$T: 'union'(var set of $$T: x, var set of $$T: y)
```

Usage: x union y

Return the union of sets x and y

```
function set of $U: array_intersect(array [$$T] of set of $$U: x)
```

Return the intersection of the sets in array x

```
function var set of int: array_intersect(array [int] of var set of int: x)
```

Return the intersection of the sets in array x

```
function set of $U: array_union(array [$$T] of set of $U: x)
```

Return the union of the sets in array x
function var set of int: array_union(array [int] of var set of int: x)

Return the union of the sets in array x

function int: card(set of $T: x)

Return the cardinality of the set x

function var int: card(var set of int: x)

Return the cardinality of the set x

function var $$E: max(var set of $$E: s)

Return the maximum of the set s

function var $$E: min(var set of $$E: s)

Return the minimum of the set s

function array [int] of int: set_to_ranges(set of int: S)

Return a sorted list of the non-overlapping ranges in S

function array [int] of float: set_to_ranges(set of float: S)

Return a sorted list of the non-overlapping ranges in S

function array [int] of bool: set_to_ranges(set of bool: S)

Return a sorted list of the non-overlapping ranges in S

4.2.4.7 Conditionals

These functions implement conditional (if-then-else-endif) constraints.
**Predicate if_then_else**

```plaintext
predicate if_then_else(array [int] of var bool: c,
                        array [int] of int: x,
                        var int: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[1..i - 1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.

**Predicate if_then_else**

```plaintext
predicate if_then_else(array [int] of var bool: c,
                        array [int] of var int: x,
                        var int: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[1..i - 1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.

**Predicate if_then_else**

```plaintext
predicate if_then_else(array [int] of var bool: c,
                        array [int] of opt int: x,
                        var opt int: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[1..i - 1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.

**Predicate if_then_else**

```plaintext
predicate if_then_else(array [int] of var bool: c,
                        array [int] of var opt int: x,
                        var opt int: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[1..i - 1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.

**Predicate if_then_else**

```plaintext
predicate if_then_else(array [int] of var bool: c,
                        array [int] of bool: x,
                        var bool: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[1..i - 1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.
Conditional constraint \( \{ c[i] \land \lnot c[i+1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.

```
predicate if_then_else(array [int] of var bool: c,
                        array [int] of var bool: x,
                        var bool: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[i+1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.

```
predicate if_then_else(array [int] of var bool: c,
                        array [int] of opt bool: x,
                        var opt bool: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[i+1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.

```
predicate if_then_else(array [int] of var bool: c,
                        array [int] of var opt bool: x,
                        var opt bool: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[i+1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.

```
predicate if_then_else(array [int] of var bool: c,
                        array [int] of float: x,
                        var float: y)
```

Conditional constraint \( \{ c[i] \land \lnot c[i+1] \rightarrow y = x[i] \} \)

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \( c \) array is always the constant true, corresponding to the else case.
predicate if_then_else(array [int] of var bool: c,  
array [int] of var float: x,  
var float: y)

Conditional constraint \{c[i] \land \neg c[1..i-1] \rightarrow y = x[i]\}

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \(c\) array is always the constant true, corresponding to the else case.

predicate if_then_else(array [int] of var bool: c,  
array [int] of opt float: x,  
var opt float: y)

Conditional constraint \{c[i] \land \neg c[1..i-1] \rightarrow y = x[i]\}

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \(c\) array is always the constant true, corresponding to the else case.

predicate if_then_else(array [int] of var bool: c,  
array [int] of set of int: x,  
var set of int: y)

Conditional constraint \{c[i] \land \neg c[1..i-1] \rightarrow y = x[i]\}

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \(c\) array is always the constant true, corresponding to the else case.

predicate if_then_else(array [int] of var bool: c,  
array [int] of var set of int: x,  
var set of int: y)

Conditional constraint \{c[i] \land \neg c[1..i-1] \rightarrow y = x[i]\}

This constraint is generated by the compiler for if-then-else expressions. The last entry in the \(c\) array is always the constant true, corresponding to the else case.

predicate if_then_else_partiality(array [int] of var bool: c,  
array [int] of var bool: d,  
var bool: b)
Conditional partiality constraint

This constraint is generated by the compiler for if-then-else expressions with potentially undefined cases. The last entry in the c array is always the constant true, corresponding to the else case. The d[i] variable represents whether case i is defined. Constrains that if b is defined, then the selected case must be defined, and if the selected case is undefined, then b must be undefined.

4.2.4.8 Array operations

These functions implement the basic operations on arrays.

\[
\text{function array [int] of } T: '++'(\text{array [int] of } T: x, \\
\text{array [int] of } T: y)
\]

Usage: x ++ y

Return the concatenation of arrays x and y

\[
\text{function array [int] of opt } T: '++'(\text{array [int] of opt } T: x, \\
\text{array [int] of opt } T: y)
\]

Usage: x ++ y

Return the concatenation of arrays x and y

\[
\text{function array [int] of var } T: '++'(\text{array [int] of var } T: x, \\
\text{array [int] of var } T: y)
\]

Usage: x ++ y

Return the concatenation of arrays x and y

\[
\text{function array [int] of var opt } T: '++'(\text{array [int] of var opt } T: x, \\
\text{array [int] of var opt } T: y)
\]

Usage: x ++ y

Return the concatenation of arrays x and y
**function array [int] of $V$: array1d(array [$U$] of $V$: $x$)**

Return array $x$ coerced to index set 1..length($x$). Coercions are performed by considering the array $x$ in row-major order.

**function array [int] of opt $V$: array1d(array [$U$] of opt $V$: $x$)**

Return array $x$ coerced to index set 1..length($x$). Coercions are performed by considering the array $x$ in row-major order.

**function array [int] of var $V$: array1d(array [$U$] of var $V$: $x$)**

Return array $x$ coerced to index set 1..length($x$). Coercions are performed by considering the array $x$ in row-major order.

**function array [int] of var opt $V$: array1d(array [$U$] of var opt $V$: $x$)**

Return array $x$ coerced to index set 1..length($x$). Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to one-dimensional array with index set $S$. Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to one-dimensional array with index set $S$. Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to one-dimensional array with index set $S$. Coercions are performed by considering the array $x$ in row-major order.
function array [E] of var opt V: array1d(set of E: S,
array [U] of var opt V: x)

Return array $x$ coerced to one-dimensional array with index set $S$. Coercions are performed by considering the array $x$ in row-major order.

function array [E,F] of V: array2d(set of E: S1,
  set of F: S2,
  array [U] of V: x)

Return array $x$ coerced to two-dimensional array with index sets $S_1$ and $S_2$. Coercions are performed by considering the array $x$ in row-major order.

function array [E,F] of opt V: array2d(set of E: S1,
  set of F: S2,
  array [U] of opt V: x)

Return array $x$ coerced to two-dimensional array with index sets $S_1$ and $S_2$. Coercions are performed by considering the array $x$ in row-major order.

function array [E,F] of var V: array2d(set of E: S1,
  set of F: S2,
  array [U] of var V: x)

Return array $x$ coerced to two-dimensional array with index sets $S_1$ and $S_2$. Coercions are performed by considering the array $x$ in row-major order.

function array [E,F] of var opt V: array2d(set of E: S1,
  set of F: S2,
  array [U] of var opt V: x)

Return array $x$ coerced to two-dimensional array with index sets $S_1$ and $S_2$. Coercions are performed by considering the array $x$ in row-major order.

function var set of T: array2set(array [int] of var T: x)

Return the set containing the elements of $x$
function set of $T$: array2set(array [int] of $T$: x)

Return the set containing the elements of x

function set of bool: array2set(array [int] of bool: x)

Return the set containing the elements of x

function set of float: array2set(array [int] of float: x)

Return the set containing the elements of x

function array [E,F,G] of $V$: array3d(set of $E$: S1,
      set of $F$: S2,
      set of $G$: S3,
      array [$U$] of $V$: x)

Return array x coerced to three-dimensional array with index sets S1, S2 and S3. Coercions are performed by considering the array x in row-major order.

function array [E,F,G] of opt $V$: array3d(set of $E$: S1,
      set of $F$: S2,
      set of $G$: S3,
      array [$U$] of opt $V$: x)

Return array x coerced to three-dimensional array with index sets S1, S2 and S3. Coercions are performed by considering the array x in row-major order.

function array [E,F,G] of var $V$: array3d(set of $E$: S1,
      set of $F$: S2,
      set of $G$: S3,
      array [$U$] of var $V$: x)

Return array x coerced to three-dimensional array with index sets S1, S2 and S3. Coercions are performed by considering the array x in row-major order.

Return array $x$ coerced to three-dimensional array with index sets $S1$, $S2$ and $S3$. Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to 4-dimensional array with index sets $S1$, $S2$, $S3$ and $S4$. Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to 4-dimensional array with index sets $S1$, $S2$, $S3$ and $S4$. Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to 4-dimensional array with index sets $S1$, $S2$, $S3$ and $S4$. Coercions are performed by considering the array $x$ in row-major order.
Return array $x$ coerced to 4-dimensional array with index sets $S_1$, $S_2$, $S_3$ and $S_4$. Coercions are performed by considering the array $x$ in row-major order.

Return array $x$ coerced to 5-dimensional array with index sets $S_1$, $S_2$, $S_3$, $S_4$ and $S_5$. Coercions are performed by considering the array $x$ in row-major order.

Return array $x$ coerced to 5-dimensional array with index sets $S_1$, $S_2$, $S_3$, $S_4$ and $S_5$. Coercions are performed by considering the array $x$ in row-major order.
Return array $x$ coerced to 5-dimensional array with index sets $S_1$, $S_2$, $S_3$, $S_4$ and $S_5$. Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to 5-dimensional array with index sets $S_1$, $S_2$, $S_3$, $S_4$ and $S_5$. Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to 6-dimensional array with index sets $S_1$, $S_2$, $S_3$, $S_4$, $S_5$ and $S_6$. Coercions are performed by considering the array $x$ in row-major order.


Return array $x$ coerced to 6-dimensional array with index sets $S_1$, $S_2$, $S_3$, $S_4$, $S_5$ and $S_6$. Coercions are performed by considering the array $x$ in row-major order.
Coercions are performed by considering the array \( x \) in row-major order.

```mini
def function array[$$E, $$F, $$G, $$H, $$I, $$J]\ of\ var\ \$V: array6d(set\ of\ $$E: S1, 
set\ of\ $$F: S2, 
set\ of\ $$G: S3, 
set\ of\ $$H: S4, 
set\ of\ $$I: S5, 
set\ of\ $$J: S6, 
array[$$U] of\ 
\var\ \$V: x)```

Return array \( x \) coerced to 6-dimensional array with index sets \( S1, S2, S3, S4, S5 \) and \( S6 \). Coercions are performed by considering the array \( x \) in row-major order.

```mini
def function array[$$E, $$F, $$G, $$H, $$I, $$J]\ of\ var\ opt\ \$V: array6d(set\ of\ $$E: S1, 
set\ of\ $$F: S2, 
set\ of\ $$G: S3, 
set\ of\ $$H: S4, 
set\ of\ $$I: S5, 
set\ of\ $$J: S6, 
array[$$U] of\ 
\var\ opt \$V: x)```

Return array \( x \) coerced to 6-dimensional array with index sets \( S1, S2, S3, S4, S5 \) and \( S6 \). Coercions are performed by considering the array \( x \) in row-major order.

```mini
def function array[$$T]\ of\ \$X: arrayXd(array[$$T] of\ var\ opt\ $$X: x, 
array[$$U] of\ \$V: y)```

Return array \( y \) coerced to array with same number of dimensions and same index sets as array \( x \). Coercions are performed by considering the array \( y \) in row-major order.
Return array y coerced to array with same number of dimensions and same index sets as array x. Coercions are performed by considering the array y in row-major order.

Return array y coerced to array with same number of dimensions and same index sets as array x. Coercions are performed by considering the array y in row-major order.

Return array y coerced to array with same number of dimensions and same index sets as array x. Coercions are performed by considering the array y in row-major order.

Return column c of array x

Return column c of array x

Return column c of array x

Return column c of array x
MiniZinc Handbook, Release 2.4.3

```plaintext
test has_element($T$: e, array [int] of $T$: x)
```

Test if e is an element of array x

```plaintext
test has_element($T$: e, array [int] of opt $T$: x)
```

Test if e is an element of array x

```plaintext
predicate has_element($T$: e, array [$$E$$] of var opt $T$: x)
```

Test if e is an element of array x

```plaintext
test has_index(int: i, array [int] of var opt $T$: x)
```

Test if i is in the index set of x

```plaintext
function set of $$E$$: index_set(array [$$E$$] of var opt $U$: x)
```

Return index set of one-dimensional array x

```plaintext
function set of $$E$$: index_set_1of2(array [$$E$$,int] of var opt $U$: x)
```

Return index set of first dimension of two-dimensional array x

```plaintext
function set of $$E$$: index_set_1of3(array [$$E$$,int,int] of var opt $U$: x)
```

Return index set of first dimension of 3-dimensional array x

```plaintext
function set of $$E$$: index_set_1of4(array [$$E$$,int,int,int] of var opt $U$: x)
```

Return index set of first dimension of 4-dimensional array x

```plaintext
function set of $$E$$: index_set_1of5(array [$$E$$,int,int,int,int] of var opt $U$: x)
```

Return index set of first dimension of 5-dimensional array x
function set of $E$: index_set_1of6(array [$$E, int, int, int, int, int] of var_.
  opt $U$: x)

Return index set of first dimension of 6-dimensional array x

function set of $E$: index_set_2of2(array [int,$$$E] of var opt $U: x)

Return index set of second dimension of two-dimensional array x

function set of $E$: index_set_2of3(array [int,$$$E, int] of var opt $U: x)

Return index set of second dimension of 3-dimensional array x

function set of $E$: index_set_2of4(array [int,$$$E, int, int] of var opt $U: x)

Return index set of second dimension of 4-dimensional array x

function set of $E$: index_set_2of5(array [int,$$$E, int, int, int] of var opt
  $U: x)

Return index set of second dimension of 5-dimensional array x

function set of $E$: index_set_2of6(array [int,$$$E, int, int, int] of var_.
  opt $U: x)

Return index set of second dimension of 6-dimensional array x

function set of $E$: index_set_3of3(array [int,int,$$$E] of var opt $U: x)

Return index set of third dimension of 3-dimensional array x

function set of $E$: index_set_3of4(array [int,int,$$$E,int] of var opt $U: x)

Return index set of third dimension of 4-dimensional array x
function set of $\mathcal{E}$: index_set_3of5(array [int,int,$\mathcal{E}$,int,int] of var opt $\U$: x)

Return index set of third dimension of 5-dimensional array $x$

function set of $\mathcal{E}$: index_set_3of6(array [int,int,$\mathcal{E}$,int,int,int] of var opt $\U$: x)

Return index set of third dimension of 6-dimensional array $x$

function set of $\mathcal{E}$: index_set_4of4(array [int,int,int,$\mathcal{E}$] of var opt $\U$: x)

Return index set of fourth dimension of 4-dimensional array $x$

function set of $\mathcal{E}$: index_set_4of5(array [int,int,int,$\mathcal{E}$,int] of var opt $\U$: x)

Return index set of fourth dimension of 5-dimensional array $x$

function set of $\mathcal{E}$: index_set_4of6(array [int,int,int,$\mathcal{E}$,int,int] of var opt $\U$: x)

Return index set of fourth dimension of 6-dimensional array $x$

function set of $\mathcal{E}$: index_set_5of5(array [int,int,int,int,$\mathcal{E}$] of var opt $\U$: x)

Return index set of fifth dimension of 5-dimensional array $x$

function set of $\mathcal{E}$: index_set_5of6(array [int,int,int,int,$\mathcal{E}$,int] of var opt $\U$: x)

Return index set of fifth dimension of 6-dimensional array $x$

function set of $\mathcal{E}$: index_set_6of6(array [int,int,int,int,$\mathcal{E}$,int] of var opt $\U$: x)

Return index set of sixth dimension of 6-dimensional array $x$
Return index set of sixth dimension of 6-dimensional array $x$

```
  test index_sets_agree(array [T] of var opt $U$: x,
                        array [T] of var opt $W$: y)
```

Test if $x$ and $y$ have the same index sets

```
  function int: length(array [T] of var opt $U$: x)
```

Return the length of array $x$

Note that the length is defined as the number of elements in the array, regardless of its dimensionality.

```
  function array [$$E$$] of $T$: reverse(array [$$E$$] of $T$: x)
```

Return the array $x$ in reverse order

The resulting array has the same index set as $x$.

```
  function array [$$E$$] of opt $T$: reverse(array [$$E$$] of opt $T$: x)
```

Return the array $x$ in reverse order

The resulting array has the same index set as $x$.

```
  function array [$$E$$] of var $T$: reverse(array [$$E$$] of var $T$: x)
```

Return the array $x$ in reverse order

The resulting array has the same index set as $x$.

```
  function array [$$E$$] of var opt $T$: reverse(array [$$E$$] of var opt $T$: x)
```

Return the array $x$ in reverse order

The resulting array has the same index set as $x$.  

4.2.4. Builtins
function array [E] of $T$: row(array [int,E] of $T$: x, int: r)

Return row r of array x

function array [E] of opt $T$: row(array [int,E] of opt $T$: x, int: r)

Return row r of array x

function array [E] of var $T$: row(array [int,E] of var $T$: x, int: r)

Return row r of array x

function array [int] of $T$: slice_1d(array [E] of $T$: x, array [int] of set of int: s, set of int: dims1)

Return slice of array x specified by sets s, coerced to new 1d array with index set dims1

function array [int] of opt $T$: slice_1d(array [E] of opt $T$: x, array [int] of set of int: s, set of int: dims1)

Return slice of array x specified by sets s, coerced to new 1d array with index set dims1

function array [int] of var $T$: slice_1d(array [E] of var $T$: x, array [int] of set of int: s, set of int: dims1)

Return slice of array x specified by sets s, coerced to new 1d array with index set dims1
<table>
<thead>
<tr>
<th>Function</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>slice_1d</code></td>
<td>Returns slice of array <code>x</code> specified by sets <code>s</code>, coerced to new 1d array with index set <code>dims1</code></td>
</tr>
<tr>
<td><code>slice_2d</code></td>
<td>Returns slice of array <code>x</code> specified by sets <code>s</code>, coerced to new 2d array with index sets <code>dims1</code> and <code>dims2</code></td>
</tr>
<tr>
<td><code>slice_2d</code></td>
<td>Returns slice of array <code>x</code> specified by sets <code>s</code>, coerced to new 2d array with index sets <code>dims1</code> and <code>dims2</code></td>
</tr>
<tr>
<td><code>slice_2d</code></td>
<td>Returns slice of array <code>x</code> specified by sets <code>s</code>, coerced to new 2d array with index sets <code>dims1</code> and <code>dims2</code></td>
</tr>
</tbody>
</table>

4.2.4. Builtins
function array [int,int,int] of $T$: slice_3d(array [$E$] of $T$: x,
array [int] of set of int: s,
set of int: dims1,
set of int: dims2,
set of int: dims3)

Returns slice of array x specified by sets s, coerced to new 3d array with index sets dims1, dims2 and dims3

function array [int,int,int] of opt $T$: slice_3d(array [$E$] of opt $T$: x,
array [int] of set of int: s,
set of int: dims1,
set of int: dims2,
set of int: dims3)

Returns slice of array x specified by sets s, coerced to new 3d array with index sets dims1, dims2 and dims3

function array [int,int,int] of var $T$: slice_3d(array [$E$] of var $T$: x,
array [int] of set of int: s,
set of int: dims1,
set of int: dims2,
set of int: dims3)

Returns slice of array x specified by sets s, coerced to new 3d array with index sets dims1, dims2 and dims3

function array [int,int,int] of var opt $T$: slice_3d(array [$E$] of var opt $T$: x,
array [int] of set of int: s,
set of int: dims1,
set of int: dims2,
set of int: dims3)

Returns slice of array x specified by sets s, coerced to new 3d array with index sets dims1, dims2 and dims3
and dims3

```mini
function array [int,int,int] of $T$: slice_4d(array [$E$] of $T$: x,
array [int] of set of int: s,
set of int: dims1,
set of int: dims2,
set of int: dims3,
set of int: dims4)
```

Return slice of array x specified by sets s, coerced to new 3d array with index sets dims1, dims2, dims3, dims4

```mini
function array [int,int,int] of opt $T$: slice_4d(array [$E$] of opt $T$: x,
array [int] of set of int: s,
set of int: dims1,
set of int: dims2,
set of int: dims3,
set of int: dims4)
```

Return slice of array x specified by sets s, coerced to new 3d array with index sets dims1, dims2, dims3, dims4

```mini
function array [int,int,int] of var $T$: slice_4d(array [$E$] of var $T$: x,
array [int] of set of int: s,
set of int: dims1,
set of int: dims2,
set of int: dims3,
set of int: dims4)
```

Return slice of array x specified by sets s, coerced to new 3d array with index sets dims1, dims2, dims3, dims4

```mini
function array [int,int,int] of var opt $T$: slice_4d(array [$E$] of var opt $T$: x,
array [int] of set of int: s,
set of int: dims1,
set of int: dims2,
set of int: dims3,
set of int: dims4)
```

4.2.4. Builtins
Return slice of array $x$ specified by sets $s$, coerced to new 3d array with index sets $\text{dims1}$, $\text{dims2}$, $\text{dims3}$, $\text{dims4}$

```
function array [int,int,int] of $T$: slice_5d(array [$E$] of $T$: $x$, array [int] of set of int: $s$, set of int: $\text{dims1}$, set of int: $\text{dims2}$, set of int: $\text{dims3}$, set of int: $\text{dims4}$, set of int: $\text{dims5}$)
```

Return slice of array $x$ specified by sets $s$, coerced to new 3d array with index sets $\text{dims1}$, $\text{dims2}$, $\text{dims3}$, $\text{dims4}$, $\text{dims5}$

```
function array [int,int,int] of opt $T$: slice_5d(array [$E$] of opt $T$: $x$, array [int] of set of int: $s$, set of int: $\text{dims1}$, set of int: $\text{dims2}$, set of int: $\text{dims3}$, set of int: $\text{dims4}$, set of int: $\text{dims5}$)
```

Return slice of array $x$ specified by sets $s$, coerced to new 3d array with index sets $\text{dims1}$, $\text{dims2}$, $\text{dims3}$, $\text{dims4}$, $\text{dims5}$

```
function array [int,int,int] of var $T$: slice_5d(array [$E$] of var $T$: $x$, array [int] of set of int: $s$, set of int: $\text{dims1}$, set of int: $\text{dims2}$, set of int: $\text{dims3}$, set of int: $\text{dims4}$, set of int: $\text{dims5}$)
```
Return slice of array $x$ specified by sets $s$, coerced to new 3d array with index sets $\dim{1}$, $\dim{2}$, $\dim{3}$, $\dim{4}$, $\dim{5}$

```plaintext
function array [int,int,int] of var opt $T$: slice_5d(array [\$E\] of var opt $\sim$\$T$: x,
array [int] of set of int: $s$,
set of int: $\dim{1}$,
set of int: $\dim{2}$,
set of int: $\dim{3}$,
set of int: $\dim{4}$,
set of int: $\dim{5}$)
```

Return slice of array $x$ specified by sets $s$, coerced to new 3d array with index sets $\dim{1}$, $\dim{2}$, $\dim{3}$, $\dim{4}$, $\dim{5}$

```plaintext
function array [int,int,int] of $T$: slice_6d(array [\$E\] of $T$: x,
array [int] of set of int: $s$,
set of int: $\dim{1}$,
set of int: $\dim{2}$,
set of int: $\dim{3}$,
set of int: $\dim{4}$,
set of int: $\dim{5}$,
set of int: $\dim{6}$
```

Return slice of array $x$ specified by sets $s$, coerced to new 3d array with index sets $\dim{1}$, $\dim{2}$, $\dim{3}$, $\dim{4}$, $\dim{5}$, $\dim{6}$

```plaintext
function array [int,int,int] of opt $T$: slice_6d(array [\$E\] of opt $T$: x,
array [int] of set of int: $s$,
set of int: $\dim{1}$,
set of int: $\dim{2}$,
set of int: $\dim{3}$,
set of int: $\dim{4}$,
set of int: $\dim{5}$,
set of int: $\dim{6}$
```

### 4.2.4. Builtins
Return slice of array \( x \) specified by sets \( s \), coerced to new 3d array with index sets \( \text{dims1} \), \( \text{dims2} \), \( \text{dims3} \), \( \text{dims4} \), \( \text{dims5} \), \( \text{dims6} \)

\[
\text{function array [int,int,int] of var} \ T: \ \text{slice}_6d(\text{array [E] of var} \ T: \ x, \ s, \ \text{set of int:} \ \text{dims1}, \ \text{set of int:} \ \text{dims2}, \ \text{set of int:} \ \text{dims3}, \ \text{set of int:} \ \text{dims4}, \ \text{set of int:} \ \text{dims5}, \ \text{set of int:} \ \text{dims6})
\]

Return slice of array \( x \) specified by sets \( s \), coerced to new 3d array with index sets \( \text{dims1} \), \( \text{dims2} \), \( \text{dims3} \), \( \text{dims4} \), \( \text{dims5} \), \( \text{dims6} \)

\[
\text{function array [int,int,int] of var opt} \ T: \ \text{slice}_6d(\text{array [E] of var opt} \ T: \ x, \ s, \ \text{set of int:} \ \text{dims1}, \ \text{set of int:} \ \text{dims2}, \ \text{set of int:} \ \text{dims3}, \ \text{set of int:} \ \text{dims4}, \ \text{set of int:} \ \text{dims5}, \ \text{set of int:} \ \text{dims6})
\]

Return slice of array \( x \) specified by sets \( s \), coerced to new 3d array with index sets \( \text{dims1} \), \( \text{dims2} \), \( \text{dims3} \), \( \text{dims4} \), \( \text{dims5} \), \( \text{dims6} \)

\[
4.2.4.9 \ \text{Array sorting operations}
\]

\[
\text{function array [int] of} \ E: \ \text{arg_sort}(\text{array [E] of int:} \ x)
\]
Returns the permutation \( p \) which causes \( x \) to be in sorted order hence \( x[ p[i]] \leq x[ p[i+1]] \).

The permutation is the stable sort hence \( x[ p[i]] = x[p[i+1]] \Rightarrow p[i] < p[i+1] \).

\[
\text{function array [int] of } \mathbb{E}: \text{arg_sort(array [\mathbb{E}] of float: } x)
\]

Returns the permutation \( p \) which causes \( x \) to be in sorted order hence \( x[ p[i]] \leq x[ p[i+1]] \).

The permutation is the stable sort hence \( x[ p[i]] = x[p[i+1]] \Rightarrow p[i] < p[i+1] \).

\[
\text{function array [\mathbb{E}] of int: sort(array [\mathbb{E}] of int: } x)
\]

Return values from array \( x \) sorted in non-decreasing order

\[
\text{function array [\mathbb{E}] of float: sort(array [\mathbb{E}] of float: } x)
\]

Return values from array \( x \) sorted in non-decreasing order

\[
\text{function array [\mathbb{E}] of bool: sort(array [\mathbb{E}] of bool: } x)
\]

Return values from array \( x \) sorted in non-decreasing order

\[
\text{function array [\mathbb{E}] of var opt } \mathbb{T}: \text{sort_by(array [\mathbb{E}] of var opt } \mathbb{T}: x, \\
\quad \text{array [\mathbb{E}] of int: } y)
\]

Return array \( x \) sorted by the values in \( y \) in non-decreasing order

The sort is stable, i.e. if \( y[i] = y[j] \) with \( i < j \), then \( x[i] \) will appear in the output before \( x[j] \).

\[
\text{function array [\mathbb{E}] of var } \mathbb{T}: \text{sort_by(array [\mathbb{E}] of var } \mathbb{T}: x, \\
\quad \text{array [\mathbb{E}] of int: } y)
\]

Return array \( x \) sorted by the values in \( y \) in non-decreasing order

The sort is stable, i.e. if \( y[i] = y[j] \) with \( i < j \), then \( x[i] \) will appear in the output before \( x[j] \).
4.2.4.10 Coercions

These functions implement coercions, or channeling, between different types.

function float: bool2float(bool: b)

Return Boolean b coerced to a float
function array [\$T\] of float: bool2float(array [\$T\] of bool: x)

Return array of Booleans x coerced to an array of floats

function array [\$T\] of var float: bool2float(array [\$T\] of var bool: x)

Return array of Booleans x coerced to an array of floats

function var float: bool2float(var bool: b)

Return Boolean b coerced to a float

function array [\$T\] of var opt float: bool2float(array [\$T\] of var opt bool: x)

Return array of Booleans x coerced to an array of floats

function int: bool2int(bool: b)

Return Boolean b coerced to an integer

function var int: bool2int(var bool: b)

Return Boolean b coerced to an integer

function array [\$T\] of var int: bool2int(array [\$T\] of var bool: b)

Return array of Booleans b coerced to an array of integers

function array [\$T\] of int: bool2int(array [\$T\] of bool: x)

Return array of Booleans x coerced to an array of integers

function array [\$T\] of set of int: bool2int(array [\$T\] of set of bool: x)

Return array of sets of Booleans x coerced to an array of sets of integers
function array [T] of var int: bool2int(array [T] of var bool: x)

Return array of Booleans x coerced to an array of integers

function array [T] of var opt int: bool2int(array [T] of var opt bool: x)

Return array of Booleans x coerced to an array of integers

function int: ceil(float: x)

Return ⌈x⌉

function int: floor(float: x)

Return ⌊x⌋

function float: int2float(int: x)

Return integer x coerced to a float

function var float: int2float(var int: x)

Return integer x coerced to a float

function array [T] of float: int2float(array [T] of int: x)

Return array of integers x coerced to an array of floats

function array [T] of var float: int2float(array [T] of var int: x)

Return array of integers x coerced to an array of floats

function array [T] of var opt float: int2float(array [T] of var opt int: x)

Return array of optional integers x coerced to an array of optional floats
**MiniZinc Handbook, Release 2.4.3**

**function int: round(float: x)**

Return x rounded to nearest integer

**function array [int] of $$E$$: set2array(set of $$E$$: x)**

Return a set of integers x coerced to an array of integers

### 4.2.4.11 String operations

These functions implement operations on strings.

**function string: '++'(string: s1, string: s2)**

Usage: s1 ++ s2

Return concatenation of s1 and s2

**function string: concat(array [$T] of string: s)**

Return concatenation of strings in array s

**function string: file_path()**

Return path of file where this function is called

**function string: format(var opt $$T$$: x)**

Convert x into a string

**function string: format(var opt set of $$T$$: x)**

Convert x into a string

**function string: format(array [$U] of var opt $$T$$: x)**
Convert x into a string

```
function string: format(int: w, int: p, var opt $T: x)
```

Formatted to-string conversion

Converts the value x into a string right justified by the number of characters given by w, or left justified if w is negative.

The maximum length of the string representation of x is given by p, or the maximum number of digits after the decimal point for floating point numbers. It is a run-time error for p to be negative.

```
function string: format(int: w, int: p, var opt set of $T: x)
```

Formatted to-string conversion

Converts the value x into a string right justified by the number of characters given by w, or left justified if w is negative.

The maximum length of the string representation of x is given by p. It is a run-time error for p to be negative.

```
function string: format(int: w, int: p, array [$_U$] of var opt $T: x)
```

Formatted to-string conversion

Converts the value x into a string right justified by the number of characters given by w, or left justified if w is negative.

The maximum length of the string representation of x is given by p. It is a run-time error for p to be negative.

```
function string: format(int: w, var opt $T: x)
```

Formatted to-string conversion

Converts the value x into a string right justified by the number of characters given by w, or left justified if w is negative.
**function string: format (int: w, var opt set of $T: x)**

Formatted to-string conversion

Converts the value x into a string right justified by the number of characters given by w, or left justified if w is negative.

**function string: format (int: w, array [SU] of var opt $T: x)**

Formatted to-string conversion

Converts the value x into a string right justified by the number of characters given by w, or left justified if w is negative.

**function string: format_justify_string (int: w, string: x)**

String justification

Returns the string x right justified by the number of characters given by w, or left justified if w is negative.

**function string: join (string: d, array [$T] of string: s)**

Join string in array s using delimiter d

**function array [int] of string: outputJSON()**

Return array for output of all variables in JSON format

**function array [int] of string: outputJSON(bool: b)**

Return array for output of all variables in JSON format, including objective if b is true

**function array [int] of string: outputJSONParameters()**

Return array for output of all parameters in JSON format
function string: show(var opt set of $T: x)

Convert x into a string

function string: show(var opt $T: x)

Convert x into a string

function string: show(array [$U] of var opt $T: x)

Convert x into a string

function string: show2d(array [int,int] of var opt $T: x)

Convert two-dimensional array x into a string

function string: show3d(array [int,int,int] of var opt $T: x)

Convert three-dimensional array x into a string

function string: showJSON(var opt $T: x)

Convert x into JSON string

function string: showJSON(array [$U] of var opt $T: x)

Convert x into JSON string

function string: show_float(int: w, int: p, var float: x)

Formatted to-string conversion for floats.

Converts the float x into a string right justified by the number of characters given by w, or left justified if w is negative. The number of digits to appear after the decimal point is given by p. It is a run-time error for p to be negative.
function string: show_int(int: w, var int: x)

Formatted to-string conversion for integers

Converts the integer x into a string right justified by the number of characters given by w, or left justified if w is negative.

function int: string_length(string: s)

Return length of s

### 4.2.4.12 Reflection operations

These functions return information about declared or inferred variable bounds and domains.

function set of int: dom(var int: x)

Return domain of x

function set of int: dom_array(array [$T] of var int: x)

Return union of all domains of the elements in array x

function set of int: dom_bounds_array(array [$T] of var int: x)

Return approximation of union of all domains of the elements in array x

function int: dom_size(var int: x)

Return cardinality of the domain of x

function $T: fix(var opt $T: x)

Check if the value of x is fixed at this point in evaluation. If it is fixed, return its value, otherwise abort.
function array [U] of $T: fix(array [U] of var opt $T: x)

Check if the value of every element of the array $x$ is fixed at this point in evaluation. If all are
fixed, return an array of their values, otherwise abort.

```
test has_bounds(var int: x)
```

Test if variable $x$ has declared, finite bounds

```
test has_bounds(var float: x)
```

Test if variable $x$ has declared, finite bounds

```
test has_ub_set(var set of int: x)
```

Test if variable $x$ has a declared, finite upper bound

```
test is_fixed(var opt $T: x)
```

Test if $x$ is fixed

```
test is_fixed(set of $T: x)
```

Test if $x$ is fixed

```
test is_fixed(var set of int: x)
```

Test if $x$ is fixed

```
test is_fixed(array [U] of var opt $T: x)
```

Test if every element of array $x$ is fixed

```
function int: lb(var int: x)
```

Return lower bound of $x$
function int: lb(var opt int: x)

Return lower bound of x

function float: lb(var float: x)

Return lower bound of x

function float: lb(var opt float: x)

Return lower bound of x

function set of int: lb(var set of int: x)

Return lower bound of x

function array [\$U] of int: lb(array [\$U] of var int: x)

Return array of lower bounds of the elements in array x

function array [\$U] of float: lb(array [\$U] of var float: x)

Return array of lower bounds of the elements in array x

function array [\$U] of set of int: lb(array [\$U] of var set of int: x)

Return array of lower bounds of the elements in array x

function int: lb_array(array [\$U] of var opt int: x)

Return minimum of all lower bounds of the elements in array x

function float: lb_array(array [\$U] of var opt float: x)

Return minimum of all lower bounds of the elements in array x

4.2.4. Builtins
<table>
<thead>
<tr>
<th>Function</th>
<th>Signature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>lb_array</code></td>
<td><code>function set of int: lb_array(array [$U] of var set of int: x)</code></td>
<td>Return intersection of all lower bounds of the elements in array x</td>
</tr>
<tr>
<td><code>ub</code></td>
<td><code>function int: ub(var int: x)</code></td>
<td>Return upper bound of x</td>
</tr>
<tr>
<td><code>ub</code></td>
<td><code>function int: ub(var opt int: x)</code></td>
<td>Return upper bound of x</td>
</tr>
<tr>
<td><code>ub</code></td>
<td><code>function float: ub(var float: x)</code></td>
<td>Return upper bound of x</td>
</tr>
<tr>
<td><code>ub</code></td>
<td><code>function float: ub(var opt float: x)</code></td>
<td>Return upper bound of x</td>
</tr>
<tr>
<td><code>ub</code></td>
<td><code>function set of int: ub(var set of int: x)</code></td>
<td>Return upper bound of x</td>
</tr>
<tr>
<td><code>ub</code></td>
<td><code>function array [$U] of int: ub(array [$U] of var int: x)</code></td>
<td>Return array of upper bounds of the elements in array x</td>
</tr>
<tr>
<td><code>ub</code></td>
<td><code>function array [$U] of float: ub(array [$U] of var float: x)</code></td>
<td>Return array of upper bounds of the elements in array x</td>
</tr>
<tr>
<td><code>ub</code></td>
<td><code>function array [$U] of set of int: ub(array [$U] of var set of int: x)</code></td>
<td>Return array of upper bounds of the elements in array x</td>
</tr>
</tbody>
</table>

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Function `int`: \[ \text{ub\_array}(\text{array}[U]\text{ of var opt int}: x) \]

Return maximum of all upper bounds of the elements in array \(x\).

Function `float`: \[ \text{ub\_array}(\text{array}[U]\text{ of var opt float}: x) \]

Return maximum of all upper bounds of the elements in array \(x\).

Function `set of int`: \[ \text{ub\_array}(\text{array}[U]\text{ of var set of int}: x) \]

Return union of all upper bounds of the elements in array \(x\).

### 4.2.4.13 Assertions and debugging functions

These functions help debug models and check that input data conforms to the expectations.

Test `abort(string: msg)`

Abort evaluation and print message `msg`.

Function `T`: \[ \text{assert(bool: b, string: msg, T: x)} \]

If \(b\) is true, return \(x\), otherwise abort with message `msg`.

Function `var T`: \[ \text{assert(bool: b, string: msg, var T: x)} \]

If \(b\) is true, return \(x\), otherwise abort with message `msg`.

Function `var opt T`: \[ \text{assert(bool: b, string: msg, var opt T: x)} \]

If \(b\) is true, return \(x\), otherwise abort with message `msg`.

Function `array [U] of T`: \[ \text{assert(bool: b, string: msg, array [U] of T: x)} \]

If \(b\) is true, return \(x\), otherwise abort with message `msg`.
If b is true, return x, otherwise abort with message msg.

```plaintext
function array [\$U\] of var $T$: assert(bool: b,
    string: msg,
    array [\$U\] of var $T$: x)
```

If b is true, return x, otherwise abort with message msg.

```plaintext
function array [\$U\] of var opt $T$: assert(bool: b,
    string: msg,
    array [\$U\] of var opt $T$: x)
```

If b is true, return x, otherwise abort with message msg.

```plaintext
test assert(bool: b, string: msg)
```

If b is true, return true, otherwise abort with message msg.

```plaintext
function $T$: trace(string: msg, $T$: x)
```

Return x, and print message msg.

```plaintext
function var $T$: trace(string: msg, var $T$: x)
```

Return x, and print message msg.

```plaintext
function var opt $T$: trace(string: msg, var opt $T$: x)
```

Return x, and print message msg.

```plaintext
test trace(string: msg)
```

Return true, and print message msg.

```plaintext
function $T$: trace_stdout(string: msg, $T$: x)
```

Return x, and print message msg.
function var $T$: trace_stdout(string: msg, var $T$: x)

Return x, and print message msg.

function var opt $T$: trace_stdout(string: msg, var opt $T$: x)

Return x, and print message msg.

test trace_stdout(string: msg)

Return true, and print message msg.

### 4.2.4.14 Functions for enums

function $\mathcal{E}$: enum_next(set of $\mathcal{E}$: e, $\mathcal{E}$: x)

Return next greater enum value of x in enum e

function var $\mathcal{E}$: enum_next(set of $\mathcal{E}$: e, var $\mathcal{E}$: x)

Return next greater enum value of x in enum e

function $\mathcal{E}$: enum_prev(set of $\mathcal{E}$: e, $\mathcal{E}$: x)

Return next smaller enum value of x in enum e

function var $\mathcal{E}$: enum_prev(set of $\mathcal{E}$: e, var $\mathcal{E}$: x)

Return next smaller enum value of x in enum e

function $\mathcal{E}$: to_enum(set of $\mathcal{E}$: X, int: x)

Convert x to enum type X

4.2.4. Builtins
4.2.4.15 Random Number Generator builtins

These functions implement random number generators from different probability distributions.

test bernoulli(float: p)

Return a boolean sample from the Bernoulli distribution defined by probability p

function int: binomial(int: t, float: p)

Return a sample from the binomial distribution defined by sample number t and probability p

function float: cauchy(float: mean, float: scale)

Return a sample from the cauchy distribution defined by mean, scale
Return a sample from the Cauchy distribution defined by mean, scale

```plaintext
function float: chisquared(int: n)
```

Return a sample from the chi-squared distribution defined by the degree of freedom n

```plaintext
function float: chisquared(float: n)
```

Return a sample from the chi-squared distribution defined by the degree of freedom n

```plaintext
function int: discrete_distribution(array [int] of int: weights)
```

Return a sample from the discrete distribution defined by the array of weights weights that assigns a weight to each integer starting from zero

```plaintext
function float: exponential(int: lambda)
```

Return a sample from the exponential distribution defined by lambda

```plaintext
function float: exponential(float: lambda)
```

Return a sample from the exponential distribution defined by lambda

```plaintext
function float: fdistribution(float: d1, float: d2)
```

Return a sample from the Fisher-Snedecor F-distribution defined by the degrees of freedom d1, d2

```plaintext
function float: fdistribution(int: d1, int: d2)
```

Return a sample from the Fisher-Snedecor F-distribution defined by the degrees of freedom d1, d2

```plaintext
function float: gamma(float: alpha, float: beta)
```

Return a sample from the gamma distribution defined by alpha, beta
function float: gamma(int: alpha, float: beta)

Return a sample from the gamma distribution defined by alpha, beta

function float: lognormal(float: mean, float: std)

Return a sample from the lognormal distribution defined by mean, std

function float: lognormal(int: mean, float: std)

Return a sample from the lognormal distribution defined by mean, std

function float: normal(float: mean, float: std)

Return a sample from the normal distribution defined by mean, std

function float: normal(int: mean, float: std)

Return a sample from the normal distribution defined by mean, std

function int: poisson(float: mean)

Return a sample from the poisson distribution defined by mean

function int: poisson(int: mean)

Return a sample from the poisson distribution defined by an integer mean

function float: tdistribution(float: n)

Return a sample from the student’s t-distribution defined by the sample size n

function float: tdistribution(int: n)

Return a sample from the student’s t-distribution defined by the sample size n
function float: uniform(float: lowerbound, float: upperbound)

Return a sample from the uniform distribution defined by lowerbound, upperbound

function int: uniform(int: lowerbound, int: upperbound)

Return a sample from the uniform distribution defined by lowerbound, upperbound

function float: weibull(float: shape, float: scale)

Return a sample from the Weibull distribution defined by shape, scale

function float: weibull(int: shape, float: scale)

Return a sample from the Weibull distribution defined by shape, scale

4.2.4.16 Special constraints

These predicates allow users to mark constraints as e.g. symmetry breaking or redundant, so that solvers can choose to implement them differently.

We cannot easily use annotations for this purpose, since annotations are propagated to all constraints in a decomposition, which may be incorrect for redundant or symmetry breaking constraints in the presence of common subexpression elimination (CSE).

predicate implied_constraint(var bool: b)

Mark b as an implied constraint (synonym for redundant_constraint)

predicate redundant_constraint(var bool: b)

Mark b as a redundant constraint

predicate symmetry_breaking_constraint(var bool: b)

Mark b as a symmetry breaking constraint
4.2.4.17 Language information

These functions return information about the MiniZinc system.

```plaintext
function int: mzn_compiler_version()
```

Return MiniZinc version encoded as an integer (major*10000+minor*1000+patch).

```plaintext
function string: mzn_version_to_string(int: v)
```

Return string representation of v given an integer major*10000+minor*1000+patch

4.2.5 FlatZinc builtns

These are the standard constraints that need to be supported by FlatZinc solvers (or redefined in the redefinitions.mzn file).

4.2.5.1 Integer FlatZinc builtns

```plaintext
predicate array_int_element(var int: b,
                            array [int] of int: as,
                            var int: c)
```

Constrains as [ b ] = c

```plaintext
predicate array_int_maximum(var int: m, array [int] of var int: x)
```

Constrains m to be the maximum value of the (non-empty) array x

```plaintext
predicate array_int_minimum(var int: m, array [int] of var int: x)
```

Constrains m to be the minimum value of the (non-empty) array x

```plaintext
predicate array_var_int_element(var int: b,
                                array [int] of var int: as,
                                var int: c)
```

Constrains as [ b ] = c
Constrains as \[ b \] = c

\[
predicate \text{int_abs}(\text{var int}: a, \text{var int}: b)
\]

Constrains \( b \) to be the absolute value of \( a \)

\[
predicate \text{int_div}(\text{var int}: a, \text{var int}: b, \text{var int}: c)
\]

Constrains \( a \div b = c \)

\[
predicate \text{int_eq}(\text{var int}: a, \text{var int}: b)
\]

Constrains \( a \) to be equal to \( b \)

\[
predicate \text{int_eq_reif}(\text{var int}: a, \text{var int}: b, \text{var bool}: r)
\]

Constrains \(( a = b ) \leftrightarrow r\)

\[
predicate \text{int_le}(\text{var int}: a, \text{var int}: b)
\]

Constrains \( a \) to be less than or equal to \( b \)

\[
predicate \text{int_le_reif}(\text{var int}: a, \text{var int}: b, \text{var bool}: r)
\]

Constrains \(( a \leq b ) \leftrightarrow r\)

\[
predicate \text{int_lin_eq}(\text{array [int] of int}: as,
            array [int] of var int: bs,
            int: c)
\]

Constrains \( c = \sum_i a[i] \times b[i] \)

\[
predicate \text{int_lin_eq_reif}(\text{array [int] of int}: as,
            array [int] of var int: bs,
            int: c)
\]

4.2.5. FlatZinc builtins
int: c,
var bool: r)

Constrains $r \leftrightarrow (c = \sum_i a[i] * b[i])$

predicate int_lin_le(array [int] of int: as,
array [int] of var int: bs,
int: c)

Constrains $\sum_i a[i] * b[i] \leq c$

predicate int_lin_le_reif(array [int] of int: as,
array [int] of var int: bs,
int: c,
var bool: r)

Constrains $r \leftrightarrow (\sum_i a[i] * b[i] \leq c)$

predicate int_lin_ne(array [int] of int: as,
array [int] of var int: bs,
int: c)

Constrains $c \neq \sum_i a[i] * b[i]$

predicate int_lin_ne_reif(array [int] of int: as,
array [int] of var int: bs,
int: c,
var bool: r)

Constrains $r \leftrightarrow (c \neq \sum_i a[i] * b[i])$

predicate int_lt(var int: a, var int: b)

Constrains $a < b$

predicate int_lt_reif(var int: a, var int: b, var bool: r)
Constrains $r \leftrightarrow (a < b)$

```
predicate int_max(var int: a, var int: b, var int: c)
```

Constrains $\text{max}(a, b) = c$

```
predicate int_min(var int: a, var int: b, var int: c)
```

Constrains $\text{min}(a, b) = c$

```
predicate int_mod(var int: a, var int: b, var int: c)
```

Constrains $a \% b = c$

```
predicate int_ne(var int: a, var int: b)
```

Constrains $a \neq b$

```
predicate int_ne_reif(var int: a, var int: b, var bool: r)
```

$r \leftrightarrow (a \neq b)$

```
predicate int_plus(var int: a, var int: b, var int: c)
```

Constrains $a + b = c$

```
predicate int_pow(var int: x, var int: y, var int: z)
```

Constrains $z = x^y$

```
predicate int_pow_fixed(var int: x, int: y, var int: z)
```

Constrains $z = x^y$

```
predicate int_times(var int: a, var int: b, var int: c)
```

Constrains $a \ast b = c$
4.2.5.2 Bool FlatZinc builtins

**Predicate**

`array_bool_and(array [int] of var bool: as, var bool: r)`

**Constraints**

\[ r \iff \bigwedge_i as[i] \]

`array_bool_element(var int: b, array [int] of bool: as, var bool: c)`

**Constraints**

\[ as[b] = c \]

`array_bool_or(array [int] of var bool: as, var bool: r)`

**Constraints**

\[ r \iff \bigvee_i as[i] \]

`array_bool_xor(array [int] of var bool: as)`

**Constraints**

\[ \oplus_i as[i] \]

`array_var_bool_element(var int: b, array [int] of var bool: as, var bool: c)`

**Constraints**

\[ as[b] = c \]

`bool2int(var bool: a, var int: b)`

**Constraints**

\[ b \in \{0,1\} \text{ and } a \iff b = 1 \]

`bool_and(var bool: a, var bool: b, var bool: r)`

**Constraints**

\[ r \iff a \land b \]

`bool_clause(array [int] of var bool: as, array [int] of var bool: bs)`
Constrains $\forall i \ as[i] \lor \forall j \ -bs[j]$

```plaintext
predicate bool_eq(var bool: a, var bool: b)
```

Constrains $a = b$

```plaintext
predicate bool_eq_reif(var bool: a, var bool: b, var bool: r)
```

Constrains $r \leftrightarrow (a = b)$

```plaintext
predicate bool_le(var bool: a, var bool: b)
```

Constrains $a \leq b$

```plaintext
predicate bool_le_reif(var bool: a, var bool: b, var bool: r)
```

Constrains $r \leftrightarrow (a \leq b)$

```plaintext
predicate bool_lin_eq(array [int] of int: as,
    array [int] of var bool: bs,
    var int: c)
```

Constrains $c = \sum_i as[i] \ast bs[i]$

```plaintext
predicate bool_lin_le(array [int] of int: as,
    array [int] of var bool: bs,
    int: c)
```

Constrains $c \leq \sum_i as[i] \ast bs[i]$

```plaintext
predicate bool_lt(var bool: a, var bool: b)
```

Constrains $a < b$

```plaintext
predicate bool_lt_reif(var bool: a, var bool: b, var bool: r)
```

Constrains $r \leftrightarrow (a < b)$
predicate bool_not(var bool: a, var bool: b)

Constrains $a \neq b$

predicate bool_or(var bool: a, var bool: b, var bool: r)

Constrains $r \leftrightarrow a \lor b$

predicate bool_xor(var bool: a, var bool: b, var bool: r)

Constrains $r \leftrightarrow a \oplus b$

predicate bool_xor(var bool: a, var bool: b)

Constrains $a \oplus b$

4.2.5.3 Set FlatZinc builtins

predicate array_set_element(var int: b,
                            array [int] of set of int: as,
                            var set of int: c)

Constrains $[b] = c$

predicate array_var_set_element(var int: b,
                                array [int] of var set of int: as,
                                var set of int: c)

Constrains $[b] = c$

predicate set_card(var set of int: S, var int: x)

Constrains $x = |S|$
predicate set_diff(var set of int: x, var set of int: y, var set of int: r)

Constrains \( r = x \setminus y \)

predicate set_eq(var set of int: x, var set of int: y)

Constrains \( x = y \)

predicate set_eq_reif(var set of int: x, var set of int: y, var bool: r)

Constrains \( r \leftrightarrow (x = y) \)

predicate set_in(var int: x, set of int: S)

Constrains \( x \in S \)

predicate set_in(var int: x, var set of int: S)

Constrains \( x \in S \)

predicate set_in_reif(var int: x, set of int: S, var bool: r)

Constrains \( r \leftrightarrow (x \in S) \)

predicate set_in_reif(var int: x, var set of int: S, var bool: r)

Constrains \( r \leftrightarrow (x \in S) \)

predicate set_intersect(var set of int: x, var set of int: y, var set of int: r)
Constrains \( r = x \cap y \)

```plaintext
predicate set_le(var set of int: x, var set of int: y)
```

Constrains \( x \leq y \) (lexicographic order of the sorted lists of elements)

```plaintext
predicate set_le_reif(var set of int: x, var set of int: y, var bool: r)
```

Constrains \( r \leftrightarrow (x \leq y) \) (lexicographic order of the sorted lists of elements)

```plaintext
predicate set_lt(var set of int: x, var set of int: y)
```

Constrains \( x < y \) (lexicographic order of the sorted lists of elements)

```plaintext
predicate set_lt_reif(var set of int: x, var set of int: y, var bool: r)
```

Constrains \( r \leftrightarrow (x < y) \) (lexicographic order of the sorted lists of elements)

```plaintext
predicate set_ne(var set of int: x, var set of int: y)
```

Constrains \( x \neq y \)

```plaintext
predicate set_ne_reif(var set of int: x, var set of int: y, var bool: r)
```

Constrains \( r \leftrightarrow (x \neq y) \)

```plaintext
predicate set_subset(var set of int: x, var set of int: y)
```

Constrains \( x \subseteq y \)
predicate set_subset_reif(var set of int: x,
                   var set of int: y,
                   var bool: r)

Constrains r \leftrightarrow (x \subseteq y)

predicate set_superset(var set of int: x, var set of int: y)

Constrains x \supseteq y

predicate set_symdiff(var set of int: x,
                      var set of int: y,
                      var set of int: r)

Constrains r to be the symmetric difference of x and y

predicate set_union(var set of int: x,
                     var set of int: y,
                     var set of int: r)

Constrains r = x \cup y

4.2.5.4 Float FlatZinc builtins

predicate array_float_element(var int: b,
                               array [int] of float: as,
                               var float: c)

Constrains as [ b ] = c

predicate array_float_maximum(var int: m, array [int] of var int: x)

Constrains m to be the maximum value of the (non-empty) array x

predicate array_float_minimum(var int: m, array [int] of var int: x)
Constrains $m$ to be the minimum value of the (non-empty) array $x$

```plaintext
predicate array_var_float_element (var int: b,
   array [int] of var float: as,
   var float: c)
```

Constrains $a[b] = c$

```plaintext
predicate float_abs(var float: a, var float: b)
```

Constrains $b$ to be the absolute value of $a$

```plaintext
predicate float_acos(var float: a, var float: b)
```

Constrains $b = \cos(a)$

```plaintext
predicate float_acosh(var float: a, var float: b)
```

Constrains $b = \cosh(a)$

```plaintext
predicate float_asin(var float: a, var float: b)
```

Constrains $b = \sin(a)$

```plaintext
predicate float_asinh(var float: a, var float: b)
```

Constrains $b = \sinh(a)$

```plaintext
predicate float_atan(var float: a, var float: b)
```

Constrains $b = \tan(a)$

```plaintext
predicate float_atanh(var float: a, var float: b)
```

Constrains $b = \tanh(a)$
predicate float_cos(var float: a, var float: b)

Constrains $b = \cos(a)$

predicate float_cosh(var float: a, var float: b)

Constrains $b = \cosh(a)$

predicate float_div(var float: a, var float: b, var float: c)

Constrains $a / b = c$

predicate float_dom(var float: x, array [int] of float: as)

Constrains the domain of $x$ using the values in $as$, using each pair of values as $[2^i - 1.. 2^i]$ for $i$ in $1.. n/2$ as a possible range

predicate float_eq(var float: a, var float: b)

Constrains $a = b$

predicate float_eq_reif(var float: a, var float: b, var bool: r)

Constrains $r \leftrightarrow (a = b)$

predicate float_exp(var float: a, var float: b)

Constrains $b = \exp(a)$

predicate float_in(var float: a, float: b, float: c)

Constrains $a \in [b, c]$

predicate float_in_reif(var float: a, float: b, float: c, var bool: r)

Constrains $r \leftrightarrow a \in [b, c]$
predicate float_le(var float: a, var float: b)

Constrains \( a \leq b \)

predicate float_le_reif(var float: a, var float: b, var bool: r)

Constrains \( r \leftrightarrow (a \leq b) \)

predicate float_lin_eq(array [int] of float: as,
                        array [int] of var float: bs,
                        float: c)

Constrains \( c = \sum_i as[i] \times bs[i] \)

predicate float_lin_eq_reif(array [int] of float: as,
                            array [int] of var float: bs,
                            float: c,
                            var bool: r)

Constrains \( r \leftrightarrow (c = \sum_i as[i] \times bs[i]) \)

predicate float_lin_le(array [int] of float: as,
                        array [int] of var float: bs,
                        float: c)

Constrains \( c \leq \sum_i as[i] \times bs[i] \)

predicate float_lin_le_reif(array [int] of float: as,
                             array [int] of var float: bs,
                             float: c,
                             var bool: r)

Constrains \( r \leftrightarrow (c \leq \sum_i as[i] \times bs[i]) \)

predicate float_lin_lt(array [int] of float: as,
                        array [int] of var float: bs,
float: c)

Constrains \( c < \sum_i a[i] \times b[i] \)

\[
predicate float_lin_lt_reif(array [int] of float: as,
  array [int] of var float: bs,
  float: c,
  var bool: r)
\]

Constrains \( r \leftrightarrow (c < \sum_i a[i] \times b[i]) \)

\[
predicate float_lin_ne(array [int] of float: as,
  array [int] of var float: bs,
  float: c)
\]

Constrains \( c \neq \sum_i a[i] \times b[i] \)

\[
predicate float_lin_ne_reif(array [int] of float: as,
  array [int] of var float: bs,
  float: c,
  var bool: r)
\]

Constrains \( r \leftrightarrow (c \neq \sum_i a[i] \times b[i]) \)

\[
predicate float_ln(var float: a, var float: b)
\]

Constrains \( b = \ln(a) \)

\[
predicate float_log10(var float: a, var float: b)
\]

Constrains \( b = \log_{10}(a) \)

\[
predicate float_log2(var float: a, var float: b)
\]

Constrains \( b = \log_{2}(a) \)
predicate float_lt(var float: a, var float: b)

Constrains \(a < b\)

predicate float_lt_reif(var float: a, var float: b, var bool: r)

Constrains \(r \leftrightarrow (a < b)\)

predicate float_max(var float: a, var float: b, var float: c)

Constrains \(\max(a, b) = c\)

predicate float_min(var float: a, var float: b, var float: c)

Constrains \(\min(a, b) = c\)

predicate float_ne(var float: a, var float: b)

Constrains \(a \neq b\)

predicate float_ne_reif(var float: a, var float: b, var bool: r)

Constrains \(r \leftrightarrow (a \neq b)\)

predicate float_plus(var float: a, var float: b, var float: c)

Constrains \(a + b = c\)

predicate float_pow(var float: x, var float: y, var float: z)

Constrains \(z = x^y\)

predicate float_sin(var float: a, var float: b)

Constrains \(b = \sin(a)\)
4.2.5.5 FlatZinc builtins added in MiniZinc 2.0.0.

These functions and predicates define built-in operations of the MiniZinc language that have been added in MiniZinc 2.0.0. Solvers that support these natively need to include a file called redefinitions-2.0.mzn in their solver library that redefines these predicates as builtins.

Constrains $m$ to be the maximum value in array $x$.  

4.2.5. FlatZinc builtins
The `array_float_minimum` predicate constrains `m` to be the minimum value in array `x`.

```
predicate array_float_minimum(var float: m, array [int] of var float: x)
```

Constrains `m` to be the minimum value in array `x`.

```
predicate array_int_maximum(var int: m, array [int] of var int: x)
```

Constrains `m` to be the maximum value in array `x`.

```
predicate array_int_minimum(var int: m, array [int] of var int: x)
```

Constrains `m` to be the minimum value in array `x`.

```
predicate bool_clause_reif(array [int] of var bool: as, array [int] of var bool: bs, var bool: b)
```

Reified clause constraint. Constrains `b ← ∨_i as[i] ∨ j bs[j]`

### 4.2.5.6 FlatZinc builtns added in MiniZinc 2.0.2.

These functions and predicates define built-in operations of the MiniZinc language that have been added in MiniZinc 2.0.2. Solvers that support these natively need to include a file called redefinitions-2.0.2.mzn in their solver library that redefines these predicates as builtins.

```
predicate array_var_bool_element_nonshifted(var int: idx, array [int] of var bool: x, var bool: c)
```

Element constraint on array with MiniZinc index set, constrains `x[i] = c` This can be overridden in a solver that can perform the index calculation more efficiently than using a MiniZinc decomposition.

```
predicate array_var_float_element_nonshifted(var int: idx, array [int] of var float: x, var float: c)
```
Element constraint on array with MiniZinc index set, constrains \( x[\text{idx}] = c \) This can be overridden in a solver that can perform the index calculation more efficiently than using a MiniZinc decomposition.

```plaintext
predicate array_var_int_element_nonshifted(var int: idx,
                                            array [int] of var int: x,
                                            var int: c)
```

Element constraint on array with MiniZinc index set, constrains \( x[\text{idx}] = c \) This can be overridden in a solver that can perform the index calculation more efficiently than using a MiniZinc decomposition.

```plaintext
predicate array_var_set_element_nonshifted(var int: idx,
                                            array [int] of var set of int: x,
                                            var set of int: c)
```

Element constraint on array with MiniZinc index set, constrains \( x[\text{idx}] = c \) This can be overridden in a solver that can perform the index calculation more efficiently than using a MiniZinc decomposition.

### 4.2.5.7 FlatZinc builtins added in MiniZinc 2.1.0.

These functions and predicates define built-in operations of the MiniZinc language that have been added in MiniZinc 2.1.0. Solvers that support these natively need to include a file called redefinitions-2.1.0.mzn in their solver library that redefines these predicates as builtins.

```plaintext
predicate float_dom(var float: x, array [int] of float: as)
```

Constrains \( x \) to take one of the values in \( \text{as} \)

```plaintext
predicate float_in(var float: x, float: a, float: b)
```

Constrains \( a \leq x \leq b \)

### 4.2.5.8 FlatZinc builtins added in MiniZinc 2.1.1.

These functions and predicates define built-in operations of the MiniZinc language that have been added in MiniZinc 2.1.1. Solvers that support these natively need to include a file called
redefinitions-2.1.1.mzn in their solver library that redefines these predicates as builtins.

```plaintext
function var $E$: max(var set of $E$: s)
```

Returns variable constrained to be equal to the maximum of the set $s$. An alternative implementation can be found in the comments of the source code.

```plaintext
function var $E$: min(var set of $E$: s)
```

Returns variable constrained to be equal to the minimum of the set $s$. An alternative implementation can be found in the comments of the source code.

### 4.2.6 Global constraints

These constraints represent high-level modelling abstractions, for which many solvers implement special, efficient inference algorithms.

#### 4.2.6.1 Counting constraints

These constraints count and restrict how many times certain values occur in an array of variables. MiniZinc will automatically generate the basic counting constraints below from expressions such as `count(i in x)(i=c) <= d`, so you can write models in this much more readable style instead of using these predicates. However, if your model contains multiple counting constraints over the same array, constraints like `distribute` or `global_cardinality` below may be useful.

```plaintext
predicate among(var int: n, array [int] of var int: x, set of int: v)
```

Requires exactly $n$ variables in $x$ to take one of the values in $v$.

```plaintext
function var int: among(array [int] of var int: x, set of int: v)
```

Returns the number of variables in $x$ that take one of the values in $v$.

```plaintext
predicate at_least(int: n, array [int] of var set of int: x, set of int: v)
```
Requires at least \( n \) variables in \( x \) to take the value \( v \).

\[ \text{predicate at_most}(\text{int: } n, \text{array [int] of var set of int: } x, \text{set of int: } v) \]

Requires at most \( n \) variables in \( x \) to take the value \( v \).

\[ \text{predicate at_most1}(\text{array [int] of var set of int: } s) \]

Requires that each pair of sets in \( s \) overlap in at most one element.

\[ \text{function var int: count}(\text{array [int] of var int: } x, \text{var int: } y) \]

Returns the number of occurrences of \( y \) in \( x \).

\[ \text{predicate count}(\text{array [int] of var int: } x, \text{var int: } y, \text{var int: } c) \]

Constrains \( c \) to be the number of occurrences of \( y \) in \( x \).

\[ \text{predicate count_eq}(\text{array [int] of var int: } x, \text{var int: } y, \text{var int: } c) \]

Constrains \( c \) to be the number of occurrences of \( y \) in \( x \).

\[ \text{predicate count_eq}(\text{array [int] of var int: } x, \text{int: } y, \text{int: } c) \]

Constrains \( c \) to be the number of occurrences of \( y \) in \( x \).

\[ \text{function var int: count_eq}(\text{array [int] of var int: } x, \text{var int: } y) \]

Returns the number of occurrences of \( y \) in \( x \).

\[ \text{predicate count_geq}(\text{array [int] of var int: } x, \text{var int: } y, \text{var int: } c) \]

Constrains \( c \) to be greater than or equal to the number of occurrences of \( y \) in \( x \).
\textbf{predicate count\_geq(array [int] of var int: x, int: y, int: c)}

Constrains \(c\) to be greater than or equal to the number of occurrences of \(y\) in \(x\).

\textbf{predicate count\_gt(array [int] of var int: x, var int: y, var int: c)}

Constrains \(c\) to be strictly greater than the number of occurrences of \(y\) in \(x\).

\textbf{predicate count\_leq(array [int] of var int: x, var int: y, var int: c)}

Constrains \(c\) to be less than or equal to the number of occurrences of \(y\) in \(x\).

\textbf{predicate count\_lt(array [int] of var int: x, var int: y, var int: c)}

Constrains \(c\) to be strictly less than the number of occurrences of \(y\) in \(x\).

\textbf{predicate count\_neq(array [int] of var int: x, var int: y, var int: c)}

Constrains \(c\) to be not equal to the number of occurrences of \(y\) in \(x\).
predicate distribute(array [int] of var int: card,
                      array [int] of var int: value,
                      array [int] of var int: base)

Requires that card[i] is the number of occurrences of value[i] in base. The values in value need not be distinct.

function array [int] of var int: distribute(array [int] of var int: value,
                                             array [int] of var int: base)

Returns an array of the number of occurrences of value[i] in base. The values in value need not be distinct.

predicate exactly(int: n,
                  array [int] of var set of int: x,
                  set of int: v)

Requires exactly n variables in x to take the value v.

predicate global_cardinality(array [int] of var int: x,
                             array [int] of int: cover,
                             array [int] of var int: counts)

Requires that the number of occurrences of cover[i] in x is counts[i].

function array [int] of var int: global_cardinality(array [int] of var int: x,
                                                     array [int] of int: cover)

Returns the number of occurrences of cover[i] in x.

predicate global_cardinality_closed(array [int] of var int: x,
                                     array [int] of int: cover,
                                     array [int] of var int: counts)

Requires that the number of occurrences of i in x is counts[i].
The elements of \( x \) must take their values from \( \text{cover} \).

```plaintext
function array [int] of var int: global_cardinality_closed(array [int] of
  var int: x,
  array [int] of int: cover)
```

Returns an array with number of occurrences of \( i \) in \( x \).

The elements of \( x \) must take their values from \( \text{cover} \).

```plaintext
predicate global_cardinality_low_up(array [int] of var int: x,
  array [int] of int: cover,
  array [int] of int: lbound,
  array [int] of int: ubound)
```

Requires that for all \( i \), the value \( \text{cover}[i] \) appears at least \( \text{lbound}[i] \) and at most \( \text{ubound}[i] \) times in the array \( x \).

```plaintext
predicate global_cardinality_low_up_closed(array [int] of var int: x,
  array [int] of int: cover,
  array [int] of int: lbound,
  array [int] of int: ubound)
```

Requires that for all \( i \), the value \( \text{cover}[i] \) appears at least \( \text{lbound}[i] \) and at most \( \text{ubound}[i] \) times in the array \( x \).

The elements of \( x \) must take their values from \( \text{cover} \).

### 4.2.6.2 All-Different and related constraints

```plaintext
predicate all_different(array [\$X\] of var int: x)
```

Constrain the array of integers \( x \) to be all different.

```plaintext
predicate all_different(array [\$X\] of var set of int: x)
```

Constrain the array of sets of integers \( x \) to be all different.
predicate all_disjoint(array [\$X\] of var set of int: S)

Constrain the array of sets of integers S to be pairwise disjoint.

predicate all_equal(array [\$X\] of var int: x)

Constrain the array of integers x to be all equal

predicate all_equal(array [\$X\] of var set of int: x)

Constrain the array of sets of integers x to be all equal

predicate alldifferent_except_0(array [\$X\] of var int: vs)

Constrain the array of integers vs to be all different except those elements that are assigned the value 0.

predicate nvalue(var int: n, array [int] of var int: x)

Requires that the number of distinct values in x is n.

function var int: nvalue(array [int] of var int: x)

Returns the number of distinct values in x.

predicate symmetric_all_different(array [int] of var int: x)

Requires the array of integers x to be all different, and for all i, x[i]=j \rightarrow x[j]=i.

4.2.6.3 Lexicographic constraints

predicate lex2(array [int,int] of var int: x)

Require adjacent rows and adjacent columns in the array x to be lexicographically ordered. Adjacent rows and adjacent columns may be equal.
**MiniZinc Handbook, Release 2.4.3**

**predicate lex_greater**

\[
\text{array [int] of var bool: } x,
\text{array [int] of var bool: } y
\]

Requires that the array \( x \) is strictly lexicographically greater than array \( y \). Compares them from first to last element, regardless of indices.

**predicate lex_greater**

\[
\text{array [int] of var int: } x,
\text{array [int] of var int: } y
\]

Requires that the array \( x \) is strictly lexicographically greater than array \( y \). Compares them from first to last element, regardless of indices.

**predicate lex_greater**

\[
\text{array [int] of var float: } x,
\text{array [int] of var float: } y
\]

Requires that the array \( x \) is strictly lexicographically greater than array \( y \). Compares them from first to last element, regardless of indices.

**predicate lex_greater**

\[
\text{array [int] of var set of int: } x,
\text{array [int] of var set of int: } y
\]

Requires that the array \( x \) is strictly lexicographically greater than array \( y \). Compares them from first to last element, regardless of indices.

**predicate lex_greataeq**

\[
\text{array [int] of var bool: } x,
\text{array [int] of var bool: } y
\]

Requires that the array \( x \) is lexicographically greater than or equal to array \( y \). Compares them from first to last element, regardless of indices.

**predicate lex_greataeq**

\[
\text{array [int] of var int: } x,
\text{array [int] of var int: } y
\]

Requires that the array \( x \) is lexicographically greater than or equal to array \( y \). Compares them from first to last element, regardless of indices.
predicate lex_greatereq(array [int] of var float: x,
array [int] of var float: y)

Requires that the array x is lexicographically greater than or equal to array y. Compares them from first to last element, regardless of indices.

predicate lex_greatereq(array [int] of var set of int: x,
array [int] of var set of int: y)

Requires that the array x is lexicographically greater than or equal to array y. Compares them from first to last element, regardless of indices.

define predicate lex_less(array [int] of var bool: x,
array [int] of var bool: y)

Requires that the array x is strictly lexicographically less than array y. Compares them from first to last element, regardless of indices.

define predicate lex_less(array [int] of var int: x,
array [int] of var int: y)

Requires that the array x is strictly lexicographically less than array y. Compares them from first to last element, regardless of indices.

define predicate lex_less(array [int] of var float: x,
array [int] of var float: y)

Requires that the array x is strictly lexicographically less than array y. Compares them from first to last element, regardless of indices.

define predicate lex_less(array [int] of var set of int: x,
array [int] of var set of int: y)

Requires that the array x is strictly lexicographically less than array y. Compares them from first to last element, regardless of indices.
**predicate lex_lesseq(array [int] of var bool: x, array [int] of var bool: y)**

Requires that the array \( x \) is lexicographically less than or equal to array \( y \). Compares them from first to last element, regardless of indices.

**predicate lex_lesseq(array [int] of var float: x, array [int] of var float: y)**

Requires that the array \( x \) is lexicographically less than or equal to array \( y \). Compares them from first to last element, regardless of indices.

**predicate lex_lesseq(array [int] of var int: x, array [int] of var int: y)**

Requires that the array \( x \) is lexicographically less than or equal to array \( y \). Compares them from first to last element, regardless of indices.

**predicate lex_lesseq(array [int] of var set of int: x, array [int] of var set of int: y)**

Requires that the array \( x \) is lexicographically less than or equal to array \( y \). Compares them from first to last element, regardless of indices.

**predicate seq_precede_chain(array [int] of var int: x)**

Requires that \( i \) precedes \( i + 1 \) in the array \( x \) for all positive \( i \).

**predicate seq_precede_chain(array [int] of var set of int: x)**

Requires that \( i \) appears in a set in array \( x \) before \( i + 1 \) for all positive \( i \).

**predicate strict_lex2(array [int,int] of var int: x)**

Require adjacent rows and adjacent columns in the array \( x \) to be lexicographically ordered. Adjacent rows and adjacent columns cannot be equal.
predicate value_precede(int: s, int: t, array [int] of var int: x)

Requires that s precede t in the array x.
Precedence means that if any element of x is equal to t, then another element of x with a lower index is equal to s.

predicate value_precede(int: s, int: t, array [int] of var set of int: x)

Requires that s precede t in the array x.
Precedence means that if an element of x contains t but not s, then another element of x with lower index contains s but not t.

predicate value_precede_chain(array [int] of int: c, array [int] of var int: x)

Requires that c[i] precedes c[i+1] in the array x.
Precedence means that if any element of x is equal to c[i+1], then another element of x with a lower index is equal to c[i].

predicate value_precede_chain(array [int] of int: c, array [int] of var set of int: x)

Requires that c[i] precedes c[i+1] in the array x.
Precedence means that if an element of x contains c[i+1] but not c[i], then another element of x with lower index contains c[i] but not c[i+1].

4.2.6.4 Sorting constraints

function array [int] of var int: arg_sort(array [int] of var int: x)

Returns the permutation p which causes x to be in sorted order hence x[p[i]] <= x[p[i+1]].
The permutation is the stable sort hence $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$.

```mini
function array [int] of var int: arg_sort(array [int] of var float: x)
```

Returns the permutation $p$ which causes $x$ to be in sorted order hence $x[p[i]] \leq x[p[i+1]]$.

The permutation is the stable sort hence $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$.

```mini
predicate arg_sort(array [int] of var int: x, array [int] of var int: p)
```

Constrains $p$ to be the permutation which causes $x$ to be in sorted order hence $x[p[i]] \leq x[p[i+1]]$.

The permutation is the stable sort hence $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$.

```mini
predicate arg_sort(array [int] of var float: x, array [int] of var int: p)
```

Constrains $p$ to be the permutation which causes $x$ to be in sorted order hence $x[p[i]] \leq x[p[i+1]]$.

The permutation is the stable sort hence $x[p[i]] = x[p[i+1]] \rightarrow p[i] < p[i+1]$.

```mini
predicate decreasing(array [int] of var bool: x)
```

Requires that the array $x$ is in decreasing order (duplicates are allowed).

```mini
predicate decreasing(array [int] of var float: x)
```

Requires that the array $x$ is in decreasing order (duplicates are allowed).

```mini
predicate decreasing(array [int] of var int: x)
```

Requires that the array $x$ is in decreasing order (duplicates are allowed).

```mini
predicate decreasing(array [int] of var set of int: x)
```
Requires that the array x is in decreasing order (duplicates are allowed).

```
predicate increasing(array [int] of var bool: x)
```

Requires that the array x is in increasing order (duplicates are allowed).

```
predicate increasing(array [int] of var float: x)
```

Requires that the array x is in increasing order (duplicates are allowed).

```
predicate increasing(array [int] of var int: x)
```

Requires that the array x is in increasing order (duplicates are allowed).

```
predicate increasing(array [int] of var set of int: x)
```

Returns a multiset of values that is the same as the multiset of values in x but in sorted order.

```
function array [int] of var int: sort(array [int] of var int: x)
```

4.2.6.5 Channeling constraints

```
predicate int_set_channel(array [int] of var int: x,
                          array [int] of var set of int: y)
```

Requires that array of int variables x and array of set variables y are related such that \((x[i] = j) \leftrightarrow (i \in y[j])\).
Constrains two arrays of int variables, f and invf, to represent inverse functions. All the values in each array must be within the index set of the other array.

\[
\text{predicate inverse(array [int] of var int: } f, \\
\text{array [int] of var int: } \text{invf})
\]

If the ith variable of the collection X is assigned to j and if j is in the index set of Y then the jth variable of the collection Y is assigned to i.

Conversely, if the jth variable of the collection Y is assigned to i and if i is in the index set of X then the ith variable of the collection X is assigned to j.

\[
\text{predicate inverse_set(array [int] of var set of int: } f, \\
\text{array [int] of var set of int: } \text{invf})
\]

Constrains two arrays of set of int variables, f and invf, so that a j in f[ i ] iff i in invf[ j ]. All the values in each array’s sets must be within the index set of the other array.

\[
\text{predicate link_set_to_booleans(var set of int: } s, \\
\text{array [int] of var bool: } b)
\]

Constrain the array of Booleans b to be a representation of the set s : i in s ↔ b[ i ]. The index set of b must be a superset of the possible values of s.

4.2.6.6 Packing constraints
predicate bin_packing(int: c,
    array [int] of var int: bin,
    array [int] of int: w)

Requires that each item \( i \) with weight \( w[i] \), be put into \( bin[i] \) such that the sum of the weights of the items in each bin does not exceed the capacity \( c \).

Assumptions:
- \( \forall i, w[i] \geq 0 \)
- \( c \geq 0 \)

predicate bin_packing_capa(array [int] of int: c,
    array [int] of var int: bin,
    array [int] of int: w)

Requires that each item \( i \) with weight \( w[i] \), be put into \( bin[i] \) such that the sum of the weights of the items in each bin \( b \) does not exceed the capacity \( c[b] \).

Assumptions:
- \( \forall i, w[i] \geq 0 \)
- \( \forall b, c[b] \geq 0 \)

predicate bin_packing_load(array [int] of var int: load,
    array [int] of var int: bin,
    array [int] of int: w)

Requires that each item \( i \) with weight \( w[i] \), be put into \( bin[i] \) such that the sum of the weights of the items in each bin \( b \) is equal to \( load[b] \).

Assumptions:
- \( \forall i, w[i] \geq 0 \)

function array [int] of var int: bin_packing_load(array [int] of var int: \_bin,
    array [int] of int: w)
Returns the load of each bin resulting from packing each item \( i \) with weight \( w[i] \) into bin \( b[i] \), where the load is defined as the sum of the weights of the items in each bin.

Assumptions:

- \( \forall i, w[i] \geq 0 \)

```plaintext
predicate diffn(array [int] of var int: x,
                array [int] of var int: y,
                array [int] of var int: dx,
                array [int] of var int: dy)
```

Constrains rectangles \( i \), given by their origins \((x[i], y[i])\) and sizes \((dx[i], dy[i])\), to be non-overlapping. Zero-width rectangles can still not overlap with any other rectangle.

```plaintext
predicate diffn_k(array [int,int] of var int: box_posn,
                   array [int,int] of var int: box_size)
```

Constrains \( k \)-dimensional boxes to be non-overlapping. For each box \( i \) and dimension \( j \), \( box\_posn[i,j] \) is the base position of the box in dimension \( j \), and \( box\_size[i,j] \) is the size in that dimension. Boxes whose size is 0 in any dimension still cannot overlap with any other box.

```plaintext
predicate diffn_nonstrict(array [int] of var int: x,
                          array [int] of var int: y,
                          array [int] of var int: dx,
                          array [int] of var int: dy)
```

Constrains rectangles \( i \), given by their origins \((x[i], y[i])\) and sizes \((dx[i], dy[i])\), to be non-overlapping. Zero-width rectangles can be packed anywhere.

```plaintext
predicate diffn_nonstrict_k(array [int,int] of var int: box_posn,
                             array [int,int] of var int: box_size)
```

Constrains \( k \)-dimensional boxes to be non-overlapping. For each box \( i \) and dimension \( j \), \( box\_posn[i,j] \) is the base position of the box in dimension \( j \), and \( box\_size[i,j] \) is the size in that dimension. Boxes whose size is 0 in at least one dimension can be packed anywhere.
predicate geost(int: k,
    array [int,int] of int: rect_size,
    array [int,int] of int: rect_offset,
    array [int] of set of int: shape,
    array [int,int] of var int: x,
    array [int] of var int: kind)

A global non-overlap constraint for k dimensional objects. It enforces that no two objects overlap.

Parameters:

• k: the number of dimensions

• rect_size: the size of each box in k dimensions

• rect_offset: the offset of each box from the base position in k dimensions

• shape: the set of rectangles defining the i-th shape.

• x: the base position of each object. x[i,j] is the position of object i in dimension j.

• kind: the shape used by each object.

predicate geost_bb(int: k,
    array [int,int] of int: rect_size,
    array [int,int] of int: rect_offset,
    array [int] of set of int: shape,
    array [int,int] of var int: x,
    array [int] of var int: kind,
    array [int] of var int: l,
    array [int] of var int: u)

A global non-overlap constraint for k dimensional objects. It enforces that no two objects overlap, and that all objects fit within a global k dimensional bounding box.

Parameters:

• k: the number of dimensions

• rect_size: the size of each box in k dimensions

• rect_offset: the offset of each box from the base position in k dimensions

• shape: the set of rectangles defining the i-th shape.
• \( x \): the base position of each object. \( x[i, j] \) is the position of object \( i \) in dimension \( j \).
• kind: the shape used by each object.
• \( l \): is an array of lower bounds, \( l[i] \) is the minimum bounding box for all objects in dimension \( i \).
• \( u \): is an array of upper bounds, \( u[i] \) is the maximum bounding box for all objects in dimension \( i \).

```plaintext
predicate geost_nonoverlap_k(array [int] of var int: x1,
                             array [int] of int: w1,
                             array [int] of var int: x2,
                             array [int] of int: w2)
```

A global non-overlap constraint for 2 dimensional objects. It enforces that no two objects overlap and zero-length objects do not appear in the middle of other objects.

Parameters:

• \( x1 \): first coordinate of each object
• \( w2 \): width in first dimension for each object
• \( x2 \): second coordinate of each object
• \( w2 \): width in second dimension for each object

```plaintext
predicate geost_smallest_bb(int: k,
                            array [int,int] of int: rect_size,
                            array [int,int] of int: rect_offset,
                            array [int] of set of int: shape,
                            array [int,int] of var int: x,
                            array [int] of var int: kind,
                            array [int] of var int: l,
                            array [int] of var int: u)
```

A global non-overlap constraint for \( k \) dimensional objects. It enforces that no two objects overlap, and that all objects fit within a global \( k \) dimensional bounding box. In addition, it enforces that the bounding box is the smallest one containing all objects, i.e., each of the \( 2k \) boundaries is touched by at least by one object.

Parameters:
• $k$: the number of dimensions

• rect_size: the size of each box in $k$ dimensions

• rect_offset: the offset of each box from the base position in $k$ dimensions

• shape: the set of rectangles defining the $i$-th shape.

• $x$: the base position of each object. $x[i, j]$ is the position of object $i$ in dimension $j$.

• kind: the shape used by each object.

• $l$: is an array of lower bounds, $l[i]$ is the minimum bounding box for all objects in dimension $i$.

• $u$: is an array of upper bounds, $u[i]$ is the maximum bounding box for all objects in dimension $i$.

```minizinc
predicate knapsack(array [int] of int: w,
    array [int] of int: p,
    array [int] of var int: x,
    var int: W,
    var int: P)
```

Requires that items are packed in a knapsack with certain weight and profit restrictions.

Assumptions:

• Weights $w$ and profits $p$ must be non-negative

• $w$, $p$ and $x$ must have the same index sets

Parameters:

• $w$: weight of each type of item

• $p$: profit of each type of item

• $x$: number of items of each type that are packed

• $W$: sum of sizes of all items in the knapsack

• $P$: sum of profits of all items in the knapsack

4.2.6.7 Scheduling constraints
predicate alternative(var opt int: s0,  
var int: d0,  
array [int] of var opt int: s,  
array [int] of var int: d)

Alternative constraint for optional tasks. Task (s0, d0) spans the optional tasks (s[i], d[i]) in the array arguments and at most one can occur.

predicate cumulative(array [int] of var int: s,  
array [int] of var int: d,  
array [int] of var int: r,  
var int: b)

Requires that a set of tasks given by start times s, durations d, and resource requirements r, never require more than a global resource bound b at any one time.

Assumptions:

- forall i, d[i] >= 0 and r[i] >= 0

predicate cumulative(array [int] of var opt int: s,  
array [int] of var int: d,  
array [int] of var int: r,  
var int: b)

Requires that a set of tasks given by start times s, durations d, and resource requirements r, never require more than a global resource bound b at any one time. Start times are optional variables, so that absent tasks do not need to be scheduled.

Assumptions:

- forall i, d[i] >= 0 and r[i] >= 0

predicate disjunctive(array [int] of var int: s,  
array [int] of var int: d)

Requires that a set of tasks given by start times s and durations d do not overlap in time. Tasks with duration 0 can be scheduled at any time, even in the middle of other tasks.

Assumptions:

- forall i, d[i] >= 0
**predicate disjunctive**

```plaintext
disjunctive(array [int] of var opt int: s,
            array [int] of var int: d)
```

Requires that a set of tasks given by start times `s` and durations `d` do not overlap in time. Tasks with duration 0 can be scheduled at any time, even in the middle of other tasks. Start times are optional variables, so that absent tasks do not need to be scheduled.

Assumptions:

- `forall i, d[i] >= 0`

**predicate disjunctive_strict**

```plaintext
disjunctive_strict(array [int] of var int: s,
                    array [int] of var int: d)
```

Requires that a set of tasks given by start times `s` and durations `d` do not overlap in time. Tasks with duration 0 CANNOT be scheduled at any time, but only when no other task is running.

Assumptions:

- `forall i, d[i] >= 0`

**predicate disjunctive_strict**

```plaintext
disjunctive_strict(array [int] of var opt int: s,
                    array [int] of var int: d)
```

Requires that a set of tasks given by start times `s` and durations `d` do not overlap in time. Tasks with duration 0 CANNOT be scheduled at any time, but only when no other task is running. Start times are optional variables, so that absent tasks do not need to be scheduled.

Assumptions:

- `forall i, d[i] >= 0`

**predicate span**

```plaintext
span(var opt int: s0,
     var int: d0,
     array [int] of var opt int: s,
     array [int] of var int: d)
```

Span constraint for optional tasks. Task `(s0, d0)` spans the optional tasks `(s[i], d[i])` in the array arguments.
4.2.6.8 Graph constraints

predicate bounded_dpath(int: N,
    int: E,
    array [int] of int: from,
    array [int] of int: to,
    array [int] of int: w,
    var int: s,
    var int: t,
    array [int] of var bool: ns,
    array [int] of var bool: es,
    var int: K)

Constrains the subgraph ns and es of a given directed graph to be a path from s to t of weight K.

Parameters:

• N: the number of nodes in the given graph
• E: the number of edges in the given graph
• from: the leaving node 1.. N for each edge
• to: the entering node 1.. N for each edge
• w: the weight of each edge
• s: the source node (which may be variable)
• t: the dest node (which may be variable)
• ns: a Boolean for each node whether it is in the subgraph
• es: a Boolean for each edge whether it is in the subgraph
• K: the cost of the path
Constrains the subgraph $ns$ and $es$ of a given directed graph to be a path from $s$ to $t$ of weight $K$.

Parameters:

- `from`: the leaving node for each edge
- `to`: the entering node for each edge
- `w`: the weight of each edge
- `s`: the source node (which may be variable)
- `t`: the dest node (which may be variable)
- `ns`: a Boolean for each node whether it is in the subgraph
- `es`: a Boolean for each edge whether it is in the subgraph
- `K`: the cost of the path

Constrains the subgraph $ns$ and $es$ of a given undirected graph to be a path from $s$ to $t$ of weight $K$.

Parameters:

- `N`: the number of nodes in the given graph
- `E`: the number of edges in the given graph

4.2.6. Global constraints
• from: the leaving node 1.. N for each edge
• to: the entering node 1.. N for each edge
• w: the weight of each edge
• s: the source node (which may be variable)
• t: the dest node (which may be variable)
• ns: a Boolean for each node whether it is in the subgraph
• es: a Boolean for each edge whether it is in the subgraph
• K: the cost of the path

```mini
predicate bounded_path(array [int] of $N: from,
    array [int] of $N: to,
    array [int] of int: w,
    var $N: s,
    var $N: t,
    array [$$N] of var bool: ns,
    array [int] of var bool: es,
    var int: K)
```

Constrains the subgraph ns and es of a given undirected graph to be a path from s to t of weight K.

Parameters:

• from: the leaving node for each edge
• to: the entering node for each edge
• w: the weight of each edge
• s: the source node (which may be variable)
• t: the dest node (which may be variable)
• ns: a Boolean for each node whether it is in the subgraph
• es: a Boolean for each edge whether it is in the subgraph
• K: the cost of the path
Constrains the subgraph \( ns \) and \( es \) of a given undirected graph to be connected.

Parameters:

- \( \text{from} \): is the leaving node for each edge
- \( \text{to} \): is the entering node for each edge
- \( ns \): is a Boolean for each node whether it is in the subgraph
- \( es \): is a Boolean for each edge whether it is in the subgraph

Constrains the set of edges \( es \) of a given directed graph to be a weighted spanning tree rooted at \( r \) of weight \( W \).

Parameters:

- \( N \): the number of nodes in the given graph
- \( E \): the number of edges in the given graph
- \( \text{from} \): the leaving node 1.. \( N \) for each edge
- \( \text{to} \): the entering node 1.. \( N \) for each edge
- \( w \): the weight of each edge
- \( r \): the root node (which may be variable)
- \( es \): a Boolean for each edge whether it is in the subgraph
• $K$: the weight of the tree

```plaintext
def predicate dag(array [int] of $N$: from,
    array [int] of $N$: to,
    array [$N$] of var bool: ns,
    array [int] of var bool: es)
```

Constrains the subgraph $ns$ and $es$ of a given directed graph to be a DAG.

Parameters:

• `from`: the leaving node for each edge
• `to`: the entering node for each edge
• `ns`: a Boolean for each node whether it is in the subgraph
• `es`: a Boolean for each edge whether it is in the subgraph

```plaintext
def predicate dconnected(array [int] of $N$: from,
    array [int] of $N$: to,
    array [$N$] of var bool: ns,
    array [int] of var bool: es)
```

Constrains the subgraph $ns$ and $es$ of a given directed graph to be connected.

Parameters:

• `from`: the leaving node for each edge
• `to`: the entering node for each edge
• `ns`: a Boolean for each node whether it is in the subgraph
• `es`: a Boolean for each edge whether it is in the subgraph

```plaintext
def predicate dpath(int: N,
    int: E,
    array [int] of int: from,
    array [int] of int: to,
    var int: s,
    var int: t,
    array [int] of var bool: ns,
```


Constrains the subgraph \( ns \) and \( es \) of a given directed graph to be a path from \( s \) to \( t \).

Parameters:

- \( N \): the number of nodes in the given graph
- \( E \): the number of edges in the given graph
- \( \text{from} \): the leaving node 1.. \( N \) for each edge
- \( \text{to} \): the entering node 1.. \( N \) for each edge
- \( s \): the source node (which may be variable)
- \( t \): the dest node (which may be variable)
- \( ns \): a Boolean for each node whether it is in the subgraph
- \( es \): a Boolean for each edge whether it is in the subgraph

```minizinc
dpath(array [int] of var bool: ns, array [int] of var bool: es)
```

Constrains the subgraph \( ns \) and \( es \) of a given directed graph to be a path from \( s \) to \( t \).

Parameters:

- \( \text{from} \): the leaving node for each edge
- \( \text{to} \): the entering node for each edge
- \( s \): the source node (which may be variable)
- \( t \): the dest node (which may be variable)
- \( ns \): a Boolean for each node whether it is in the subgraph
- \( es \): a Boolean for each edge whether it is in the subgraph
predicate dreachable(int: N,
    int: E,
    array [int] of int: from,
    array [int] of int: to,
    var int: r,
    array [int] of var bool: ns,
    array [int] of var bool: es)

Constrains the subgraph ns and es of a given directed graph to be reachable from r.

Parameters:

• N: the number of nodes in the given graph
• E: the number of edges in the given graph
• from: the leaving node 1.. N for each edge
• to: the entering node 1.. N for each edge
• r: the root node (which may be variable)
• ns: a Boolean for each node whether it is in the subgraph
• es: a Boolean for each edge whether it is in the subgraph

predicate dreachable(array [int] of $N: from,
                        array [int] of $N: to,
                        var $N: r,
                        array [$N] of var bool: ns,
                        array [int] of var bool: es)

Constrains the subgraph ns and es of a given directed graph to be reachable from r.

Parameters:

• from: the leaving node for each edge
• to: the entering node for each edge
• r: the root node (which may be variable)
• ns: a Boolean for each node whether it is in the subgraph
• es: a Boolean for each edge whether it is in the subgraph
predicate dsteiner(int: N,
    int: E,
    array [int] of int: from,
    array [int] of int: to,
    array [int] of int: w,
    var int: r,
    array [int] of var bool: ns,
    array [int] of var bool: es,
    var int: K)

Constrains the subgraph ns and es of a given directed graph to be a weighted spanning tree rooted at r of weight W.

Parameters:

• N: the number of nodes in the given graph
• E: the number of edges in the given graph
• from: the leaving node 1.. N for each edge
• to: the entering node 1.. N for each edge
• w: the weight of each edge
• r: the root node (which may be variable)
• ns: a Boolean for each node whether it is in the subgraph
• es: a Boolean for each edge whether it is in the subgraph
• K: the weight of the tree

predicate dtree(int: N,
    int: E,
    array [int] of int: from,
    array [int] of int: to,
    var int: r,
    array [int] of var bool: ns,
    array [int] of var bool: es)

Constrains the subgraph ns and es of a given directed graph to be a tree rooted at r.

Parameters:
• $N$: the number of nodes in the given graph
• $E$: the number of edges in the given graph
• from: the leaving node $1..N$ for each edge
• to: the entering node $1..N$ for each edge
• $r$: the root node (which may be variable)
• $ns$: a Boolean for each node whether it is in the subgraph
• $es$: a Boolean for each edge whether it is in the subgraph

```
predicate dtree(array [int] of $N$: from,
                 array [int] of $N$: to,
                 var $N$: r,
                 array [$N$] of var bool: ns,
                 array [int] of var bool: es)
```

Constrains the subgraph $ns$ and $es$ of a given directed graph to be at tree rooted at $r$.

Parameters:
• from: the leaving node for each edge
• to: the entering node for each edge
• r: the root node (which may be variable)
• ns: a Boolean for each node whether it is in the subgraph
• es: a Boolean for each edge whether it is in the subgraph

```
predicate path(int: N,
               int: E,
               array [int] of int: from,
               array [int] of int: to,
               var int: s,
               var int: t,
               array [int] of var bool: ns,
               array [int] of var bool: es)
```

Constrains the subgraph $ns$ and $es$ of a given undirected graph to be a path from $s$ to $t$. 
Parameters:

- \( N \): the number of nodes in the given graph
- \( E \): the number of edges in the given graph
- \( \text{from} \): the leaving node 1.. \( N \) for each edge
- \( \text{to} \): the entering node 1.. \( N \) for each edge
- \( s \): the source node (which may be variable)
- \( t \): the dest node (which may be variable)
- \( \text{ns} \): a Boolean for each node whether it is in the subgraph
- \( \text{es} \): a Boolean for each edge whether it is in the subgraph

```minizinc
define predicate path(array [int] of $\$N$: from,
                      array [int] of $\$N$: to,
                      var $\$N$: s,
                      var $\$N$: t,
                      array [$\$N$] of var bool: ns,
                      array [int] of var bool: es)
```

Constrains the subgraph \( \text{ns} \) and \( \text{es} \) of a given undirected graph to be a path from \( s \) to \( t \).

Parameters:

- \( \text{from} \): the leaving node for each edge
- \( \text{to} \): the entering node for each edge
- \( s \): the source node (which may be variable)
- \( t \): the dest node (which may be variable)
- \( \text{ns} \): a Boolean for each node whether it is in the subgraph
- \( \text{es} \): a Boolean for each edge whether it is in the subgraph

```minizinc
define predicate reachable(int: N,
                          int: E,
                          array [int] of int: from,
                          array [int] of int: to,
                          var int: r,
```

4.2.6. Global constraints
Constrains the subgraph $ns$ and $es$ of a given undirected graph to be reachable from $r$.

Parameters:

- $N$: the number of nodes in the given graph
- $E$: the number of edges in the given graph
- $from$: the leaving node 1.. $N$ for each edge
- $to$: the entering node 1.. $N$ for each edge
- $r$: the root node (which may be variable)
- $ns$: a Boolean for each node whether it is in the subgraph
- $es$: a Boolean for each edge whether it is in the subgraph

```plaintext
predicate reachable(array [int] of var bool: from,
                     array [int] of var bool: to,
                     var $N$: r,
                     array [$N$] of var bool: ns,
                     array [int] of var bool: es)
```

Constrains the subgraph $ns$ and $es$ of a given undirected graph to be reachable from $r$.

Parameters:

- $from$: the leaving node for each edge
- $to$: the entering node for each edge
- $r$: the root node (which may be variable)
- $ns$: a Boolean for each node whether it is in the subgraph
- $es$: a Boolean for each edge whether it is in the subgraph

```plaintext
predicate steiner(int: N,
                   int: E,
                   array [int] of int: from,
                   array [int] of int: to,
                   ...
```


array [int] of int: w,
array [int] of var bool: ns,
array [int] of var bool: es,
var int: K)

Constrains the set of edges es of a given undirected graph to be a weighted spanning tree of weight W.

Parameters:

• N: the number of nodes in the given graph
• E: the number of edges in the given graph
• from: the leaving node 1.. N for each edge
• to: the entering node 1.. N for each edge
• w: the weight of each edge
• ns: a Boolean for each node whether it is in the subgraph
• es: a Boolean for each edge whether it is in the subgraph
• K: the weight of the tree

predicate subgraph(int: N,
                     int: E,
                     array [int] of int: from,
                     array [int] of int: to,
                     array [int] of var bool: ns,
                     array [int] of var bool: es)

Constrains that ns and es is a subgraph of a given directed graph.

Parameters:

• N: the number of nodes in the given graph
• E: the number of edges in the given graph
• from: the leaving node 1.. N for each edge
• to: the entering node 1.. N for each edge
• ns: a Boolean for each node whether it is in the subgraph

• es: a Boolean for each edge whether it is in the subgraph

```plaintext
predicate subgraph(array [int] of $N$: from,
            array [int] of $N$: to,
            array [$N$] of var bool: ns,
            array [int] of var bool: es)
```

Constrains that ns and es is a subgraph of a given directed graph.

Parameters:

• from: the leaving node for each edge

• to: the entering node for each edge

• ns: a Boolean for each node whether it is in the subgraph

• es: a Boolean for each edge whether it is in the subgraph

```plaintext
predicate tree(int: N,
            int: E,
            array [int] of int: from,
            array [int] of int: to,
            var int: r,
            array [int] of var bool: ns,
            array [int] of var bool: es)
```

Constrains the subgraph ns and es of a given undirected graph to be a tree rooted at r.

Parameters:

• N: the number of nodes in the given graph

• E: the number of edges in the given graph

• from: the leaving node 1.. N for each edge

• to: the entering node 1.. N for each edge

• r: the root node (which may be variable)

• ns: a Boolean for each node whether it is in the subgraph

• es: a Boolean for each edge whether it is in the subgraph
predicate tree(array [int] of $\mathbb{N}$: from,
    array [int] of $\mathbb{N}$: to,
    var $\mathbb{N}$: r,
    array [$\mathbb{N}$] of var bool: ns,
    array [int] of var bool: es)

Constrains the subgraph ns and es of a given undirected graph to be at tree rooted at r.

Parameters:

- from: the leaving node for each edge
- to: the entering node for each edge
- r: the root node (which may be variable)
- ns: a Boolean for each node whether it is in the subgraph
- es: a Boolean for each edge whether it is in the subgraph

predicate weighted_spanning_tree(int: N,
    int: E,
    array [int] of int: from,
    array [int] of int: to,
    array [int] of int: w,
    array [int] of var bool: es,
    var int: K)

Constrains the set of edges es of a given undirected graph to be a weighted spanning tree of weight W.

Parameters:

- N: the number of nodes in the given graph
- E: the number of edges in the given graph
- from: the leaving node 1.. N for each edge
- to: the entering node 1.. N for each edge
- w: the weight of each edge
• es: a Boolean for each edge whether it is in the subgraph

• K: the weight of the tree

4.2.6.9 Extensional constraints (table, regular etc.)

```plaintext
predicate cost_mdd(array [int] of var int: x,
  int: N,
  array [int] of int: level,
  int: E,
  array [int] of int: from,
  array [int] of set of int: label,
  array [int] of int: cost,
  array [int] of int: to,
  var int: totalcost)
```

Requires that x defines a path in the cost MDD with total edge weight totalcost.

Parameters:

• N: the number of nodes, the root node is node 1

• level: the level of each node, the root is level 1, T is level length (x)+1

• E: the number of edges

• from: the leaving node (1.. N) for each edge

• label: the set of value of the x variable for each edge

• cost: the cost for each edge

• to: the entering node for each edge, where 0 = T node

• totalcost: the total cost of the path defined by x

```plaintext
predicate cost_regular(array [int] of var int: x,
  int: Q,
  int: S,
  array [int,int] of int: d,
  int: q0,
  set of int: F,
```

Chapter 4.2. The MiniZinc library
The sequence of values in array $x$ (which must all be in the range $1..S$) is accepted by the DFA of $Q$ states with input $1..S$ and transition function $d$ (which maps $(1..Q,1..S) \rightarrow 0..Q$) and initial state $q_0$ (which must be in $1..Q$) and accepting states $F$ (which all must be in $1..Q$). We reserve state $0$ to be an always failing state. Each edge has an associated cost $c$, and $C$ is the sum of costs taken on the accepting path for $x$.


Requires that $x$ defines a path from root to true node $T$ through the (deterministic) MDD defined by

The MDD must be deterministic, i.e., there cannot be two edges with the same label leaving the same node.

Parameters:

- $N$: the number of nodes, the root node is node 1
- $level$: the level of each node, the root is level 1, $T$ is level $\text{length}(x)+1$
- $E$: the number of edges
- $from$: the leaving node ($1..N$) for each edge
- $label$: the set of values of the $x$ variable for each edge
- $to$: the entering node for each edge, where $0 = T$ node

array [int] of set of int: label,
array [int] of int: to

Requires that \( x \) defines a path from root to true node \( T \) through the (nondeterministic) MDD defined by

The MDD can be nondeterministic, i.e., there can be two edges with the same label leaving the same node.

Parameters:

- \( N \): the number of nodes, the root node is node 1
- \( \text{level} \): the level of each node, the root is level 1, \( T \) is level length \( (x) + 1 \)
- \( E \): the number of edges
- \( \text{from} \): the leaving node (1.. \( N \)) for each edge
- \( \text{label} \): the set of values of the \( x \) variable for each edge
- \( \text{to} \): the entering node for each edge, where 0 = \( T \) node

predicate regular(array [int] of var int: x,
int: Q,
int: S,
array [int,int] of int: d,
int: q0,
set of int: F)

The sequence of values in array \( x \) (which must all be in the range 1.. \( S \)) is accepted by the DFA of \( Q \) states with input 1.. \( S \) and transition function \( d \) (which maps (1.. \( Q \), 1.. \( S \)) \(-> 0.. \( Q \)) and initial state \( q0 \) (which must be in 1.. \( Q \)) and accepting states \( F \) (which all must be in 1.. \( Q \)). We reserve state 0 to be an always failing state.

predicate regular(array [int] of var int: x,
int: Q,
set of int: S,
array [int,int] of int: d,
int: q0,
set of int: F)
The sequence of values in array \( x \) (which must all be in the set \( S \)) is accepted by the DFA of \( Q \) states with input \( S \) and transition function \( d \) (which maps \((1.. Q, S) \rightarrow 0.. Q\)) and initial state \( q_0 \) (which must be in \( 1.. Q \)) and accepting states \( F \) (which all must be in \( 1.. Q \)). We reserve state 0 to be an always failing state.

\[
\text{predicate regular(array [int] of var int: x, string: r)}
\]

The sequence of values in array \( x \) is accepted by the regular expression \( r \). This constraint generates it’s DFA equivalent.

Regular expressions can use the following syntax:

- **Selection:**
  - Concatenation: “12 34”, 12 followed by 34. (Characters are assumed to be the part of the same number unless split by syntax or whitespace.)
  - Union: “7|11”, a 7 or 11.
  - Groups: “7(6|8)”, a 7 followed by a 6 or an 8.
  - Wildcard: “.”, any value within the domain.
  - Classes: “[3-6 7]”, a 3,4,5,6, or 7.
  - Negated classes: “[^3 5]”, any value within the domain except for a 3 or a 5.

- **Quantifiers:**
  - Asterisk: “12*”, 0 or more times a 12.
  - Question mark: “5?”, 0 or 1 times a 5. (optional)
  - Plus sign: “42+”, 1 or more time a 42.
  - Exact: “1{3}”, exactly 3 times a 1.
  - At least: “9{5,}”, 5 or more times a 9.
  - Between: “7{3,5}”, at least 3 times, but at most 5 times a 7.

Members of enumerated types can be used in place of any integer (e.g., “A B”, A followed by B). Enumerated identifiers still use whitespace for concatenation.
The sequence of values in array $x$ (which must all be in the range $1..S$) is accepted by the NFA of $Q$ states with input $1..S$ and transition function $d$ (which maps $(1..Q,1..S)\rightarrow\text{set of }1..Q$) and initial state $q\emptyset$ (which must be in $1..Q$) and accepting states $F$ (which all must be in $1..Q$).

```plaintext
predicate regular_nfa(array [int] of var int: x,
                     int: Q,
                     set of int: S,
                     array [int,int] of set of int: d,
                     int: q\emptyset,
                     set of int: F)
```

The sequence of values in array $x$ (which must all be in the range $S$) is accepted by the NFA of $Q$ states with input $S$ and transition function $d$ (which maps $(1..Q,S)\rightarrow\text{set of }1..Q$) and initial state $q\emptyset$ (which must be in $1..Q$) and accepting states $F$ (which all must be in $1..Q$).

```plaintext
predicate regular_nfa(array [int] of var int: x,
                     int: Q,
                     set of int: S,
                     array [int,int] of set of int: d,
                     int: q\emptyset,
                     set of int: F)
```

Represents the constraint $x$ in $t$ where we consider each row in $t$ to be a tuple and $t$ as a set of tuples.

```plaintext
predicate table(array [int] of var bool: x, array [int,int] of bool: t)
```

Represents the constraint $x$ in $t$ where we consider each row in $t$ to be a tuple and $t$ as a set of tuples.

```plaintext
predicate table(array [int] of var int: x, array [int,int] of int: t)
```

4.2.6.10 Machine learning constraints

```plaintext
predicate neural_net(array [int] of var float: inputs,
                      array [int] of int: input_ids,
                      array [int] of var float: outputs,
                      array [int] of int: output_ids,
```

Chapter 4.2. The MiniZinc library
constrain the output layer of a neural net to take the value defined by the input layer. the arguments are inputs: an array of float variables input_ids: array[int] of node outputs: an array of float variables output_ids: array[int] of node bias: array[node] of float edge_weight: array[edge] of float (dummy one at end!) edge_parent: array[edge] of neuron (start neuron for edge) first_edge: array[node] of 1..m+1 neuron_type: {NT_RELU, NT_STEP, NT_LINEAR, NT_SOFTPLUS }

4.2.6.11 Deprecated constraints

predicate at_least(int: n, array [int] of var int: x, int: v)

Requires at least \( n \) variables in \( x \) to take the value \( v \).

This constraint is deprecated. Use \( \text{count}(i \in x)(i=v) \geq n \) instead.

predicate at_most(int: n, array [int] of var int: x, int: v)

Requires at most \( n \) variables in \( x \) to take the value \( v \).

This constraint is deprecated. Use \( \text{count}(i \in x)(i=v) \leq n \) instead.

predicate exactly(int: n, array [int] of var int: x, int: v)

Requires exactly \( n \) variables in \( x \) to take the value \( v \).

This constraint is deprecated. Use \( \text{count}(i \in x)(i=v) = n \) instead.

4.2.6.12 Other declarations
**MiniZinc Handbook, Release 2.4.3**

**function** var \$E: arg_max(array \[\$E\] of var int: x)

Returns the index of the maximum value in the array \(x\). When breaking ties the least index is returned.

**function** var \$E: arg_max(array \[\$E\] of var bool: x)

Returns the index of the maximum value in the array \(x\). When breaking ties the least index is returned.

**function** var \$E: arg_max(array \[\$E\] of var float: x)

Returns the index of the maximum value in the array \(x\). When breaking ties the least index is returned.

**function** var \$E: arg_min(array \[\$E\] of var int: x)

Returns the index of the minimum value in the array \(x\). When breaking ties the least index is returned.

**function** var \$E: arg_min(array \[\$E\] of var bool: x)

Returns the index of the minimum value in the array \(x\). When breaking ties the least index is returned.

**function** var \$E: arg_min(array \[\$E\] of var float: x)

Returns the index of the minimum value in the array \(x\). When breaking ties the least index is returned.

**predicate** circuit(array \[int\] of var int: x)

Constrains the elements of \(x\) to define a circuit where \(x[i] = j\) means that \(j\) is the successor of \(i\).

**predicate** disjoint(var set of int: s1, var set of int: s2)

Requires that sets \(s1\) and \(s2\) do not intersect.

**predicate** maximum(var int: m, array \[int\] of var int: x)
Constrains \( m \) to be the maximum of the values in \( x \).

Assumptions: \( |x| > 0 \).

\[
\text{predicate maximum(var float: } m, \text{ array [int] of var float: } x)\]

Constrains \( m \) to be the maximum of the values in \( x \).

Assumptions: \( |x| > 0 \).

\[
\text{predicate maximum_arg(array [int] of var int: } x, \text{ var int: } i)\]

Constrain \( i \) to be the index of the maximum value in the array \( x \). When breaking ties the least index is returned.

Assumption: \( |x| > 0 \)

\[
\text{predicate maximum_arg(array [int] of var bool: } x, \text{ var int: } i)\]

Constrain \( i \) to be the index of the maximum value in the array \( x \). When breaking ties the least index is returned.

Assumption: \( |x| > 0 \)

\[
\text{predicate maximum_arg(array [int] of var float: } x, \text{ var int: } i)\]

Constrain \( i \) to be the index of the maximum value in the array \( x \). When breaking ties the least index is returned.

Assumption: \( |x| > 0 \)

\[
\text{predicate member(array [int] of var bool: } x, \text{ var bool: } y)\]

Requires that \( y \) occurs in the array \( x \).

\[
\text{predicate member(array [int] of var float: } x, \text{ var float: } y)\]

Requires that \( y \) occurs in the array \( x \).
\begin{verbatim}
predicate member(array [int] of var int: x, var int: y)
\end{verbatim}
Requires that \( y \) occurs in the array \( x \).

\begin{verbatim}
predicate member(array [int] of var set of int: x, var set of int: y)
\end{verbatim}
Requires that \( y \) occurs in the array \( x \).

\begin{verbatim}
predicate member(var set of int: x, var int: y)
\end{verbatim}
Requires that \( y \) occurs in the set \( x \).

\begin{verbatim}
predicate minimum(var float: m, array [int] of var float: x)
\end{verbatim}
Constrains \( m \) to be the minimum of the values in \( x \).
Assumptions: \( |x| > 0 \).

\begin{verbatim}
predicate minimum(var int: m, array [int] of var int: x)
\end{verbatim}
Constrains \( m \) to be the minimum of the values in \( x \).
Assumptions: \( |x| > 0 \).

\begin{verbatim}
predicate minimum_arg(array [int] of var int: x, var int: i)
\end{verbatim}
Constrain \( i \) to be the index of the minimum value in the array \( x \). When breaking ties the least index is returned.
Assumption: \( |x| > 0 \)

\begin{verbatim}
predicate minimum_arg(array [int] of var bool: x, var int: i)
\end{verbatim}
Constrain \( i \) to be the index of the minimum value in the array \( x \). When breaking ties the least index is returned.
Assumption: \( |x| > 0 \)
predicate minimum_arg(array [int] of var float: x, var int: i)

Constrain i to be the index of the minimum value in the array x. When breaking ties the least index is returned.

Assumption: | x | > 0

predicate network_flow(array [int,1..2] of int: arc,
array [int] of int: balance,
array [int] of var int: flow)

Defines a network flow constraint.

Parameters:

- arc: a directed arc of the flow network. Arc i connects node arc [i,1] to node arc [i,2].
- balance: the difference between input and output flow for each node.
- flow: the flow going through each arc.

predicate network_flow_cost(array [int,1..2] of int: arc,
array [int] of int: balance,
array [int] of int: weight,
array [int] of var int: flow,
var int: cost)

Defines a network flow constraint with cost.

Parameters:

- arc: a directed arc of the flow network. Arc i connects node arc [i,1] to node arc [i,2].
- balance: the difference between input and output flow for each node.
- weight: the unit cost of the flow through the arc.
- flow: the flow going through each arc.
- cost: the overall cost of the flow.
**predicate** partition_set(array [int] of var set of int: S,  
set of int: universe)

Constrains the sets in array S to partition the universe.

**function** var float: piecewise_linear(var float: x,  
array [int] of float: xi,  
array [int] of float: vi)

Return a piecewise-linear interpolation of the given point sequence as a function of x

**predicate** piecewise_linear(var float: x,  
var float: y,  
array [int] of float: xi,  
array [int] of float: vi)

Constrains y (x) to be a piecewise-linear interpolation of the provided point sequence.

**predicate** range(array [int] of var int: x,  
var set of int: s,  
var set of int: t)

Requires that the image of function x (represented as an array) on set of values s is t. ub(s) must be a subset of index_set(x) otherwise an assertion failure will occur.

**function** var set of int: range(array [int] of var int: x,  
var set of int: s)

Returns the image of function x (represented as an array) on set of values s. ub(s) must be a subset of index_set(x) otherwise an assertion failure will occur.

**predicate** roots(array [int] of var int: x,  
var set of int: s,  
var set of int: t)

Requires that x[i] in t for all i in s
function var set of int: roots(array [int] of var int: x,
    var set of int: t)

Returns s such that x[i] in t for all i in s

predicate sliding_sum(int: low,
    int: up,
    int: seq,
    array [int] of var int: vs)

Requires that in each subsequence vs[i], ..., vs[i + seq - 1] the sum of the values belongs to the interval [low, up].

predicate subcircuit(array [int] of var int: x)

Constrains the elements of x to define a subcircuit where x[i] = j means that j is the successor of i and x[i] = i means that i is not in the circuit.

predicate sum_pred(var int: i,
    array [int] of set of int: sets,
    array [int] of int: cs,
    var int: s)

Requires that the sum of cs[i1..cs[iN]] equals s, where i1..iN are the elements of the ith set in sets.

Nb: not called ‘sum’ as in the constraints catalog because ‘sum’ is a MiniZinc built-in function.

predicate sum_set(array [int] of int: vs,
    array [int] of int: ws,
    var set of int: x,
    var int: s)

Requires that the sum of the weights ws[i1..ws[iN]] equals s, where vs[i1..vs[iN]] are the elements appearing in set x.
4.2.7 Additional declarations for Gecode

These annotations and predicates are available for the Gecode solver. In order to use them in a model, include the file “gecode.mzn”.

4.2.7.1 Additional Gecode search annotations

Parameters

- `annotation activity_max`
  Select variable with largest activity count

- `annotation activity_min`
  Select variable with smallest activity count

- `annotation activity_size_max`
  Select variable with largest activity count divided by domain size

- `annotation activity_size_min`
  Select variable with smallest activity count divided by domain size

- `annotation afc_max`
  Select variable with largest accumulated failure count

- `annotation afc_min`
  Select variable with smallest accumulated failure count

- `annotation afc_size_max`
  Select variable with smallest accumulated failure count
Select variable with largest accumulated failure count divided by domain size

```
annotationafc_size_min
```

Select variable with smallest accumulated failure count divided by domain size

```
annotationrandom
```

Select random variable

**Functions and Predicates**

```
annotationbool_default_search(ann: varsel, ann: valsel)
```

Specify default search strategy for Boolean variables to use variable selection strategy varsel, and value choice strategy valsel.

```
annotationfloat_default_search(ann: varsel, ann: valsel)
```

Specify default search strategy for float variables to use variable selection strategy varsel, and value choice strategy valsel.

```
annotationint_default_search(ann: varsel, ann: valsel)
```

Specify default search strategy for integer variables to use variable selection strategy varsel, and value choice strategy valsel.

```
annotationrelax_and_reconstruct(array[int] of var int: x,  
               int: percentage)
```

Simple large neighbourhood search strategy: upon restart, for each variable in x, the probability of it being fixed to the previous solution is percentage (out of 100).

```
annotationrelax_and_reconstruct(array[int] of var int: x,  
               int: percentage,  
               array[int] of int: y)
```
Simple large neighbourhood search strategy: upon restart, for each variable in $x$, the probability of it being fixed to the previous solution is $\text{percentage}$ (out of 100). Start from an initial solution $y$.

\[
\text{annotation set_default_search(ann: varsel, ann: valsel)}
\]

Specify default search strategy for set variables to use variable selection strategy $\text{varsel}$, and value choice strategy $\text{valsel}$.

### 4.2.7.2 Additional Gecode constraints

\[
\text{predicate among_seq(array [int] of var int: x, set of int: S, int: l, int: m, int: n)}
\]

Every subsequence of $x$ of length $l$ has at least $m$ and at most $n$ occurrences of the values in $S$

\[
\text{predicate among_seq(array [int] of var bool: x, bool: b, int: l, int: m, int: n)}
\]

Every subsequence of $x$ of length $l$ has at least $m$ and at most $n$ occurrences of the values in $S$

\[
\text{predicate circuit_cost(array [int] of int: c, array [int] of var int: x, var int: z)}
\]

Constrains the elements of $x$ to define a circuit where $x[i] = j$ means that $j$ is the successor of $i$. Additionally, constrain $z$ to be the cost of the circuit. Each edge cost is defined by array $c$.

\[
\text{predicate circuit_cost_array(array [int] of int: c, array [int] of var int: x, array [int] of var int: y)}
\]
var int: z)

Constrains the elements of \( x \) to define a circuit where \( x[i] = j \) means that \( j \) is the successor of \( i \). Additionally, constrain \( z \) to be the cost of the circuit. Each edge cost is defined by array \( c \). The variables \( y[i] \) are constrained to be the edge cost of the node \( x[i] \).

**predicate gecode_array_set_element_intersect**

\[
\text{Constrain } z \text{ to be the intersection of all sets in } y \text{ that are selected by } x : i \in z \iff \forall j \in x : (i \in y[j])
\]

**predicate gecode_array_set_element_intersect_in**

\[
\text{Constrain } z \text{ to be a subset of } u \text{, and } z \text{ to be the intersection of all sets in } y \text{ that are selected by } x : i \in z \iff \forall j \in x : (i \in y[j])
\]

**predicate gecode_array_set_element_partition**

\[
\text{Constrain } z \text{ to be the disjoint union of all sets in } y \text{ that are selected by } x : i \in z \iff \exists j \in x : (i \in y[j]) \text{ and } i \in x \land j \in x \land i \neq j \Rightarrow y[i] \cap y[j] = \emptyset
\]

### 4.2.8 Additional declarations for Chuffed

These annotations and predicates are available for the Chuffed solver. In order to use them in a model, include the file “chuffed.mzn”.

---

4.2.8. Additional declarations for Chuffed 429
4.2.8.1 Additional Chuffed search annotations

Parameters

- **annotation largest_smallest**
  Choose the variable with the largest smallest value in its domain

- **annotation random_order**
  Choose a random variable

- **annotation smallest_largest**
  Choose the variable with the smallest largest value in its domain

Functions and Predicates

- **annotation priority_search(array [int] of var int: x,
array [int] of ann: search,
ann: select,
ann: explore)**

Specify search on a group of searches search, for which variables x act as delegates for the search selection with variable selection strategy select, and exploration strategy explore.
CHAPTER 4.3

Interfacing Solvers to FlatZinc

This document describes the interface between the MiniZinc system and FlatZinc solvers.

4.3.1 Specification of FlatZinc

This document is the specification of the FlatZinc modelling language. It also includes a definition of the standard command line options a FlatZinc solver should support in order to work with the minizinc driver program (and the MiniZinc IDE).

FlatZinc is the target constraint modelling language into which MiniZinc models are translated. It is a very simple solver independent problem specification language, requiring minimal implementation effort to support.

Throughout this document: \( r_1, r_2 \) denote float literals; \( x_1, x_2, \ldots, x_k, i, j, k \) denote int literals; \( y_1, y_2, \ldots, y_k, y_i \) denote literal array elements.

4.3.1.1 Comments

Comments start with a percent sign % and extend to the end of the line. Comments can appear anywhere in a model.
4.3.1.2 Types

There are three varieties of types in FlatZinc.

- **Parameter** types apply to fixed values that are specified directly in the model.
- **Variable** types apply to values computed by the solver during search. Every parameter type has a corresponding variable type; the variable type being distinguished by a `var` keyword.
- **Annotations** and **strings**: annotations can appear on variable declarations, constraints, and on the solve goal. They provide information about how a variable or constraint should be treated by the solver (e.g., whether a variable should be output as part of the result or whether a particular constraint should implemented using domain consistency). Strings may appear as arguments to annotations, but nowhere else.

**Parameter types**

Parameters are fixed quantities explicitly specified in the model (see rule `<par-type>` in Section 4.3.6).

<table>
<thead>
<tr>
<th>Type</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>bool</td>
<td>true or false</td>
</tr>
<tr>
<td>float</td>
<td>float</td>
</tr>
<tr>
<td>int</td>
<td>int</td>
</tr>
<tr>
<td>set of int</td>
<td>subset of int</td>
</tr>
<tr>
<td>array [1.. n] of bool</td>
<td>array of bools</td>
</tr>
<tr>
<td>array [1.. n] of float</td>
<td>array of floats</td>
</tr>
<tr>
<td>array [1.. n] of int</td>
<td>array of ints</td>
</tr>
<tr>
<td>array [1.. n] of set of int</td>
<td>array of sets of ints</td>
</tr>
</tbody>
</table>

A parameter may be used where a variable is expected, but not vice versa.

In predicate declarations the following additional parameter types are allowed.
<table>
<thead>
<tr>
<th>Type</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_a \ldots r_b$</td>
<td>bounded float</td>
</tr>
<tr>
<td>$x_a \ldots x_b$</td>
<td>int in range</td>
</tr>
<tr>
<td>${ x_a, x_b, \ldots , x_k }$</td>
<td>int in set</td>
</tr>
<tr>
<td>set of $x_a \ldots x_b$</td>
<td>subset of int range</td>
</tr>
<tr>
<td>set of ${ x_a, x_b, \ldots , x_k }$</td>
<td>subset of int range</td>
</tr>
<tr>
<td>array $[1..n]$ of $r_a \ldots r_b$</td>
<td>array of floats in range</td>
</tr>
<tr>
<td>array $[1..n]$ of $x_a \ldots x_b$</td>
<td>array of ints in range</td>
</tr>
<tr>
<td>array $[1..n]$ of set of $x_a \ldots x_b$</td>
<td>array of sets of ints in range</td>
</tr>
<tr>
<td>array $[1..n]$ of set of ${ x_a, x_b, \ldots , x_k }$</td>
<td>array of subsets of set</td>
</tr>
</tbody>
</table>

A range $x_a \ldots x_b$ denotes a closed interval $\{ x | x_a \leq x \leq x_b \}$ (same for float ranges).

An array type appearing in a predicate declaration may use just int instead of $1..n$ for the array index range in cases where the array argument can be of any length.

**Variable types**

Variables are quantities decided by the solver (see rules `<basic-var-type>` and `<array-var-type>` in Section 4.3.6).
In predicate declarations the following additional variable types are allowed.

<table>
<thead>
<tr>
<th>Variable type</th>
</tr>
</thead>
<tbody>
<tr>
<td>var set of int</td>
</tr>
<tr>
<td>array [1.. n] of var set of int</td>
</tr>
</tbody>
</table>

An array type appearing in a predicate declaration may use just `int` instead of `1.. n` for the array index range in cases where the array argument can be of any length.

**The string type**

String literals and literal arrays of string literals can appear as annotation arguments, but not elsewhere. Strings have the same syntax as in C programs (namely, they are delimited by double quotes and the backslash character is used for escape sequences).

Examples:

```
""
% The empty string.

"Hello."

"Hello, \n World\t"quoted!"
% A string with an embedded newline, tab and _
  quotes.
```

### 4.3.1.3 Values and expressions

(See rule `<expr>` in Section 4.3.6)

Examples of literal values:

Type Literals bool true, false float 2.718, -1.0, 3.0e8 int -42, 0, 69 set of int {}, {2, 3, 5}, 1..10 arrays [], [ yₐ, ..., yₖ ]

where each array element `yᵢ` is either: a non-array literal, or the name of a non-array parameter or variable, `v`. For example:

```
[1, 2, 3]          % Just literals
[x, y, z]          % x, y, and z are variables or parameters.
[x, 3]             % Mix of identifiers and literals
```

Section 4.3.6 gives the regular expressions specifying the syntax for float and int literals.
4.3.1.4 FlatZinc models

A FlatZinc model consists of:

1. zero or more external predicate declarations (i.e., a non-standard predicate that is supported directly by the target solver);
2. zero or more parameter declarations;
3. zero or more variable declarations;
4. zero or more constraints;
5. a solve goal

in that order.

FlatZinc uses the UTF-8 character set. Non-ASCII characters can only appear in string literals.

FlatZinc syntax is case sensitive (foo and Foo are different names). Identifiers start with a letter ([A-Za-z]) and are followed by any sequence of letters, digits, or underscores ([A-Za-z0-9_]). Additionally, identifiers of variable or parameter names may start with an underscore. Identifiers that correspond to the names of predicates, predicate parameters and annotations cannot have leading underscores.

The following keywords are reserved and cannot be used as identifiers: annotation, any, array, bool, case, constraint, diff, div, else, elseif, endif, enum, false, float, function, if, in, include, int, intersect, let, list, maximize, minimize, mod, not, of, satisfy, subset, superset, output, par, predicate, record, set, solve, string, symdiff, test, then, true, tuple, union, type, var, where, xor. Note that some of these keywords are not used in FlatZinc. They are reserved because they are keywords in Zinc and MiniZinc.

FlatZinc syntax is insensitive to whitespace.

4.3.1.5 Predicate declarations

(See rule <predicate-item> in Section 4.3.6)

Predicates used in the model that are not standard FlatZinc must be declared at the top of a FlatZinc model, before any other lexical items. Predicate declarations take the form

```
<predicate-item> ::= "predicate" <identifier> "(" [ <pred-param-type> :<identifier> "," <identifier> "," ... ] ")" ";"
```
Annotations are not permitted anywhere in predicate declarations.

It is illegal to supply more than one predicate declaration for a given <identifier>.

Examples:

```mini
% m is the median value of {x, y, z}.
% predicate median_of_3(var int: x, var int: y, var int: z, var int: m);

% all_different([x1, .., xn]) iff
% for all i, j in 1..n: xi != xj.
% predicate all_different(array [int] of var int: xs);

% exactly_one([x1, .., xn]) iff
% there exists an i in 1..n: xi = true
% and for all j in 1..n: j != i -> xj = false.
% predicate exactly_one(array [int] of var bool: xs);
```

4.3.1.6 Parameter declarations

(See rule param_decl in Section 4.3.6)

Parameters have fixed values and must be assigned values:

```mini
<par-decl-item> ::= <par-type> "::" <var-par-identifier> "=" <par-expr> ";"
```

where <par-type> is a parameter type, <var-par-identifier> is an identifier, and <par-expr> is a literal value (either a basic integer, float or bool literal, or a set or array of such literals).

Annotations are not permitted anywhere in parameter declarations.

Examples:

```mini
float: pi = 3.141;
array [1..7] of int: fib = [1, 1, 2, 3, 5, 8, 13];
bool: beer_is_good = true;
```
4.3.1.7 Variable declarations

(See rule \texttt{var\_decl} in Section 4.3.6)

Variables have variable types and can be declared with optional assignments. The assignment can fix a variable to a literal value, or create an alias to another variable. Arrays of variables always have an assignment, defining them in terms of an array literal that can contain identifiers of variables or constant literals. Variables may be declared with zero or more annotations.

\[
<\text{var-decl-item}> ::= <\text{basic-var-type}> "::" <\text{var-par-identifier}> <\text{annotations}> \left[ "=" <\text{basic-expr}> \right] ";"
\]

where \texttt{<basic-var-type>} and \texttt{<array-var-type>} are variable types, \texttt{<var-par-identifier>} is an identifier, \texttt{<annotations>} is a (possibly empty) set of annotations, \texttt{<basic-expr>} is an identifier or a literal, and \texttt{<array-literal>} is a literal array value.

Examples:

\begin{verbatim}
var 0..9: digit;
var bool: b;
var set of 1..3: s;
var 0.0..1.0: x;
var int: y :: mip; \% 'mip' annotation: y should be a MIP variable.
array [1..3] of var 1..10: b = [y, 3, digit];
\end{verbatim}

4.3.1.8 Constraints

(See rule \texttt{constraint-item} in Section 4.3.6)

Constraints take the following form and may include zero or more annotations:

\[
<\text{constraint-item}> ::= "constraint" <\text{identifier}> "(" [ <\text{expr}> "," \ldots ] ")" <\text{annotations}> ";"
\]

The arguments expressions (\texttt{<expr>}) can be literal values or identifiers.

Examples:
constraint int_le(0, x);  % 0 <= x
constraint int_lt(x, y);  % x < y
constraint int_le(y, 10);  % y <= 10
  % 'domain': use domain consistency for this constraint:
  % 2x + 3y = 10
constraint int_lin_eq([2, 3], [x, y], 10) :: domain;

4.3.1.9 Solve item

(See rule <solve-item> in Section 4.3.6)

A model finishes with a solve item, taking one of the following forms:

<solve-item> ::= "solve" <annotations> "satisfy" "$";"
  | "solve" <annotations> "minimize" <basic-expr> "$";"
  | "solve" <annotations> "maximize" <basic-expr> "$";"

The first alternative searches for any satisfying assignment, the second one searches for an assignment minimizing the given expression, and the third one for an assignment maximizing the expression. The <basic-expr> can be either a variable identifier or a literal value (if the objective function is constant).

A solution consists of a complete assignment where all variables in the model have been given a fixed value.

Examples:

solve satisfy;  % Find any solution using the default strategy.

solve minimize w;  % Find a solution minimizing w, using the default strategy.

  % First label the variables in xs in the order x[1], x[2], ...
  % trying values in ascending order.
solve ::: int_search(xs, input_order, indomain_min, complete)
satisfy;  % Find any solution.

  % First use first-fail on these variables, splitting domains
% at each choice point.
solve :: int_search([x, y, z], first_fail, indomain_split, complete)
  maximize x; % Find a solution maximizing x.

4.3.1.10 Annotations

Annotations are optional suggestions to the solver concerning how individual variables and constraints should be handled (e.g., a particular solver may have multiple representations for int variables) and how search should proceed. An implementation is free to ignore any annotations it does not recognise, although it should print a warning on the standard error stream if it does so. Annotations are unordered and idempotent: annotations can be reordered and duplicates can be removed without changing the meaning of the annotations.

An annotation is prefixed by ::, and either just an identifier or an expression that looks like a predicate call:

\[
\begin{align*}
\text{<annotations>} &::= ["::" \text{<annotation> }]^
\text{<annotation>} &::= \text{<identifier>}
  \quad | \quad \text{<identifier>} "(" \text{<ann-expr> "," ... ")"
\text{<ann-expr>} &::= \text{<expr>}
  \quad | \quad \text{<annotation>}
\end{align*}
\]

The arguments of the second alternative can be any expression or other annotations (without the leading ::).

Search annotations

While an implementation is free to ignore any or all annotations in a model, it is recommended that implementations at least recognise the following standard annotations for solve items.

\[
\text{seq_search}([<searchannotation>, ...])
\]

allows more than one search annotation to be specified in a particular order (otherwise annotations can be handled in any order).

A <searchannotation> is one of the following:
```plaintext
int_search(<vars>, <varchoiceannotation>, <assignmentannotation>, ...
<-<strategyannotation>)

bool_search(<vars>, <varchoiceannotation>, <assignmentannotation>, ...
<-<strategyannotation>)

set_search(<vars>, <varchoiceannotation>, <assignmentannotation>, ...
<-<strategyannotation>)
```

where `<vars>` is the identifier of an array variable or an array literal specifying the variables to be assigned (ints, bools, or sets respectively). Note that these arrays may contain literal values.

`<varchoiceannotation>` specifies how the next variable to be assigned is chosen at each choice point. Possible choices are as follows (it is recommended that implementations support the starred options):

<table>
<thead>
<tr>
<th>Choice</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>input_order</td>
<td>Choose variables in the order they appear in <code>vars</code>.</td>
</tr>
<tr>
<td>first_fail</td>
<td>Choose the variable with the smallest domain.</td>
</tr>
<tr>
<td>anti_first_fail</td>
<td>Choose the variable with the largest domain.</td>
</tr>
<tr>
<td>smallest</td>
<td>Choose the variable with the smallest value in its domain.</td>
</tr>
<tr>
<td>largest</td>
<td>Choose the variable with the largest value in its domain.</td>
</tr>
<tr>
<td>occurrence</td>
<td>Choose the variable with the largest number of attached constraints.</td>
</tr>
<tr>
<td>most_constrained</td>
<td>Choose the variable with the smallest domain, breaking ties using the number of constraints.</td>
</tr>
<tr>
<td>max_regret</td>
<td>Choose the variable with the largest difference between the two smallest values in its domain.</td>
</tr>
<tr>
<td>dom_w_deg</td>
<td>Choose the variable with the smallest value of domain size divided by weighted degree, where the weighted degree is the number of times the variables been in a constraint which failed.</td>
</tr>
</tbody>
</table>

`<assignmentannotation>` specifies how the chosen variable should be constrained. Possible choices are as follows (it is recommended that implementations support at least the starred options):
### MiniZinc Handbook, Release 2.4.3

<table>
<thead>
<tr>
<th>Assignment Strategy</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>indomain_min</code></td>
<td>Assign the smallest value in the variable’s domain.</td>
</tr>
<tr>
<td><code>indomain_max</code></td>
<td>Assign the largest value in the variable’s domain.</td>
</tr>
<tr>
<td><code>indomain_middle</code></td>
<td>Assign the value in the variable’s domain closest to the mean of its current bounds.</td>
</tr>
<tr>
<td><code>indomain_median</code></td>
<td>Assign the middle value in the variable’s domain.</td>
</tr>
<tr>
<td><code>indomain_random</code></td>
<td>Nondeterministically assign values to the variable in ascending order.</td>
</tr>
<tr>
<td><code>indomain_split</code></td>
<td>Bisect the variable’s domain, excluding the upper half first.</td>
</tr>
<tr>
<td><code>indomain_reverse_split</code></td>
<td>Bisect the variable’s domain, excluding the lower half first.</td>
</tr>
<tr>
<td><code>indomain_interval</code></td>
<td>If the variable’s domain consists of several contiguous intervals, reduce the domain to the first interval. Otherwise just split the variable’s domain.</td>
</tr>
</tbody>
</table>

Of course, not all assignment strategies make sense for all search annotations (e.g., `bool_search` and `indomain_split`).

Finally, `<strategyannotation>` specifies a search strategy; implementations should at least support complete (i.e., exhaustive search).

#### Output annotations

Model output is specified through variable annotations. Non-array output variables are annotated with `output_var`. Array output variables are annotated with `output_array([x_1 .. x_2, ...])` where `x_1 .. x_2, ...` are the index set ranges of the original MiniZinc array (which may have had multiple dimensions and/or index sets that do not start at 1). See Section 4.3.2 for details on the output format.

#### Variable definition annotations

To support solvers capable of exploiting functional relationships, a variable defined as a function of other variables may be annotated thus:

```plaintext
var int: x :: is_defined_var;

...

constraint int_plus(y, z, x) :: defines_var(x);
```
Intermediate variables

Intermediate variables introduced during conversion of a MiniZinc model to FlatZinc may be annotated thus:

```
var int: X_INTRODUCED_3 :: var_is_introduced;
```

This information is potentially useful to the solver’s search strategy.

Constraint annotations

Annotations can be placed on constraints advising the solver how the constraint should be implemented. Here are some constraint annotations supported by some solvers:

<table>
<thead>
<tr>
<th>Annotation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>boundsZ</td>
<td>Use integer bounds propagation.</td>
</tr>
<tr>
<td>boundsR</td>
<td>Use real bounds propagation.</td>
</tr>
<tr>
<td>boundsD</td>
<td>A tighter version of boundsZ where support for the bounds must exist.</td>
</tr>
<tr>
<td>domain</td>
<td>Use domain propagation.</td>
</tr>
<tr>
<td>priority(k)</td>
<td>where k is an integer constant indicating propagator priority.</td>
</tr>
</tbody>
</table>

4.3.2 Output

An implementation can produce three types of output: solutions, statistics, and errors.

4.3.2.1 Solution output

An implementation must output values for all and only the variables annotated with output_var or output_array (output annotations must not appear on parameters). Output must be printed to the standard output stream.

For example:
All non-error output must be sent to the standard output stream.

Output must take the following form:

\[
<\text{var-par-identifier}> = <\text{basic-literal-expr}>; \\
\]

or, for array variables,

\[
<\text{var-par-identifier}> = \text{array}<\text{N}>d(<a>..<b>, ..., [<y1>, <y2>, ... <yk>]); \\
\]

where \(<\text{N}>\) is the number of index sets specified in the corresponding \text{output_array} annotation, \(<a>..<b>, ...\) are the index set ranges, and \(<y1>, <y2>, ... <yk>\) are literals of the element type.

Using this format, the output of a FlatZinc model solution is suitable for input to a MiniZinc model as a data file (this is why parameters are not included in the output).

Implementations must ensure that all model variables (not just the output variables) have satisfying assignments before printing a solution.

The output for a solution must be terminated with ten consecutive minus signs on a separate line: "----------".

Multiple solutions may be output, one after the other, as search proceeds. How many solutions should be output depends on the mode the solver is run in as controlled by the \(-a\) command line flag (see Section 4.3.4).

If at least one solution has been found and search then terminates having explored the whole search space, then ten consecutive equals signs should be printed on a separate line: "=========".

If no solutions have been found and search terminates having explored the whole search space, then "====UNSATISFIABLE====" should be printed on a separate line.

If the objective of an optimization problem is unbounded, then "====UNBOUNDED====" should be printed on a separate line.

If no solutions have been found and search terminates having not explored the whole search space, then "====UNKNOWN====" should be printed on a separate line.
Implementations may output further information about the solution(s), or lack thereof, in the form of FlatZinc comments.

Examples:

Asking for a single solution to this model:

```mini
var 1..3: x :: output_var;
solve satisfy
```

might produce this output:

```mini
x = 1;
```

Asking for all solutions to this model:

```mini
array [1..2] of var 1..3: xs :: output_array([1..2]);
constraint int_lt(xs[1], xs[2]); % x[1] < x[2].
solve satisfy
```

might produce this output:

```
xs = array1d(1..2, [1, 2]);
xs = array1d(1..2, [1, 3]);
xs = array1d(1..2, [2, 3]);
```

Asking for a single solution to this model:

```mini
var 1..10: x :: output_var;
solve maximize x;
```

should produce this output:
The row of equals signs indicates that a complete search was performed and that the last result printed is the optimal solution.

Running a solver on this model with some termination condition (such as a very short time-out):

```
var 1..10: x :: output_var;
solve maximize x;
```

might produce this output:

```
  x = 1;
  --------
  x = 2;
  --------
  x = 3;
  --------
```

Because the output does not finish with =========, search did not finish, hence these results must be interpreted as approximate solutions to the optimization problem.

Asking for a solution to this model:

```
var 1..3: x :: output_var;
var 4..6: y :: output_var;
constraint int_lt(y, x);  % y < x.
solve satisfy;
```

should produce this output:

```
=======UNSATISFIABLE=======
```

indicating that a complete search was performed and no solutions were found (i.e., the problem is unsatisfiable).
4.3.2.2 Statistics output

FlatZinc solvers can output statistics in a standard format so that it can be read by scripts, for example, in order to run experiments and automatically aggregate the results. Statistics should be printed to the standard output stream in the form of FlatZinc comments that follow a specific format. Statistics can be output at any time during the solving, i.e., before the first solution, between solutions, and after the search has finished. Statistics output corresponding to a solution should be the last one before its separator.

Each value should be output on a line of its own in the following format:

```
%%%mzn-stat: <name>=<value>
```

Each block of statistics is terminated by a line of its own with the following format:

```
%%%mzn-stat-end
```

Example

```
%%%mzn-stat: objective=1e+308
%%%mzn-stat: objectiveBound=0
%%%mzn-stat: nodes=0
%%%mzn-stat: solveTime=2.3567
%%%mzn-stat-end

(no feasible solution found yet but something can be printed...)

%%%mzn-stat: objective=12345
%%%mzn-stat: objectiveBound=122
%%%mzn-stat: nodes=35
%%%mzn-stat: solveTime=78.5799
%%%mzn-stat-end

(the corresponding feasible solution with value 12345 goes here or before its statistics but above the separator)

--------

%%%mzn-stat: objective=379
%%%mzn-stat: objectiveBound=379
```

The `<name>` describes the kind of statistics gathered, and the `<value>` can be any value of a MiniZinc type. The following names are considered standard statistics:

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>nodes</td>
<td>int</td>
<td>Number of search nodes</td>
</tr>
<tr>
<td>openNodes</td>
<td>int</td>
<td>Number of open search nodes</td>
</tr>
<tr>
<td>objective</td>
<td>float</td>
<td>Current objective value</td>
</tr>
<tr>
<td>objectiveBound</td>
<td>float</td>
<td>Dual bound on the objective value</td>
</tr>
<tr>
<td>failures</td>
<td>int</td>
<td>Number of leaf nodes that were failed</td>
</tr>
<tr>
<td>restarts</td>
<td>int</td>
<td>Number of times the solver restarted the search</td>
</tr>
<tr>
<td>variables</td>
<td>int</td>
<td>Number of variables</td>
</tr>
<tr>
<td>intVariables</td>
<td>int</td>
<td>Number of integer variables created</td>
</tr>
<tr>
<td>boolVariables</td>
<td>int</td>
<td>Number of bool variables created</td>
</tr>
<tr>
<td>floatVariables</td>
<td>int</td>
<td>Number of float variables created</td>
</tr>
<tr>
<td>setVariables</td>
<td>int</td>
<td>Number of set variables created</td>
</tr>
<tr>
<td>propagators</td>
<td>int</td>
<td>Number of propagators created</td>
</tr>
<tr>
<td>propagations</td>
<td>int</td>
<td>Number of propagator invocations</td>
</tr>
<tr>
<td>peakDepth</td>
<td>int</td>
<td>Peak depth of search tree</td>
</tr>
<tr>
<td>nogoods</td>
<td>int</td>
<td>Number of nogoods created</td>
</tr>
<tr>
<td>backjumps</td>
<td>int</td>
<td>Number of backjumps</td>
</tr>
<tr>
<td>peakMem</td>
<td>float</td>
<td>Peak memory (in Mbytes)</td>
</tr>
<tr>
<td>initTime</td>
<td>float</td>
<td>Initialisation time (in seconds)</td>
</tr>
<tr>
<td>solveTime</td>
<td>float</td>
<td>Solving time (in seconds)</td>
</tr>
</tbody>
</table>
4.3.2.3 Error and warning output

Errors and warnings must be output to the standard error stream. When an error occurs, the implementation should exit with a non-zero exit code, signaling failure.

4.3.3 Solver-specific Libraries

Constraints in FlatZinc can call standard predicates as well as solver-specific predicates. Standard predicates are the ones that the MiniZinc compiler assumes to be present in all solvers. Without further customisation, the compiler will try to compile the entire model into a set of these standard predicates.

Solvers can use custom predicates and redefine standard predicates by supplying a solver specific library of predicate declarations. Examples of such libraries can be found in the binary distribution of MiniZinc, inside the share/minizinc/gecode and share/minizinc/chuffed directories.

The solver-specific library needs to be made available to the MiniZinc compiler by specifying its location in the solver’s configuration file, see Section 4.3.5.

4.3.3.1 Standard predicates

FlatZinc solvers need to support the predicates listed as FlatZinc builtins in the library reference documentation, see Section 4.2.5.

Any standard predicate that is not supported by a solver needs to be redefined. This can be achieved by placing a file called redefinitions.mzn in the solver’s MiniZinc library, which can contain alternative definitions of predicates, or define them as unsupported using the abort predicate.

Example for a redefinitions.mzn:

```plaintext
% Redefine float_sinh function in terms of exp
predicate float_sinh(var float: a, var float: b) =
    b == (exp(a)-exp(-a))/2.0;

% Mark float_tanh as unsupported
predicate float_tanh(var float: a, var float: b) =
    abort("The builtin float_tanh is not supported by this solver.");
```
The redefinition can use the full MiniZinc language. Note, however, that redefining built-in predicates in terms of MiniZinc expressions can lead to problems if the MiniZinc compiler translates the high-level expression back to the redefined builtin.

The reference documentation (Section 4.2.5) also contains sections on builtins that were added in later versions of MiniZinc. In order to maintain backwards compatibility with solvers that don’t support these, they are organised in redefinition files with a version number attached, such as redefinitions-2.0.mzn. In order to declare support for these builtins, the solver-specific library must contain the corresponding redefinitions file, with the predicates either redefined in terms of other predicates, or declared as supported natively by the solver by providing a predicate declaration without a body.

Example for a redefinitions-2.0.mzn that declares native support for the predicates added in MiniZinc 2.0:

```plaintext
predicate bool_clause_reif(array[int] of var bool: as, 
array[int] of var bool: bs, 
var bool: b);
predicate array_int_maximum(var int: m, array[int] of var int: x);
predicate array_float_maximum(var float: m, array[int] of var float: x);
predicate array_int_minimum(var int: m, array[int] of var int: x);
predicate array_float_minimum(var float: m, array[int] of var float: x);
```

### 4.3.3.2 Solver-specific predicates

Many solvers have built-in support for some of the constraints in the MiniZinc standard library. But without declaring which constraints they support, MiniZinc will assume that they don’t support any except for the standard FlatZinc builtins mentioned in the section above.

A solver can declare that it supports a non-standard constraint by overriding one of the files of the standard library in its own solver-specific library. For example, assume that a solver supports the all_different constraint on integer variables. In the standard library, this constraint is defined in the file fzn_all_different_int.mzn, with the following implementation:

```plaintext
predicate fzn_all_different_int(array[int] of var int: x) = 
forall(i,j in index_set(x) where i < j) ( x[i] != x[j] );
```

A solver, let’s call it OptiSolve, that supports this constraint natively can place a file with the same name, fzn_all_different_int.mzn, in its library, and redefine it as follows:
When a MiniZinc model that contains the all_different constraint is now compiled with the OptiSolve library, the generated FlatZinc will contain calls to the newly defined predicate optisolve_alldifferent.

Note: The solver-specific library has been reorganised for MiniZinc version 2.3.0. Previously, a solver library would contain e.g. the file bin_packing.mzn in order to override the bin_packing constraint. With version 2.3.0, this is still possible (in order to maintain backwards compatibility). However, the predicate bin_packing from file bin_packing.mzn now delegates to the predicate fzn_bin_packing in fzn_bin_packing.mzn. This enables the bin_packing predicate to check that the arguments are correct using assertions, before delegating to the solver-specific predicate. If your solver still uses the old library layout (i.e., overriding bin_packing.mzn instead of fzn_bin_packing.mzn), you should consider updating it to the new standard.

### 4.3.3.3 Reified and half-reified predicates

A reified constraint is a constraint that is not simply enforced, but whose truth value is bound to a Boolean variable. For example, a MiniZinc expression var bool: b = all_different(x); would constrain b to be true if and only if the variables x take pairwise different values.

If a predicate is called in such a reified context, the MiniZinc compiler will try to find a version of the predicate with _reif added to its identifier and an additional var bool argument. For the above example, the compiler will try to generate the following FlatZinc code:

```plaintext
var bool: b;
constraint all_different_reif(x, b);
```

If the _reif predicate does not exist, the compiler will try to use the definition of the original predicate. However, this may not be ideal: the original definition may make use of free variables in a let expression (which is not allowed in reified contexts), or it may lead to inefficient solving.

Solver libraries should therefore provide reified versions of constraints whenever possible. The library contains files fzn_<constraintname>_reif.mzn for this purpose.

When a reified constraint is used in a positive context (see pred-context), the MiniZinc compiler can use a special version, called a half-reified predicate and identified by an _imp suffix,
instead of the _reif predicate. Half-reified predicates essentially represent constraints that are implied by a Boolean variable rather than being equivalent to one. This typically leads to simpler translations or more efficient propagation (e.g., a half-reified all_different only needs to check whether it is false, but it never has to implement the negation of the actual constraint).

For example, constraint \( y = 0 \) \( \lor \) all_different(\( x \)) might be translated as follows:

```plaintext
var bool: X_INTRODUCED_1;
var bool: X_INTRODUCED_2;
constraint int_eq_imp(y, 0, X_INTRODUCED_1);
constraint all_different_imp(x, X_INTRODUCED_2);
constraint array_bool_or([X_INTRODUCED_1, X_INTRODUCED_2]);
```

MiniZinc will decide whether to use half-reification case by case based on the availability of the _imp predicate. As for reified constraints, it may be beneficial to provide specialised half-reified versions if the solver supports them.

### 4.3.4 Command-Line Interface and Standard Options

In order to work with the minizinc command line driver, a FlatZinc solver must be an executable (which can include e.g. shell scripts) that can be invoked as follows:

```
$ <executable-name> [options] model.fzn
```

where `<executable-name>` is the name of the executable. Solvers may support the following standard options:

- `-a`
  
  Instructs the solver to report all solutions in the case of satisfaction problems, or print intermediate solutions of increasing quality in the case of optimisation problems.

- `-n <i>`
  
  Instructs the solver to stop after reporting \( i \) solutions (only used with satisfaction problems).

- `-f`
  
  Instructs the solver to conduct a “free search”, i.e., ignore any search annotations. The solver is not required to ignore the annotations, but it is allowed to do so.

- `-s`
  
  Print statistics during and/or after the search for solutions. Statistics should be printed as
FlatZinc comments to the standard output stream. See below for a standard format for statistics.

-v
Print log messages (verbose solving) to the standard error stream. If solvers choose to print to standard output instead, all messages must be valid comments (i.e., start with a % character).

-p <i>
Run with i parallel threads (for multi-threaded solvers).

-r <i>
Use i as the random seed (for any random number generators the solver may be using).

-t <ms>
Wall time limit ms milliseconds.

4.3.5 Solver Configuration Files

In order for a solver to be available to MiniZinc, it has to be described in a solver configuration file. This is a simple file, in JSON or .dzn format, that contains some basic information such as the solver’s name, version, where its library of global constraints can be found, and a path to its executable. Examples are given in section Solver Backends in User Manual.

A solver configuration file must have file extension .msc (for MiniZinc Solver Configuration), and can be placed in any of the following locations:

- In the minizinc/solvers/ directory of the MiniZinc installation. If you install MiniZinc from the binary distribution, this directory can be found at /usr/share/minizinc/solvers on Linux systems, inside the MiniZincIDE application on macOS system, and in the Program Files\MiniZinc IDE (bundled) folder on Windows.

- In the directory $HOME/.minizinc/solvers on Linux and macOS systems, and the Application Data directory on Windows systems.

- In any directory listed on the MZN_SOLVER_PATH environment variable (directories are separated by : on Linux and macOS, and by ; on Windows systems).

- In any directory listed in the mzn_solver_path option of the global or user-specific configuration file (see Section 3.1.4)

- Alternatively, you can use the MiniZinc IDE to create solver configuration files, see Section 3.2.5.2 for details.
Solver configuration files must be valid JSON or .dzn files. As a JSON file, it must be an object with certain fields. As a .dzn file, it must consist of assignment items.

For example, a simple solver configuration in JSON format could look like this:

```json
{
  "name": "My Solver",
  "version": "1.0",
  "id": "org.myorg.my_solver",
  "executable": "fzn-mysolver"
}
```

The same configuration in .dzn format would look like this:

```dzn
name = "My Solver";
version = "1.0";
id = "org.myorg.my_solver";
executable = "fzn-mysolver";
```

Here is a list of all configuration options recognised by the configuration file parser. Any valid configuration file must at least contain the fields name, version, id, and executable.

- **name** (string, required): The name of the solver (displayed, together with the version, when you call minizinc --solvers, and in the MiniZinc IDE).

- **version** (string, required): The version of the solver.

- **id** (string, required): A unique identifier for the solver, “reverse domain name” notation.

- **executable** (string, required): The executable for this solver that can run FlatZinc files. This can be just a file name (in which case the solver has to be on the current PATH), or an absolute path to the executable, or a relative path (which is interpreted relative to the location of the configuration file).

- **mznlib** (string, default ":"): The solver-specific library of global constraints and redefinitions. This should be the name of a directory (either an absolute path or a relative path, interpreted relative to the location of the configuration file). For solvers whose libraries are installed in the same location as the MiniZinc standard library, this can also take the form -G<solverlib>, e.g., -Ggecode (this is mostly the case for solvers that ship with the MiniZinc binary distribution).

- **tags** (list of strings, default empty): Each solver can have one or more tags that describe its features in an abstract way. Tags can be used for selecting a solver using the --solver
option. There is no fixed list of tags, however we recommend using the following tags if they match the solver’s behaviour:

- "cp": for Constraint Programming solvers
- "mip": for Mixed Integer Programming solvers
- "float": for solvers that support float variables
- "api": for solvers that use the internal C++ API

• stdFlags (list of strings, default empty): Which of the standard solver command line flags are supported by this solver. The standard flags are -a, -n, -s, -v, -p, -r, -f, -t.

• extraFlags (list of list of strings, default empty): Extra command line flags supported by the solver. Each entry must be a list of four strings. The first string is the name of the option (e.g. "--special-algorithm"). The second string is a description that can be used to generate help output (e.g. "which special algorithm to use"). The third string specifies the type of the argument ("int", "bool", "float", "string" or "opt"). The fourth string is the default value. The following types have an additional extended syntax:
  
  - "int:n:m" where n and m are integers, gives lower and upper bounds for the supported values
  - "float:n:m" where n and m are floating point numbers, gives lower and upper bounds for the supported values
  - "bool:onstring:offstring" specifies strings to add to the command line flag to turn it on (onstring) and off (offstring). E.g., ["-interrupt","whether to catch Ctrl-C","bool:false:true","true"] specifies a command line option that can be called as -interrupt true or -interrupt false. The standard behaviour (just "bool") means that the option is either added to the command line or not.
  - "opt:first option:second option:....:last option" specifies a list of possible values for the option

• supportsMzn (bool, default false): Whether the solver can run MiniZinc directly (i.e., it implements its own compilation or interpretation of the model).

• supportsFzn (bool, default true): Whether the solver can run FlatZinc. This should be the case for most solvers

• needsSolns2Out (bool, default true): Whether the output of the solver needs to be passed through the MiniZinc output processor.

• needsMznExecutable (bool, default false): Whether the solver needs to know the location
of the MiniZinc executable. If true, it will be passed to the solver using the `mzn-executable` option.

- `needsStdlibDir` (bool, default false): Whether the solver needs to know the location of the MiniZinc standard library directory. If true, it will be passed to the solver using the `stdlib-dir` option.

- `isGUIApplication` (bool, default false): Whether the solver has its own graphical user interface, which means that MiniZinc will detach from the process and not wait for it to finish or to produce any output.

### 4.3.6 Grammar

This is the full grammar for FlatZinc. It is a proper subset of the MiniZinc grammar (see Section 4.1.14). However, instead of specifying all the cases in the MiniZinc grammar that do not apply to FlatZinc, the BNF syntax below contains only the relevant syntactic constructs. It uses the same notation as in Section 4.1.2.

```plaintext
% A FlatZinc model
<model> ::= [ <predicate-item> ]*
[ <par-decl-item> ]*
[ <var-decl-item> ]*
[ <constraint-item> ]*
<solve-item>

% Predicate items
<predicate-item> ::= "predicate" <identifier> "(" [ <pred-param-type> :␣ários]
<identifier> "," ... ")" ";" 

% Identifiers
<identifier> ::= [A-Za-z][A-Za-z0-9_]*

<basic-par-type> ::= "bool"
| "int"
| "float"
| "set of int"

<par-type> ::= <basic-par-type>
```
### Chapter 4.3. Interfacing Solvers to FlatZinc

| array | [ <index-set> ] | of | <basic-par-type> |

<basic-par-type> ::= "var" "bool"
| "var" "int"
| "var" <int-literal> "." <int-literal>
| "var" "{" <int-literal> "," ... "}" |
| "var" "float"
| "var" <float-literal> "." <float-literal>
| "var" "set" "of" <int-literal> "." <int-literal>
| "var" "set" "of" "{ [ <int-literal> "," ... ] "}" |

<array-var-type> ::= "array" [ <index-set> ] "of" <basic-var-type> |

<index-set> ::= "1" "." <int-literal> |

<basic-pred-param-type> ::= <basic-par-type>
| <basic-var-type>
| <int-literal> "." <int-literal>
| <float-literal> "." <float-literal>
| "{" <int-literal> "," ... "}" |
| "set" "of" <int-literal> .. <int-literal>
| "set" "of" "{ [ <int-literal> "," ... ] "}" |
| "var" "set" "of" "int" |

<pred-param-type> ::= <basic-pred-param-type>
| "array" [ <pred-index-set> ] "of" .. <basic-pred-param-type>

<pred-index-set> ::= <index-set>
| "int"

<basic-literal-expr> ::= <bool-literal>
| <int-literal>
| <float-literal>
| <set-literal>

<basic-expr> ::= <basic-literal-expr>
| <var-par-identifier>
<expr> ::= <basic-expr> 
           | <array-literal>

<par-expr> ::= <basic-literal-expr> 
              | <par-array-literal>

<var-par-identifier> ::= [A-Za-z][A-Za-z0-9_]*

% Boolean literals
<bool-literal> ::= "false" 
               | "true"

% Integer literals
<int-literal> ::= [0-9]+ 
                | 0x[0-9A-Fa-f]+ 
                | 0o[0-7]+

% Float literals
<float-literal> ::= [0-9]+.[0-9]+ 
                 | [0-9]+.[0-9]+[Ee][-+]?[0-9]+ 
                 | [0-9]+[Ee][-+]?[0-9]+

% Set literals
<set-literal> ::= "{" [ <int-literal> "," ... ] "}" 
               | <int-literal> "." <int-literal> 
               | "{" [ <float-literal> "," ... ] "}" 
               | <float-literal> "." <float-literal>

$array-literal$ ::= "[" [ <basic-expr> "," ... ] "]"

<par-array-literal> ::= "[" [ <basic-literal-expr> "," ... ] "]"

% Parameter declarations
<par-decl-item> ::= <par-type> ":" <var-par-identifier> "=" <par-expr> ";

% Variable declarations
<var-decl-item> ::= <basic-var-type> "::= <var-par-identifier> <annotations>
| "=" <basic-expr> "]" ;
| <array-var-type> "::= <var-par-identifier> <annotations>
| "=" <array-literal> "]" ;

% Constraint items

<constraint-item> ::= "constraint" <identifier> "(" <expr> "," ... ")" ";"

% Solve item

<solve-item> ::= "solve" <annotations> "satisfy" ";" 
| "solve" <annotations> "minimize" <basic-expr> ";" 
| "solve" <annotations> "maximize" <basic-expr> ";"

% Annotations

<annotations> ::= [ ":=" <annotation> ]*

<annotation> ::= <identifier> 
| <identifier> "(" <ann-expr> "," ... ")"

<ann-expr> ::= <expr> 
| <annotation>

% End of FlatZinc grammar
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